MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
DEPARTMENT OF NUCLEAR ENGINEERING

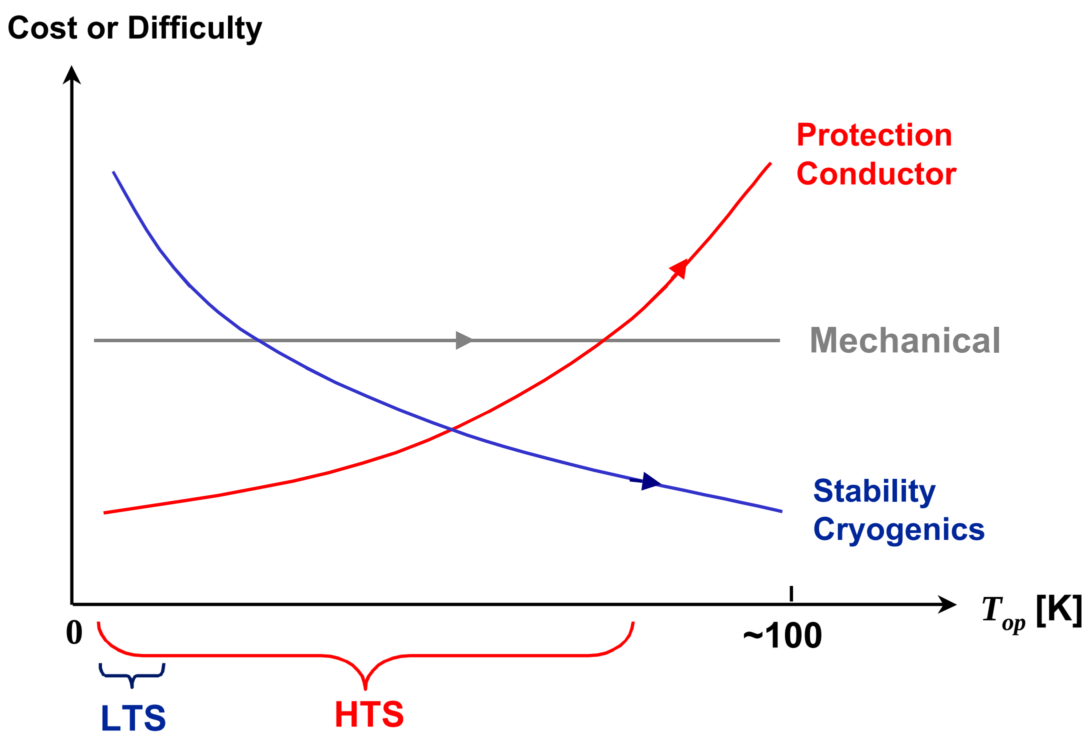
2.64J/22.68J Spring Term 2003

April 24, 2003

Lecture 8: Stability

- **Key magnet issues vs.** T_{op}
- **Magnetic and thermal diffusion**
- **Winding pack; stability; power density equation**
- **Disturbances; energy density spectra; energy or stability margin**
- $E(J)$ **&** $V(I)$ **characteristics of "high-current" superconductor**
- **Important stability issues: cryostable & adiabatic magnets**
- **Cryostability; Stekly criterion; Equal area criterion; CICC**
- **Heat transfer data**
- **High-performance (adiabatic) magnets; MPZ concept**
- **Mechanical disturbances—premature quenches & training**
- **AE technique**
- **Stability of HTS**

Key Magnet Issues vs T_{op}



Magnetic Diffusion

$$\nabla \times \vec{H} = \vec{J}_f \quad \Rightarrow \quad (1\text{-D}) \quad \frac{\partial H_y}{\partial x} = J_z = \frac{E_z}{\rho_e}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} = \mu_0 \frac{\partial H_y}{\partial t}$$

$$\rho_e \frac{\partial^2 H_y}{\partial x^2} = \mu_0 \frac{\partial H_y}{\partial t} \qquad D_{mg} = \frac{\rho_e}{\mu_0}$$

$$\frac{\rho_e}{\mu_0} \frac{\partial^2 H_y}{\partial x^2} \equiv D_{mg} \frac{\partial^2 H_y}{\partial x^2} = \frac{\partial H_y}{\partial t} \qquad \tau_{mg} = \frac{1}{D_{mg}} \left(\frac{2a}{\pi}\right)^2 = \frac{\mu_0}{\rho_e} \left(\frac{2a}{\pi}\right)^2$$

$$D_{mg} = \frac{\rho_e}{\mu_0}$$

$$\tau_{mg} = \frac{1}{D_{mg}} \left(\frac{2a}{\pi}\right)^2 = \frac{\mu_0}{\rho_e} \left(\frac{2a}{\pi}\right)^2$$

Thermal Diffusion

$$k\frac{\partial^2 T}{\partial x^2} = C\frac{\partial T}{\partial t}$$

$$\frac{k}{C}\frac{\partial^2 T}{\partial x^2} \equiv D_{th}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

$$\boxed{D_{th} = \frac{k}{C}}$$

Data for Cu & Nb-Ti

	K [W/m K]	C [J/m³K]	ρ [Ω m]	D_{th} [m²/s]	D_{mg} [m²/s]
Copper	**~400**	**~1000**	**$\sim 3\times10^{-10}$**	**~0.4**	**$\sim 3\times10^{-3}$**
Nb-Ti	**$\sim 10^{-3}$**	**~500**	**$\sim 5\times10^{-7}$**	**$\sim 2\times10^{-6}$**	**~0.5**

$$\frac{[D_{th}]_{Cu}}{[D_{th}]_{SC}} \sim 2\times10^{5}$$

Temperature wave travels much faster in Cu—no temperature concentration in Cu.

$$\frac{[D_{mg}]_{Cu}}{[D_{mg}]_{SC}} \sim 6\times10^{-3}$$

Magnetic wave travels much slowly in Cu—no flux motion induced local heating concentration in Cu.

Best to use Nb-Ti with Cu

Winding Pack

$$A_{wp} = A_{str} + A_{sc} + A_m + A_{ins} + A_q$$

Coolant

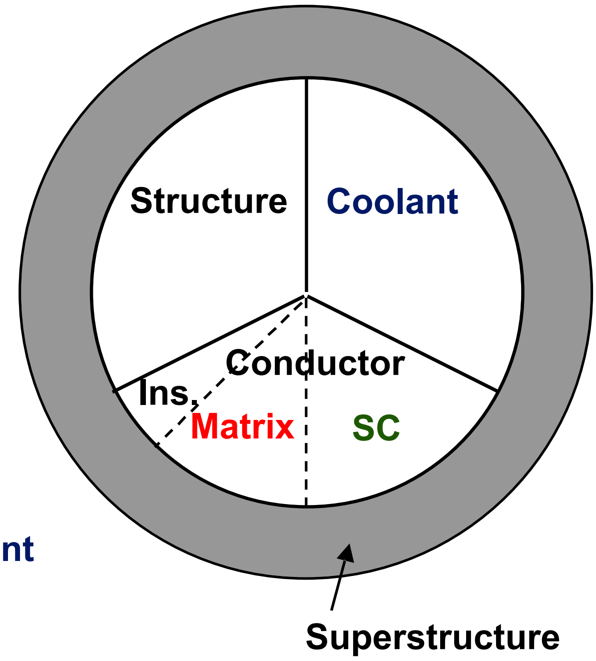
Insulation

Matrix (e.g. Cu)

Superconductor

Structure (e.g. epoxy, conduit)

Winding Pack Cross Section



- $J_c = \dfrac{I_c}{A_{sc}}$
- $J_{nonCu} = \dfrac{I_c}{A_{cd} - A_m}$ **(Nb$_3$Sn; HTS)**

  $= J_c$ **(Nb - Ti)**
- $J_{eng} = J_e = \dfrac{I_c}{A_{cd}}$
- $J_m = \dfrac{I_{op}}{A_m}$ **(Qunech propagation & protection)**
- $J_{overall} = \dfrac{I_{op}}{A_{wpk}}$

Stability

- **Stability of a superconducting magnet refers to the phenomenon of the magnet to operate reliably despite the presence of disturbance events.**

$$\Delta T_{cd} \leq \text{limit set by design}$$

- **Disturbance may be *mechanica*l, *electromagnetic, thermal,* or even *nuclear* in origin.**
  - **Distributed: energy density [J/m^3]**
  - **Point: energy [J]**

Power Density [W/m³] *Equation*

$$\dot{e}_h = g_k + g_j + g_d - g_q \qquad (6.1)$$

Cooling: $\dfrac{f_p P_{cd}}{A_{cd}} q(T)$

Disturbance: g_d

Joule heating: $\rho_{cd}(T) J_{cd}^2$

Conduction: $\nabla \cdot \left[k_{cd}(T) \nabla T \right] \rightarrow \dfrac{\partial}{\partial x}\left[k_{cd}(T) \dfrac{\partial T}{\partial x} \right]$

Storage: $C_{cd}(T) \dfrac{\partial T}{\partial t}$

$$C_{cd}(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k_{cd}(T) \frac{\partial T}{\partial x} \right] + \rho_{cd}(T) J_{cd}^2 + g_d - \frac{f_p P_{cd}}{A_{cd}} q(T)$$

Stability Concepts Derived from Eq. 6.1

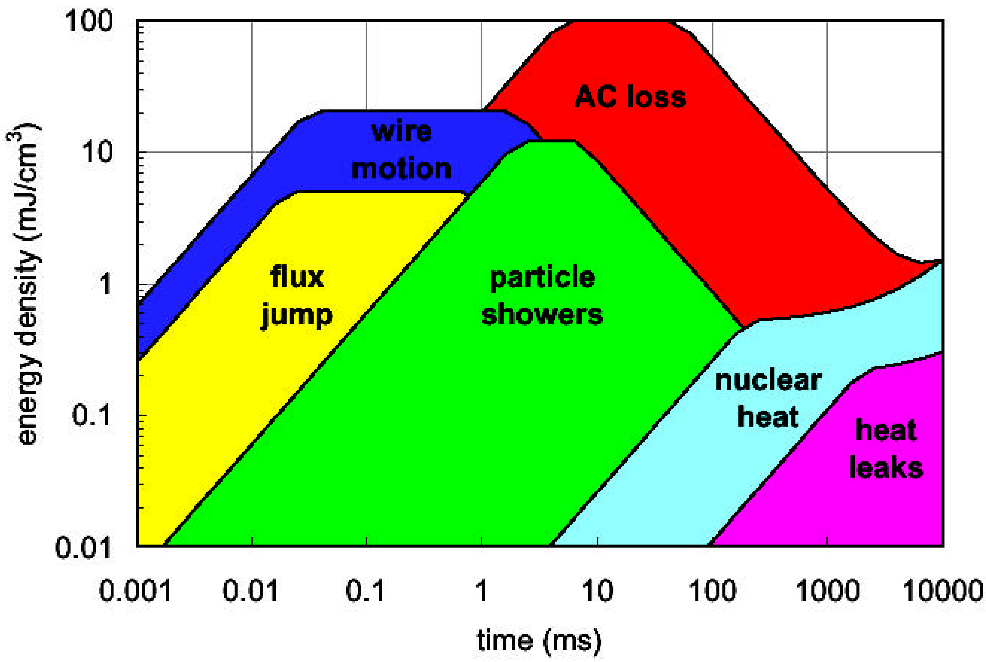
$\dot{e}_h$	g_k	g_j	g_d	g_q	***Application***
√	**0**	**0**	√	**0**	**Flux jump**
0	**0**	√	**0**	√	**Cryostability ***
√	√	√	**0**	√	**Dynamic stability**
0	√	√	**0**	√	**"Equal area" ***
0	√	√	**0**	**0**	**MPZ * †**
√	**0**	√	**0**	**0**	**Protection**
√	√	√	**0**	**0**	**Adiabatic NZP ****

*** Covered in today's lecture**
† Minimum Propagation Zone
**** Normal Zone Propagation**

Sources of Disturbance

- **Mechanical—*Lorentz force; thermal contraction***
  - **Wire motion/"micro-slip"**
  - **Structure deformation**
  - **Cracking epoxy; debonding**
- **Electrical/Magnetic —*Time-varying current/field***
  - **Current transients, includes AC current**
  - **Field transients, includes AC field**
  - **Flux motion, e.g., flux jump**
- **Thermal**
  - **Conduction, through leads**
  - **Cooling blockage (poor ventilation)**
- **Nuclear radiation**
  - **Neutron flux in fusion machines**
  - **Particle showers in accelerators**

Disturbance Energy Density Spectra

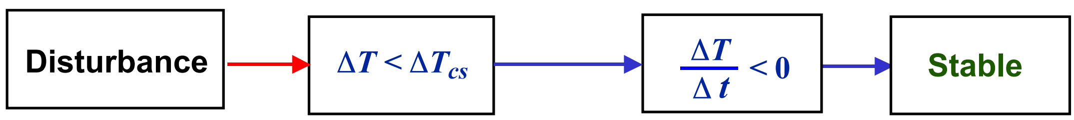


Courtesy of Luca Bottura (CERN, Geneva)

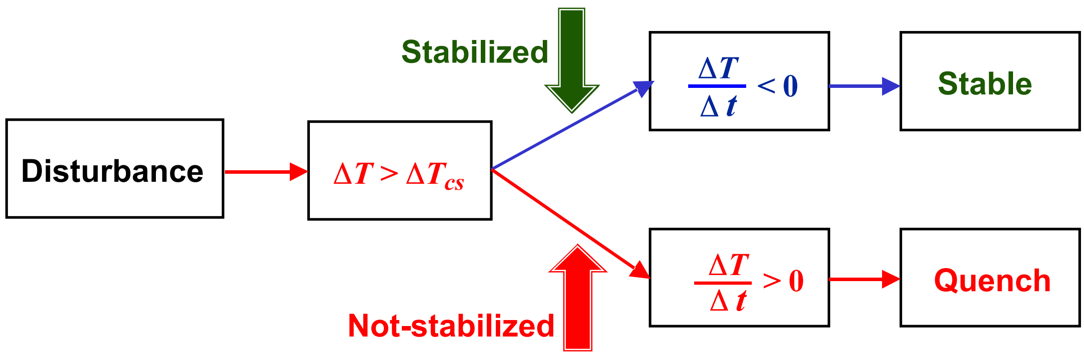
Energy Margin or Stability Margin: Δe_h; Δe_{min}

Minimum energy density (or energy) that leads to a quench

- **Disturbance energy < Δe_h**



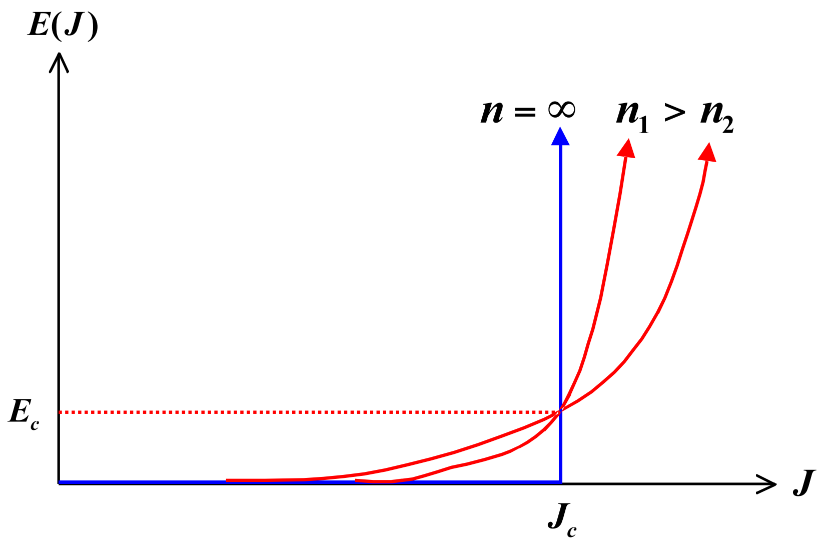
- **Disturbance energy > Δe_h**



E(J) & V(I) Characteristics of "High-Current" Superconductor

$$E(J) = E_c\left(\frac{J}{J_c}\right)^n \qquad V(I) = V_c\left(\frac{I}{I_c}\right)^n$$

- ***n***: **"Index" of a superconductor: "ideal" superconductor, *n*=**



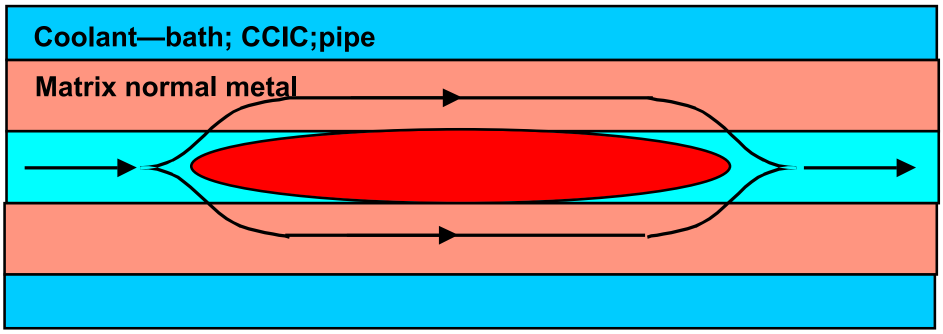
Typical Values of n for "Magnet-Grade" Conductors

- **Nb-Ti: 20--100**
  (50--100 for NMR/MRI persistent-mode magnets)
- **Nb3Sn: 20--80**
  (40--80 for NMR/MRI persistent-mode magnets)
- **BSCCO2223: 10--25**

Important Stability Issues—Cryostable & Adiabatic Magnets

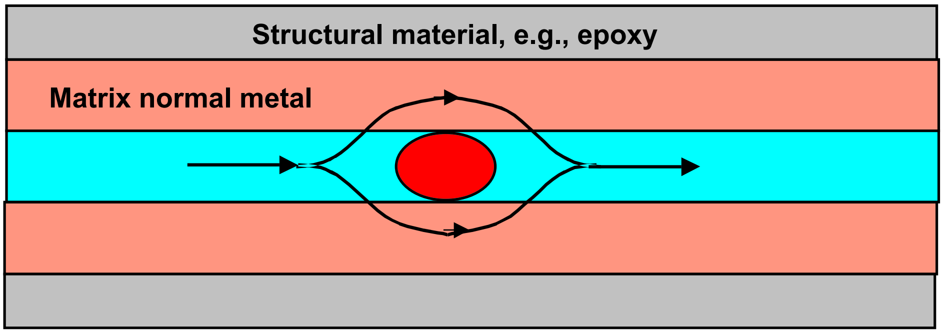
- **Cryostable Magnets**
  - **Circuit model for a well-cooled composite conductor.**
  - **“Current-sharing.”**
  - **Cryostability (Stekly)—“linear”cooling.**
  - **Cryostability—nonlinear cooling.**
  - **“Equal Area” stability.**
  - **CICC**
  - **Cooling data—bath & forced-flow.**
- **Adiabatic Magnets**
  - **Premature quenches; training.**
  - **Minimum propagation zone (MPZ).**
  - **Disturbance spectra.**
  - **Acoustic emission (AE) technique.**

Cryostable Magnets



- **Normal zone size >> "MPZ;" in bath-cooled magnets, can extend over the entire magnet volume.**
- **Best design: sufficient cooling with the minimum coolant space.**

Adiabatic Magnets



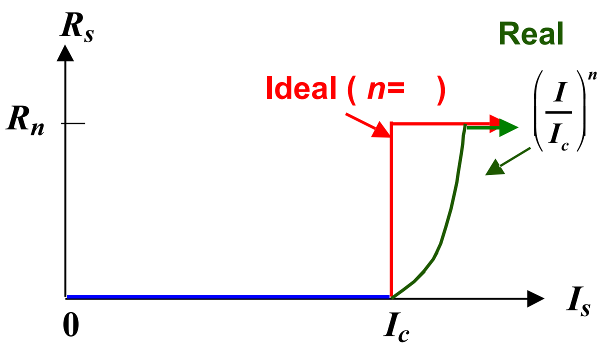
- "Localized" normal zone permitted, but its size < "MPZ" (~mm in most windings).
- **Best design: elimination of all disturbances.**

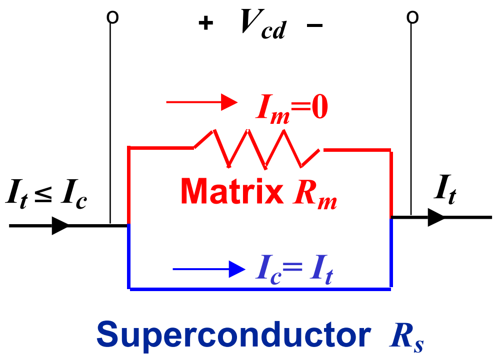
Cryostability

Circuit Model for Ideal Superconductor ($n =$)— $I_t \leq I_c$

$$V_{cd} = 0$$

$$G_j = V_{cd} I_t = 0$$





Cryostability

Circuit Model for Ideal Superconductor ($n =$)— $I_t \geq I_c$

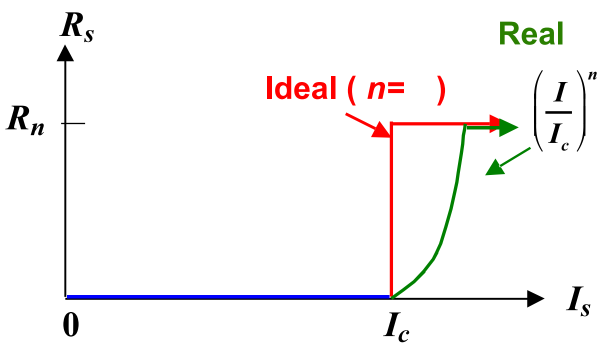
$$V_{cd} = R_m I_m = R_m (I_t - I_c) = R_s I_c$$

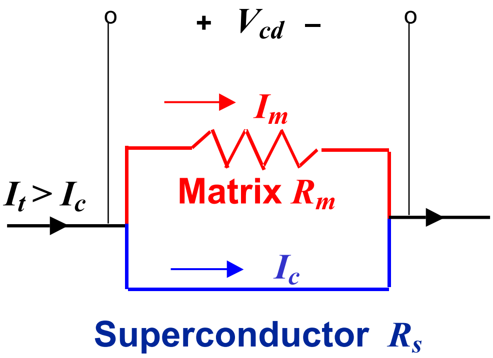
$$R_s = R_m \left(\frac{I_t}{I_c} - 1 \right)$$

$$G_j = V_{cd} I_t$$

$$G_j = R_m I_t (I_t - I_c)$$

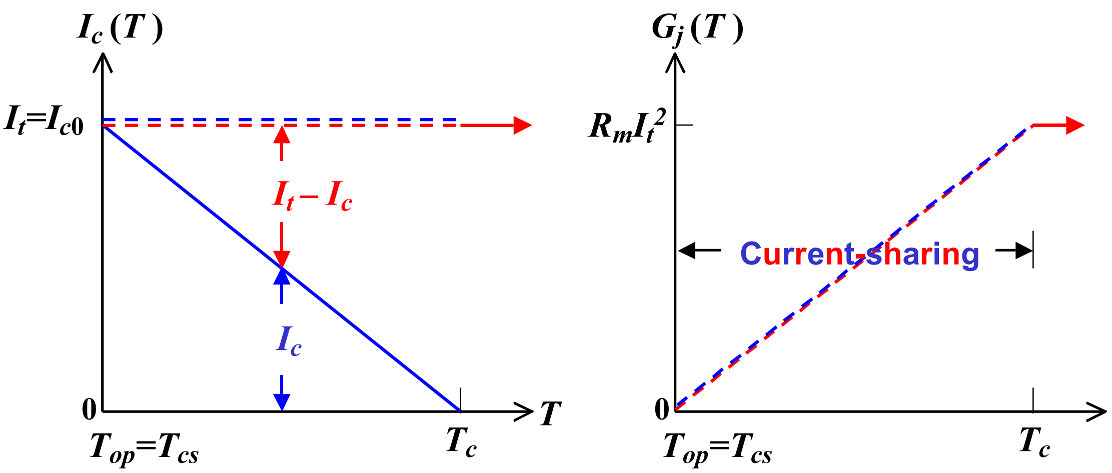
At I_c, superconductor will have R_s between 0 and $R_n >> R_m$ to satisfy the circuit requirements





Temperature Dependent $G_j(T)$—Stekly Model

- $I_t = I_c(T_{\varphi}) = I_{c0}$
- **Ideal superconductor (*n* =)**



Cryostability—Stekly Criterion

$$G_j(T) = R_m I_t^2 \left(\frac{T - T_{cs}}{T_c - T_{cs}} \right) \qquad (T_{cs} \le T \le T_c)$$

Note: $T_b \cong T_{op}$

$$G_q(T) = f_p P_{cd} h_q (T - T_b)$$

$$g_q(T) \cong \frac{f_p P_{cd} h_q (T - T_{op})}{A_{cd}}$$

$$\frac{f_p P_{cd} h_q (T - T_{op})}{A_{cd}} \ge \frac{\rho_m I_{c_0}^2}{A_{cd} A_m} \left(\frac{T - T_{op}}{T_c - T_{op}} \right)$$

$$\frac{f_p P_{cd} A_m h_q (T_c - T_{op})}{\rho_m I_{c_0}^2} \ge 1$$

$$\alpha_{sk} = \frac{f_p P_{cd} A_m h_q (T_c - T_{op})}{\rho_m I_{c_0}^2} \qquad (6.8)$$

Note that the conventional definition of α_{sk} is ***upside down*** *of Eq. 6.8*

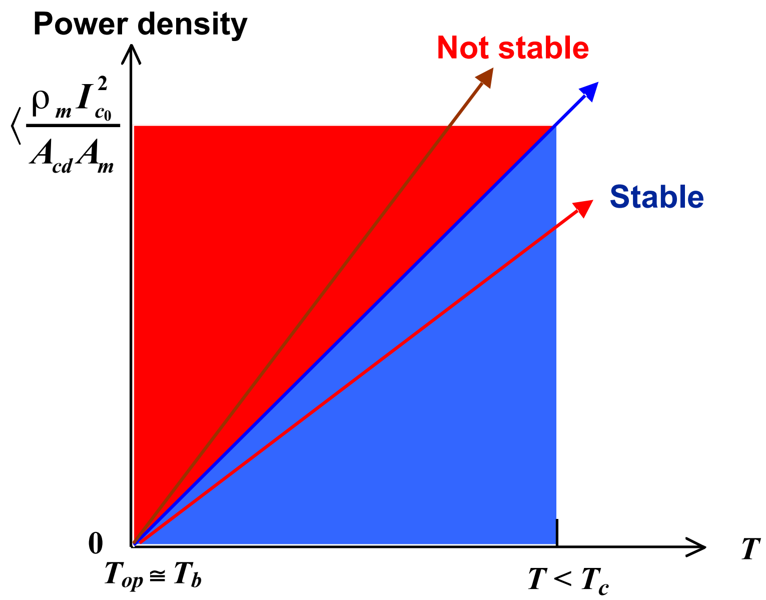
$$A_m = \frac{\alpha_{sk} \rho_m I_{c_0}^2}{f_p P_{cd} h_q (T_c - T_{op})}$$

$$\left[J_{c_0} \right]_{cd} \equiv \frac{I_{c_0}}{A_{cd}} = \frac{I_{c_0}}{A_s + A_m} = \frac{\left[J_{c_0} \right]_s}{1 + \gamma_{c/s}}$$

Stekly (continued)

Dissipation: $\dfrac{\rho_m I_{c_0}^2}{A_{cd} A_m}\left(\dfrac{T - T_{op}}{T_c - T_{\varphi}}\right)$

Cooling: $\dfrac{f_p P_{cd} h_q (T - T_{op})}{A_{cd}}$

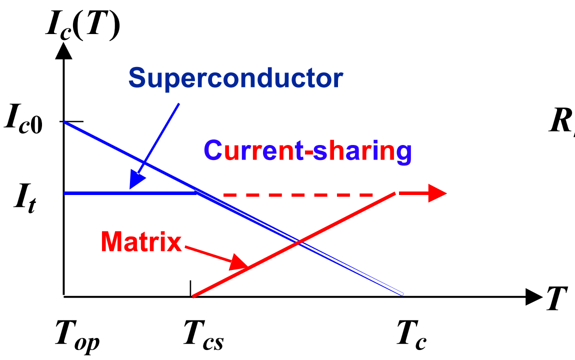


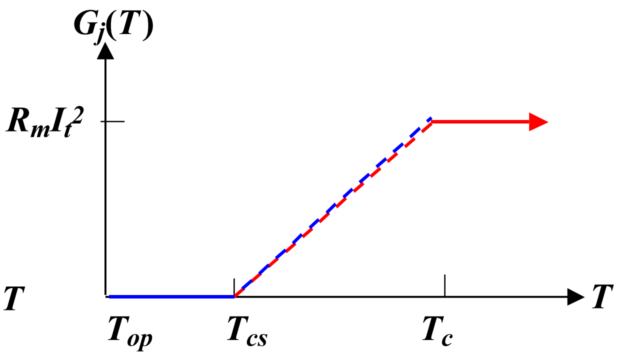
General Case ($I_t < I_{c0}$ or $T_{cs} > T_{op}$)

$$G_j(T) = 0 \qquad (T_{op} \le T \le T_{cs})$$

$$G_j(T) = R_m I_t \left[I_t - I_{c0}\left(\frac{T_c - T}{T_c - T_{op}} \right) \right] \qquad (T_{cs} \le T \le T_c)$$

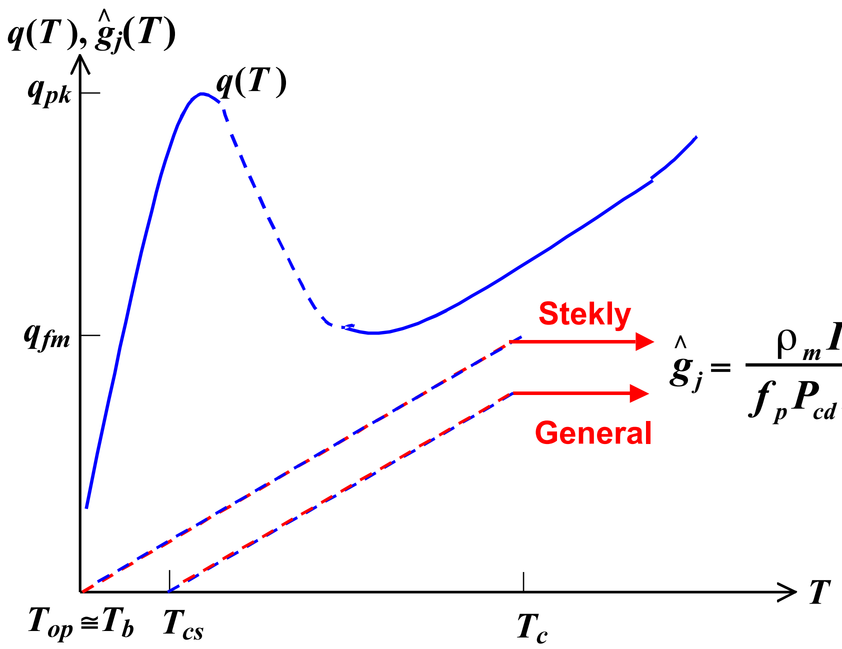
$$I_{c0} = I_t \left(\frac{T_c - T_{op}}{T_c - T_{cs}} \right) \quad \rightarrow \quad G_j(T) = R_m I_t^2 \left(\frac{T - T_{cs}}{T_c - T_{cs}} \right) \qquad (T_{cs} \le T \le T_c)$$





Nonlinear Cooling Curves

$$[J_{c0}]_{cd} = [J_t]_{cd} = [J_\varphi]_{cd} = \sqrt{\frac{f_p P_{cd} A_m q_{fm}}{\rho_m (A_s + A_m)^2}} \cong \sqrt{\frac{f_p P_{cd} q_{fm}}{\rho_m A_m}} \quad [\text{A/m}^3]$$



$$\hat{g}_j = \frac{\rho_m I_t^2}{f_p P_{cd} A_m} \quad [\text{w/m}^2]$$

Example

$$f_p = 0.5;\ P_{cd} = 2.6\,\text{cm};\ A_m + A_s \equiv A_{cd} = 0.3\ \text{cm}^2;\ \frac{A_m}{A_s} \equiv \gamma = 10;$$

$$A_m = \left(\frac{\gamma}{1+\gamma}\right) A_{cd} = 0.273\ \text{cm}^2;\ \rho_m = 3\times 10^{-8}\ \Omega\,\text{cm};\ q_{fm} = 0.3\ \text{W/cm}^2;$$

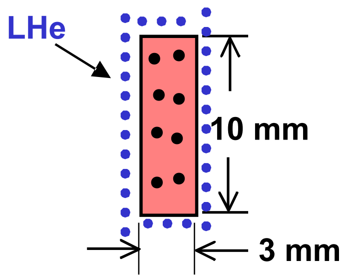
$$[J_\varphi]_{cd} \le \sqrt{\frac{f_p P_{cd} A_m q_{fm}}{\rho_m (A_s + A_m)^2}} = \sqrt{\frac{(0.5)(2.6\,\text{cm})(0.273\ \text{cm}^2)(0.3\ \text{W/cm}^2)}{(3\times 10^{-8}\ \Omega\ \text{cm})(0.3\text{cm}^2)^2}}$$

$$\le 6280\ \text{A/cm}^2$$

$$[J_\varphi]_s = (1+\gamma)[J_{op}]_{cd} \le 69{,}080\ \text{A/cm}^2$$

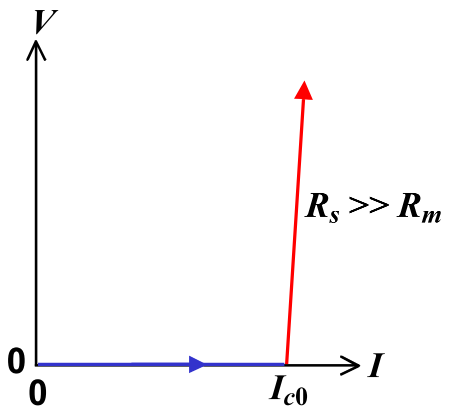
$$I_{op} \le A_{cd}[J_{op}]_{cd} = 1884\ \text{A} \quad (\gamma = 10)$$

$$I_{op} \le A_{cd}[J_\varphi]_{cd} = 1803\ \text{A} \quad (\gamma = 5)$$



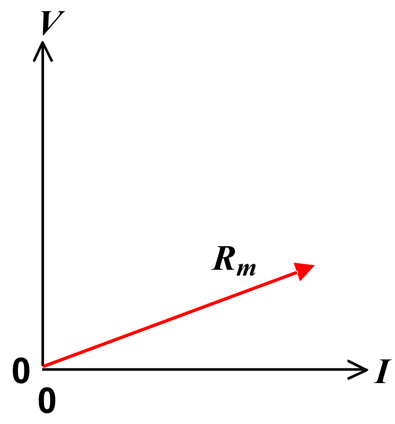
V vs. I Traces of a Composite Conductor

SC alone at $T = T_b$



R_s / R_m **~1000-10000**

Matrix alone at $T = T_b$



V vs. I Traces of a Composite Conductor

$$G_j(T_{op}+\Delta T)=VI=R_m\left\{I-I_{c_0}\left[\frac{T_c-(T_{op}+\Delta T)}{T_c-T_{op}}\right]\right\}I=R_m I\left[(I-I_{c_0})+\frac{I_{c_0}\Delta T}{T_c-T_{op}}\right]$$

$$f_p P_{cd}\ell h_q(T-T_{op})=f_p P_{cd}\ell h_q\Delta T$$

$$\Delta T=\frac{R_m I(I-I_{c_0})(T_c-T_{op})}{f_d P_{cd}\ell h_q(T_c-T_{op})-R_m I_{c_0} I}$$

$$V=R_m\left\{(I-I_{c_0})+\left(\frac{I_{c_0}}{T_c-T_{op}}\right)\times\left[\frac{R_m I(I-I_{c_0})(T_c-T_{op})}{f_p P_{cd}\ell h_q(T_c-T_{op})-R_m I_{c_0} I}\right]\right\}$$

$$=R_m(I-I_{c_0})+\frac{R_m^2 I(I-I_{c_0})(T_c-T_{op})}{f_p P_{cd}\ell h_q(T_c-T_{op})-R_m I_{c_0} I}$$

V vs. I Traces: Dimensionless Expressions

$$v = \frac{V}{R_m I_{c_0}}$$

$$i = \frac{I}{I_{c_0}}$$

$$\alpha_{sk} = \frac{f_p P_{cd} \ell h_q (T_c - T_{op})}{R_m I_{c_0}^2} = \frac{f_p P_{cd} A_m h_q (T_c - T_{op})}{\rho_m I_{c0}^2}$$

$$v(i) = (i-1) + \frac{i(i-1)}{\alpha_{sk} - i} = \frac{\alpha_{sk}(i-1)}{\alpha_{sk} - i}$$

$$i(v) = \frac{\alpha_{sk}(v+1)}{\alpha_{sk} + v}$$

Numerical Illustrations

$I_{c0} = 1000\,\text{A}$; $T_c = 5.2\,\text{K}$; $T_{op} = 4.2\,\text{K}$; $A_m = 2\times 10^{-5}\,\text{m}^2$; $\rho_m = 4\times 10^{-10}\,\Omega\,\text{m}$;

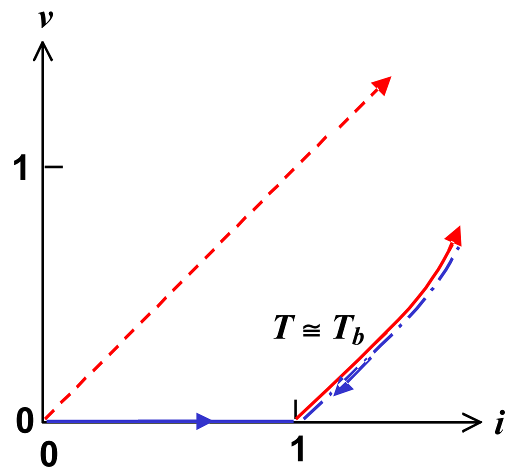
$h_q = 10^4\,\text{W/m}^2$; $P_{cd} = 2\times 10^{-2}\,\text{m}^2$

f_m=1: *Entire* **conductor surface exposed to coolant**

$$\alpha_{sk} = \frac{(1)(2\times 10^{-2}\,\text{m})(2\times 10^{-5}\,\text{m}^2)(10^4\,\text{W/m}^2\text{K})(5.2\,\text{K} - 4.2\,\text{K})}{(4\times 10^{-10}\,\Omega\,\text{m})(1000\,\text{A})^2} = 10$$

v vs. i *for* $\alpha_{sk} = 10$

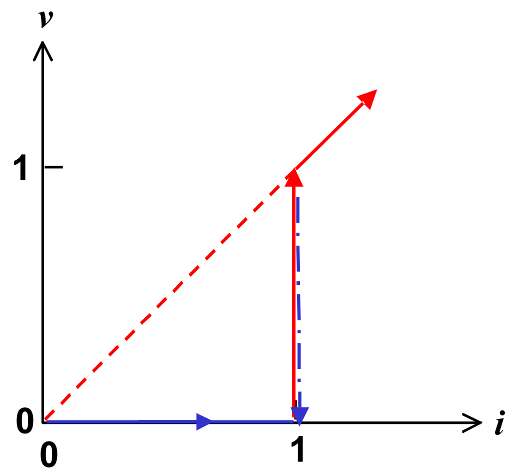
i	1	1.1	1.5	2
v	0	0.11	0.59	1.25



Numerical Illustrations

f_m=0.1: **10% of conductor surface exposed to coolant**

$$\alpha_{sk} = \frac{(0.1)(2\times 10^{-2}\ \mathrm{m})(2\times 10^{-5}\ \mathrm{m}^2)(10^4\ \mathrm{W/m^2K})(5.2\ \mathrm{K} - 4.2\ \mathrm{K})}{(4\times 10^{-10}\ \Omega\mathrm{m})(1000\ \mathrm{A})^2} = 1$$



Numerical Illustrations

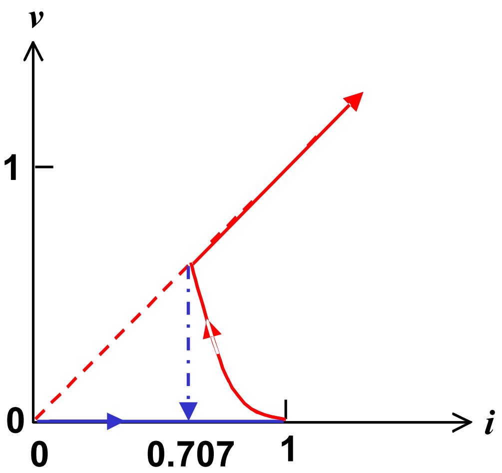
f_m=0.05: Only 5% of conductor surface exposed to coolant

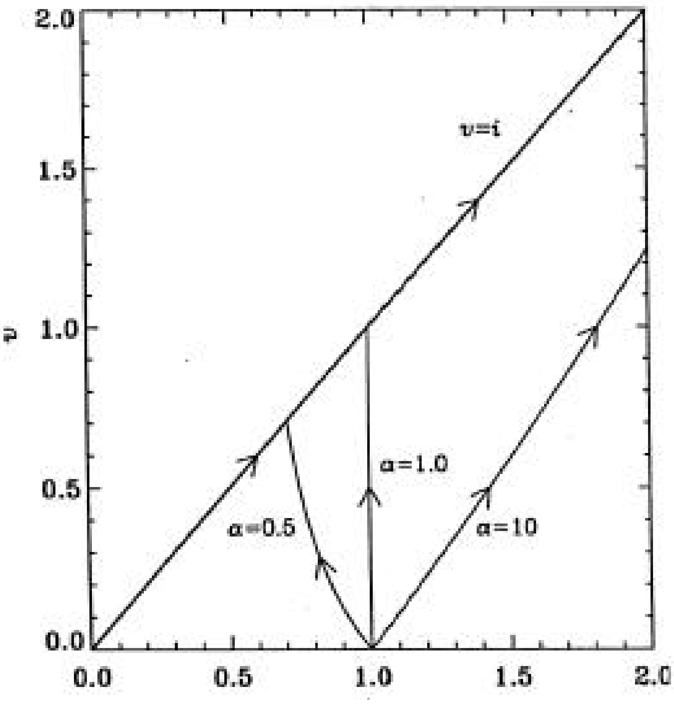
v vs. i *for* $\alpha_{sk} = 0.5$

v	0	0.125	0.25	0.5	0.625	0.707
i	1	0.9	0.833	0.75	0.722	0.707

$$i = \frac{0.5(i+1)}{0.5+i} \quad \text{or} \quad v = \frac{0.5(v+1)}{0.5+v}$$

$$i = v = \sqrt{0.5} = 0.707$$





2.0
1.5
1.0
0.5
0.0
v
v=i
α=0.5
α=1.0
α=10
0.0
0.5
1.0
1.5
2.0

"Equal-Area" Criterion

$$0 = g_k(T) + g_j(T) + 0 - g_q(T)$$

$$g_k(T) = \frac{d}{dx}\left[k_{cd}(T)\frac{dT}{dx}\right] = g_q(T) - g_j(T) \qquad S(T) = k_{cd}(T)\frac{dT}{dx}$$

$$\frac{dS(T)}{dx} = g_q(T) - g_j(T) = \frac{dS(T)}{dT}\frac{dT}{dx} = \frac{dS(T)}{dT}\frac{S(T)}{k_{cd}(T)}$$

$$S(T)\frac{dS(T)}{dT} = k_{cd}(T)\left[g_q(T) - g_j(T)\right]$$

$$\frac{1}{2}\left[S^2(T_2) - S^2(T_1)\right] = k_0\int_{T_1}^{T_2}\left[g_q(T) - g_j(T)\right]dT$$

$S(T_1) = S(T_2) = 0$ **because** $dT/dx\big|_{x_1} = dT/dx\big|_{x_2} = 0$

$$\boxed{\int_{T_1}^{T_2}\left[g_q(T) - g_j(T)\right]dT = 0}$$

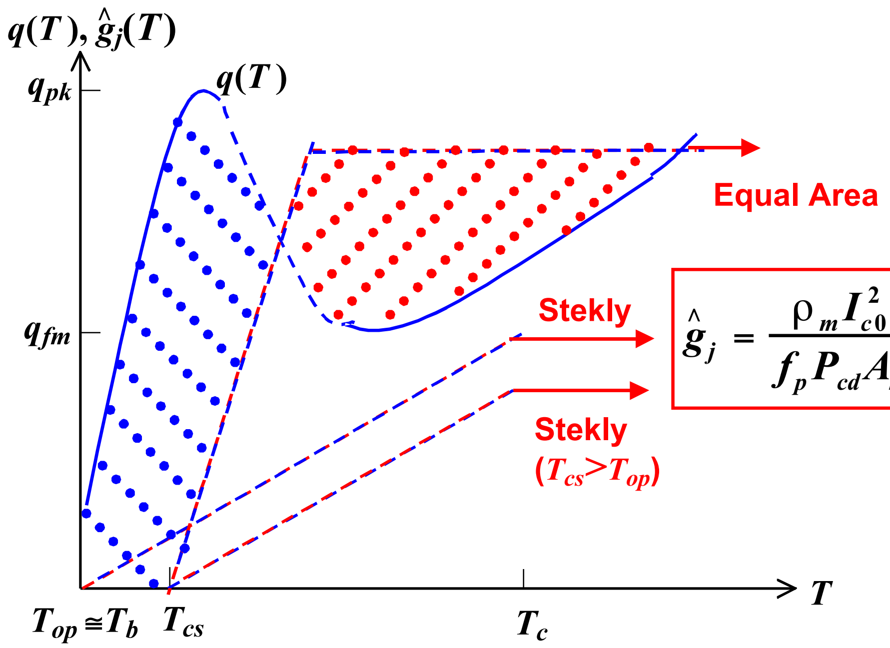
Equal Area (continued)

2-D Case

$$\nabla \cdot \left[k_{cd}(T) \nabla T \right] = \frac{d}{dr}\left[k_{cd}(T) \frac{dT}{dr} \right] + \frac{1}{r} k_{cd}(T) \frac{dT}{dr}$$

$$\frac{d}{dr}\left[k_{cd}(T) \frac{dT}{dr} \right] = g_q(T) - g_j(T) - \frac{1}{r} k_{cd}(T) \frac{dT}{dr}$$

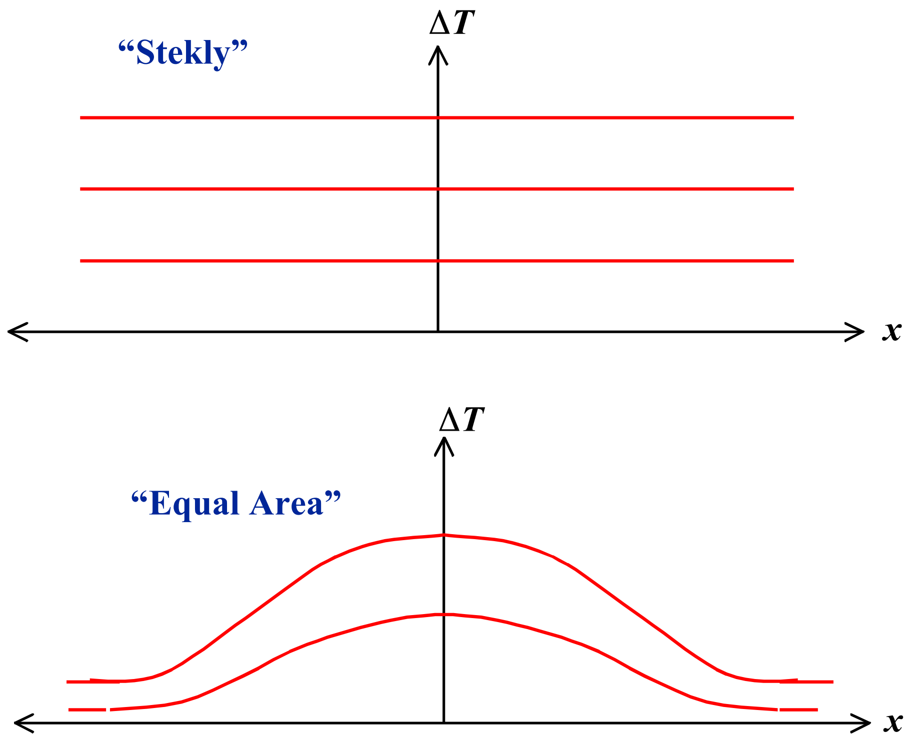
Because the last term dT/dr is negative, it has the same effect as that of increasing cooling or decreasing Joule heating.



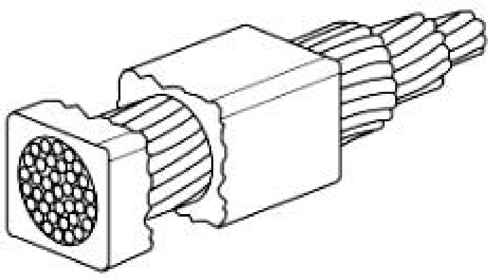
$$\hat{g}_j = \frac{\rho_m I_{c0}^2}{f_p P_{cd} A_m}$$

$[I_{c0}]_{\text{equal area}} > [I_{c0}]_{\text{Stekly}}$

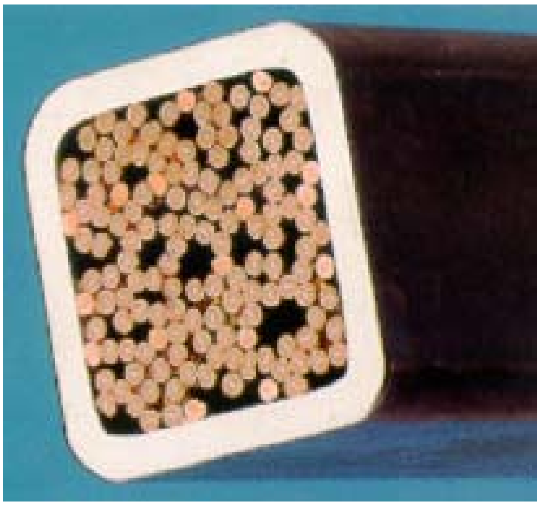
Recovery Processes



CICC

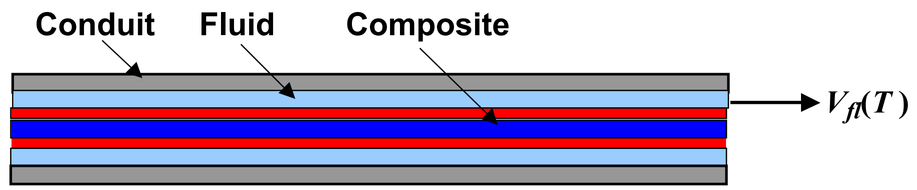


Transposed 37-Strand Cable (c. 1970)



Courtesy of Luca Bottura (CERN, Geneva)

Power Density Equation—CICC

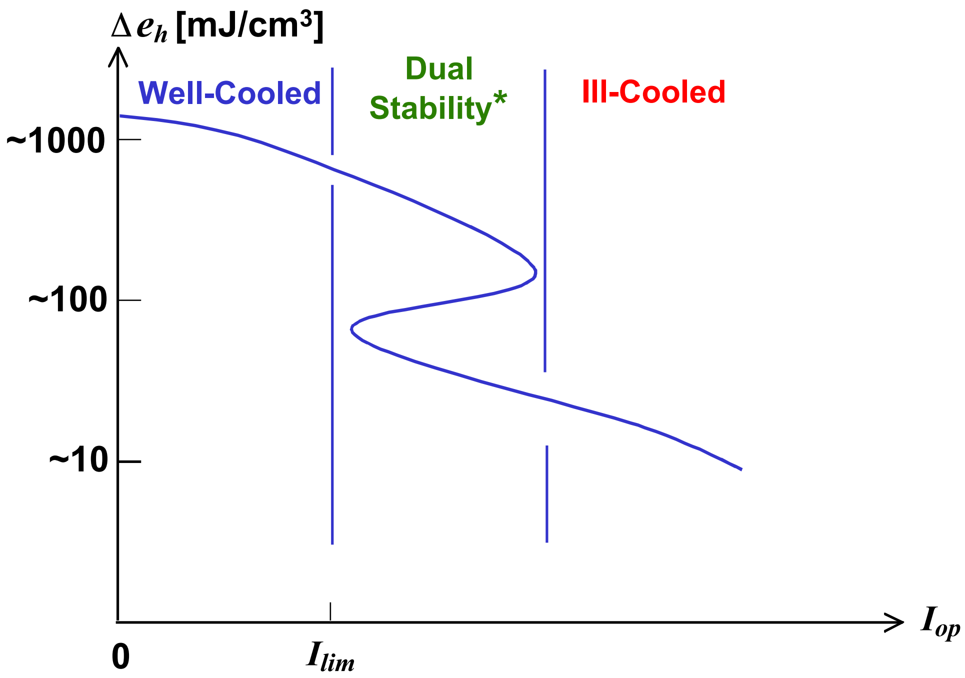


$$C_{cd}(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k_{cd}(T)\frac{\partial T}{\partial x}\right] + \rho_{cd}(T)J_{cd}^2 + g_d - \frac{f_p P_{cd}}{A_{cd}}q(T)$$

$$C_{fl}(T)\frac{\partial T_{fl}}{\partial t} = \frac{f_p P_{cd}}{A_{fl}}q(T) = \frac{f_p P_{cd}}{A_{fl}}h_{fl}\times(T - T_{fl})$$

$$h_{fl} = 0.0259\frac{k_{fl}}{D_{hy}}Re^{0.8}Pr^{0.4}\left(\frac{T_{fl}}{T}\right)^{0.716}$$

Stability Margin vs. Current in CICC



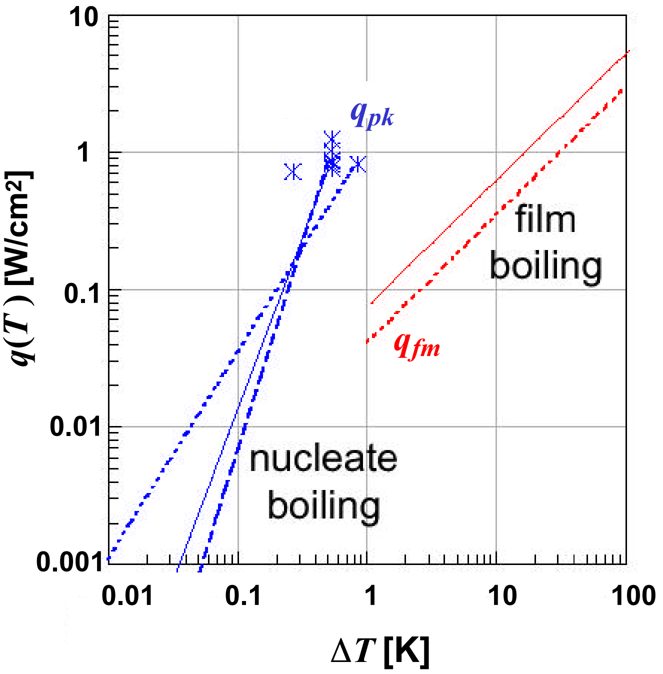
* J.W. Lue, J.R. Miller, L. Dresner (*J. Appl. Phys.* Vol. 51, 1980)

Stability Approach for CICC

- **Because CICC is mostly applied for "large" ("expensive") magnets, cryostability (Stekly) is the accepted approach.**
- **Choose $I_{op} \leq I_{lim}$ given by:**

$$I_{lim} = \sqrt{\frac{f_p P_{cd} A_m h_q (T_c - T_{op})}{\rho_m}}$$

Helium Heat Transfer Data—Nucleate & Film Boiling

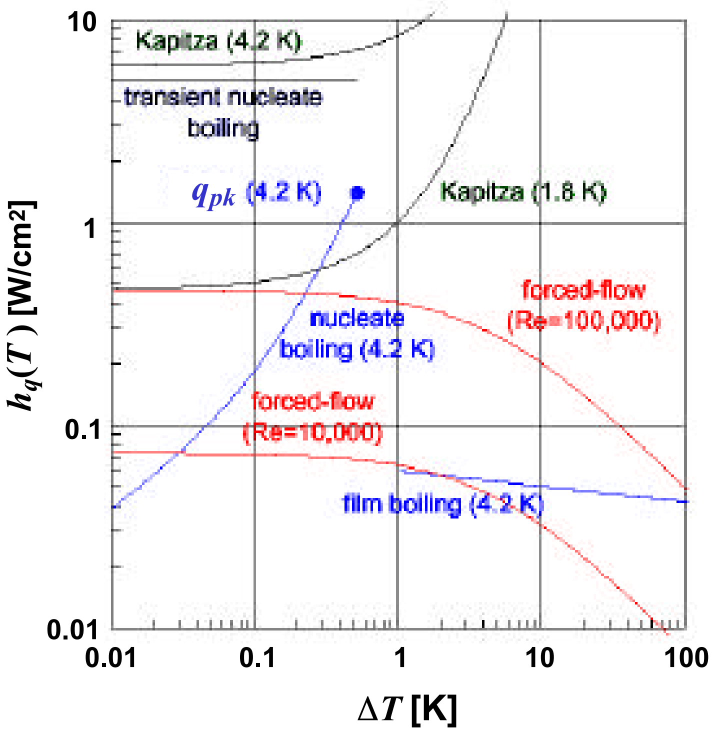


Approximate N_2 data multiply:

- ***y*-scale by 10-30**
- ***x*-scale by 10**

Based on data from Luca Bottura (CERN, Geneva)

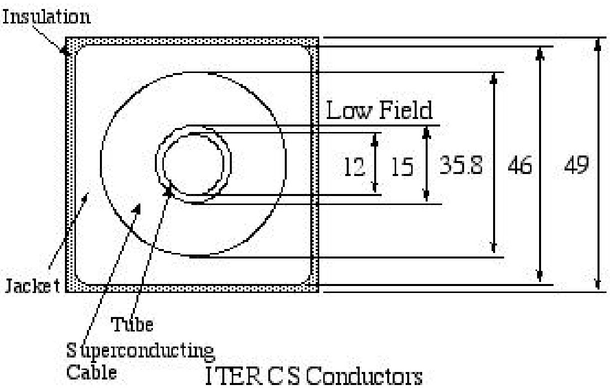
Helium Heat Transfer Coefficient Data



Based on data from Luca Bottura (CERN, Geneva)

Computation of I_{lim} for ITER CS (Central Solenoid) CICC

Strand dia. (d_{st})	0.81 mm
Cu/non-Cu (γ)	1.45
Cable	3×4 ×4 ×4 ×4
# Strands (N_{st})	1152
$A_m = A_{cu}$	3.51×10^{-4} m^2
f_p	5/6
$P_{cd} = P_{st}$	2.93 m
h_q	600 W/m^2 K
T_c **(13 T);** Tb	11.2 K; 4.5 K
ρ_m **(13 T)**	6.8×10^{-10} Ω m



$$A_{cu} = \frac{\pi d_{st}^2}{4} N_s \left(\frac{\gamma}{\gamma + 1}\right) \qquad P_{st} = \pi d_{st} N_{st}$$

$$I_{lim} = \sqrt{\frac{(5/6)(2.93\ \text{m})(3.51\times 10^{-4}\ \text{m}^2)(600\ \text{W/m}^2\text{K})(11.2\ \text{K} - 4.5\ \text{K})}{6.8\times 10^{-10}\ \Omega\text{m})}}$$

$$= 71\ \text{kA}$$

$$I_{op}\,(40 - 46\ \text{kA}) < I_{lim} < I_c\ (96\ \text{kA})$$

This CICC design as with CICC designs for the rest of ITER coils are highly stable, i.e. very conservative.

High-Performance ("Adiabatic") Magnet

- **J_{over} enhanced by:**
  - **Combining superconductor and high-strength normal metal (stability; protection; mechanical).**
  - **Eliminating *local* coolant* and impregnating the entire winding space unoccupied by conductor with epoxy, making the entire winding as one monolithic structural entity.**
    (Presence of cooling in the winding makes the winding mechanically weak and takes up the conductor space.)
- **High-performance approach universally used for NMR, MRI, HEP dipoles & quadrupoles in which $R \times J \times B$ manageable with a combination of "composite conductor" & "monolithic entity."**

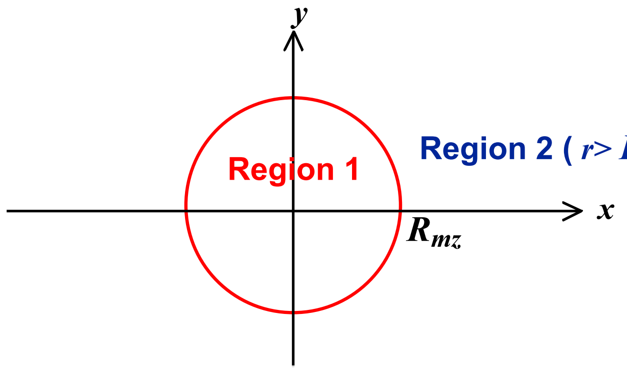
*** The conductor *always* requires cooling but not necessarily exposed directly to the coolant.**

*MPZ Concept**

- **Even in an adiabatic magnet, a small normal-state region may remain stable: g_j is balance by thermal conduction to the outside region.**

Corollary: Even an adiabatic magnet can tolerate a disturbance without being driven to a quench

How large is this normal state region, R_{mz}, and a permissible disturbance energy?



* A.P. Martinelli, S.L. Wipf (*IEEE*, 1971)

$$0 = g_k + g_j + 0 + 0$$

$$\frac{k_{wd}}{r^2}\frac{d}{dr}\left(r^2\frac{dT_1}{dr}\right) = -g_j \qquad \textbf{(Region 1)}$$

$$\frac{k_{wd}}{r^2}\frac{d}{dr}\left(r^2\frac{dT_2}{dr}\right) = 0 \qquad \textbf{(Region 2)}$$

$$T_1(r) = -\frac{g_j}{6k_{wd}}r^2 - \frac{A}{r} + B; \qquad T_2(r) = -\frac{C}{r} + D$$

Boundary conditions:

At $r = \infty, T_2 = T_{op} \Rightarrow D = T_{op}$; $\quad r = 0, T_1 \neq \infty \Rightarrow A = 0$; $\quad r = R_{mz}, dT_1/dr = dT_2/dr$:

$$\left.\frac{dT_1}{dr}\right|_{r=R_{mz}} = -\frac{g_j R_{mg}}{3k_{wd}} = \left.\frac{dT_2}{dr}\right|_{r=R_{mz}} = \frac{C}{R_{mz}^2} \Rightarrow C = -\frac{g_j R_{mz}^3}{3k_{wd}}$$

At $r = R_{mz}, T_1 = T_2 \Rightarrow T_1(r) = B - \frac{g_j r^2}{6k_{wd}};\ T_2(r) = T_{op} + \frac{g_j R_{mz}^3}{3k_{wd} r} \Rightarrow B = T_{op} + \frac{g_j R_{mz}^2}{2k_{wd}}$

$$\boxed{T_1(r) = T_{op} + \frac{g_j R_{mz}^2}{2k_{wd}}\left[1 - \frac{1}{3}\left(\frac{r}{R_{mz}}\right)^2\right]}$$

Total Joule Heating G_j

$$G_j = \frac{4pR_{mz}^3}{3} g_j = -\frac{4pR_{m\,z}^3}{3} k_{wd} \frac{dT_1}{dr}\bigg|_{r=R_{mz}} = -4pR_{m\,z}^2 k_{wd} \left(\frac{g_j R_{mz}^2}{2k_{wd}}\right)\left(-\frac{2}{3R_{mz}}\right) = \frac{4\pi R_{m\,z}^3}{3} g_j$$

Because $g_j = \rho_m J_m^2$ **and** $T_1 = T_c$ **at** $r = R_{mz}$ **:**

$$T_c = T_{op} + \frac{\rho_m J_m^2 R_{mz}^2}{3k_{wd}}$$

$$R_{m\,z} = \sqrt{\frac{3k_{wd}\,(T_c - T_{op})}{\rho_m J_m^2}}$$

With $k_{wd} = 400\,\text{W/mK}; T_c = 6\text{K};\ T_{op} = 4\text{K};\ \rho_m = 2\times 10^{-10}\,\Omega\text{m};\ J_m = 300\times 10^6\,\text{A/m}^2$ **:**

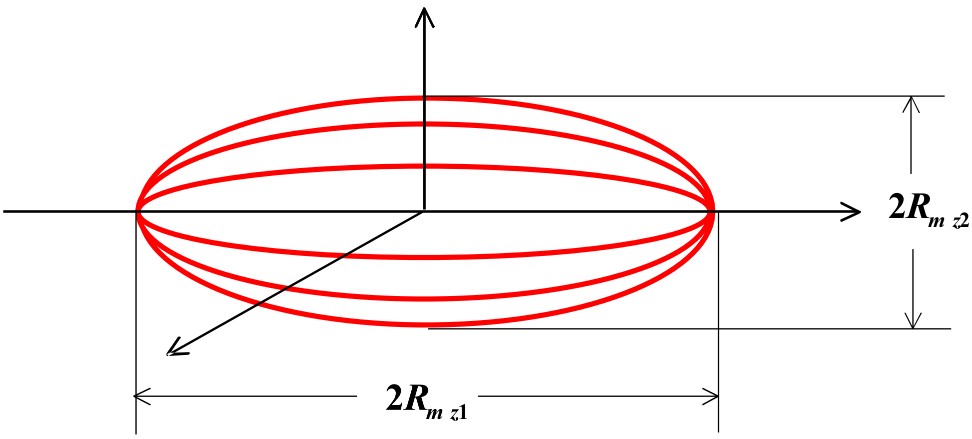
$$R_{mz\,1} = \sqrt{\frac{3(400\text{W/mK})(6\text{K} - 4\text{K})}{(2\times 10^{-10}\,\Omega\text{m})(300\times 10^6\,\text{A/m}^2)}} \cong 12\text{mm}$$

$$R_{mz\,2} = \sqrt{\frac{k_{wd2}}{k_{wd1}}} R_{mz1} \cong \sqrt{\frac{0.1\text{W/mK}}{400\text{W/mK}}} R_{mz1} \cong 0.2\text{mm}$$

With $k_{wd} = 400$ W/m K; $T_c = 6$ K; $T_{op} = 4$ K; $\rho_m = 2\times10^{-10}\,\Omega$ m; $J_m = 300\times10^6$ A/m^2 :

$$R_{mz\,1} = \sqrt{\frac{3(400\,\mathrm{W/m\,K})(6\,\mathrm{K}-4\,\mathrm{K})}{(2\times10^{-10}\,\Omega\mathrm{m})(300\times10^6\,\mathrm{A/m^2})}} \cong 12\,\mathrm{mm}$$

$$R_{mz\,2} = \sqrt{\frac{k_{wd\,2}}{k_{wd\,1}}}R_{mz\,1} \cong \sqrt{\frac{0.1\,\mathrm{W/m\,K}}{400\,\mathrm{W/m\,K}}}R_{mz\,1} \cong 0.2\,\mathrm{mm}$$



Energy margin or stability margin, ΔE_h :

$$\Delta E_h = \nu_{mz} \int_{T_{op}}^{T_c} C_{wd}(T)\,dT = \frac{4\pi R_{mz2}^2 R_{mz1}}{3}\left[h(T_c) - h(T_{op})\right]_{cu}$$

With $R_{mz2} \cong 2 \times 10^{-4}$ **m,** $R_{mz1} \cong 1.2 \times 10^{-2}$ **m,** $\left[h(T_c)\right]_{cu} = 3.9\,\mathrm{kJ/m^3}$**,**

and $\left[h(T_{op})\right]_{cu} = 1.3\,\mathrm{kJ/m^3}$ **:**

$$\nu_{mz} = \frac{4\pi\,(2\times 10^{-4}\,\mathrm{m})^2(1.2\times 10^{-2}\,\mathrm{m})}{3} \cong 2 \times 10^{-9}\,\mathrm{m^3}$$

$$\Delta E_h = 5.2\times 10^{-6}\,\mathrm{J} \sim 10\,\mu\mathrm{J}$$

$$\Delta e_h = 2.6\,\mathrm{kJ/m^3} = 2.6\,\mathrm{mJ/cm^3}$$

Cryostable vs. Adiabatic (High-Performance) Magnets

Basic power density equation:

$$\dot{e}_h = g_k + g_j + g_d - g_q$$

Cryostable Magnets

$$\dot{e}_h = 0; g_k << g_j ; g_d << g_j \quad \Rightarrow \quad g_j \cong g_q$$

$$\left[J_{c0}\right]_{cd} = \sqrt{\frac{f_p P_{cd} A_m q_{fm}}{\rho_m (A_s + A_m)}} << \left[J_{c0}\right]_s$$

Cryostable Magnets (continued)

Observations

- **$[J_{c0}]_{cd}$ determined by external parameters.**
- **Applied for "large magnets" (e.g. fusion) in which non-conducting structural materials occupy a significant fraction of the winding pack, making the condition $[J_{c0}]_{cd} << [J_{c0}]_s$ not as important as in adiabatic magnets.**
- **Note that coolant, though a key component in a cryostable magnet, is a structural detriment.**

Adiabatic (High-Performance) Magnets

$$\dot{e}_h \cong g_k \cong g_j \cong g_d \cong g_q \cong 0$$

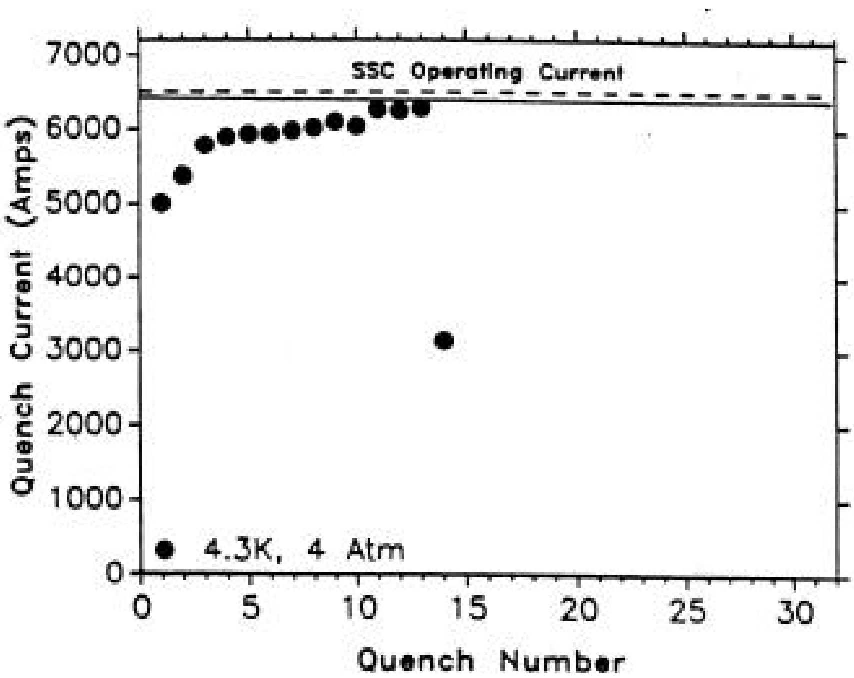
Observations

- **Elimination of coolant within the winding: 1) enhances the overall current density; 2) makes the winding pack structurally robust.**
- **$[J_{c0}]_{cd}$ no longer limited by external parameters but primarily by $[J_{c0}]_s$.**
- **Disturbances (g_d) must be eradicated or their energies minimized—impregnation of the winding with epoxy resin a widely used technique.**
- **High-performance magnets: NMR, MRI, HEP.**

Mechanical Disturbances—Premature Quenches & Training

- **Important for adiabatic magnets; not so for cryostable magnets.**
- **Through the use of AE (acoustic emission) technique, it has been demonstrated that virtually every premature quench in adiabatic magnets is induced by a mechanical event, primarily either conductor motion or epoxy fracture.**
- **Mechanical events usually obey the "Kaiser effect," mechanical behavior in cyclic loading in which mechanical disturbances, e.g., conductor motion, epoxy fracture, appear only when the loading responsible for events exceeds the maximum level achieved in the previous loading sequence.**
  - **An adiabatic magnet thus generally "trains" and improves its performance progressively to the point where it finally reaches its design operating current.**

Training Sequence for an SSC Dipole



S. Ige (MIT ME Dept. Ph.D. Thesis, 1989)

AE Technique

Acoustic signals emitted by sudden mechanical events in a body being loaded or unloaded, e.g., a magnet being charged or discharged; useful for detection and location of a premature quench caused by conductor motion or epoxy fracture event in adiabatic magnets.

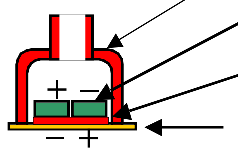
Piezoelectric Effect

The coupling of mechanical and electric effects in which a strain in a certain class of crystals, e.g. quartz, induces an electric potential and vice versa. Discovered by P. Curie in 1880.

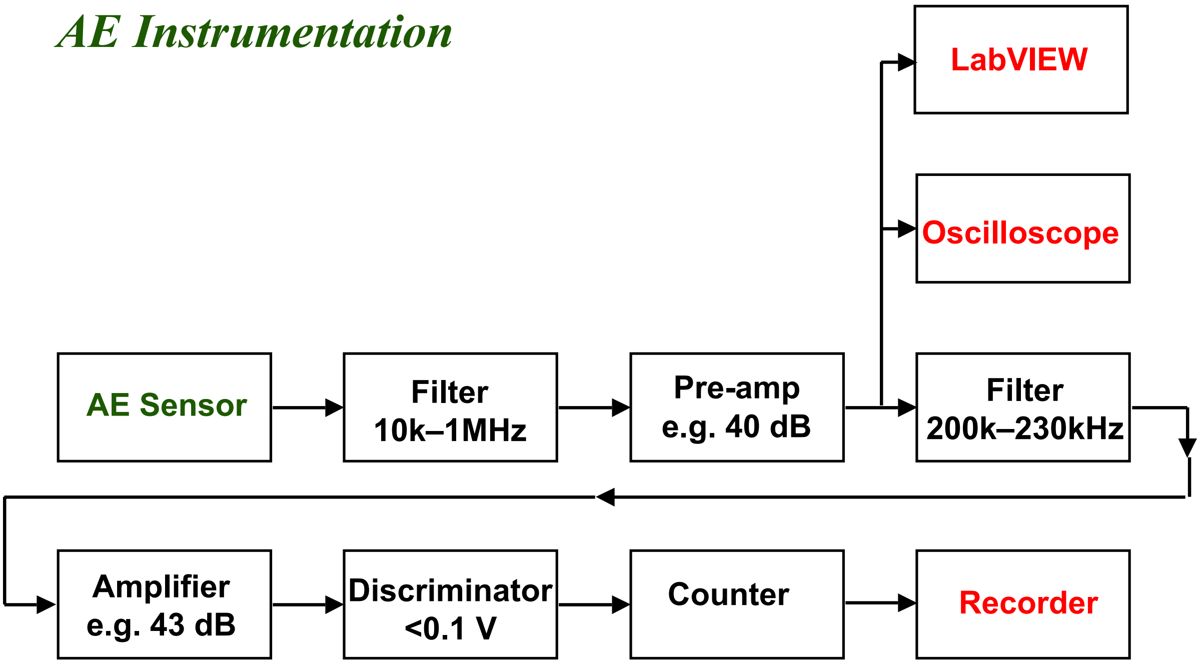
AE Sensor

- **Commercially available sensor: expensive ($500-$1000) and even those claimed to be for use in "low temperatures" generally fail.**
- **Home-made (FBML) differential sensor: reasonable ($50/disk + labor); withstands 4.2 K↔RT cycles.**

Differential Sensor



AE Instrumentation

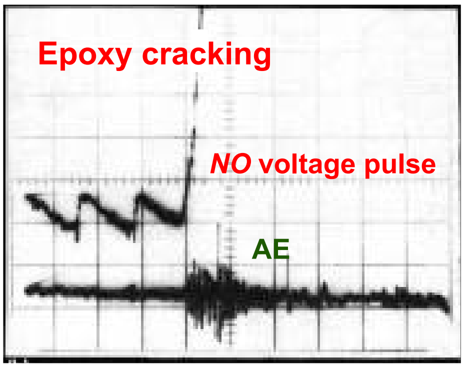


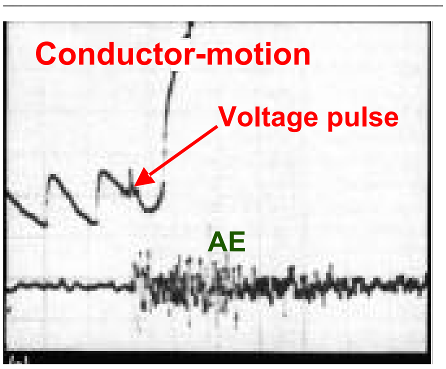
Identification of Quench Causes

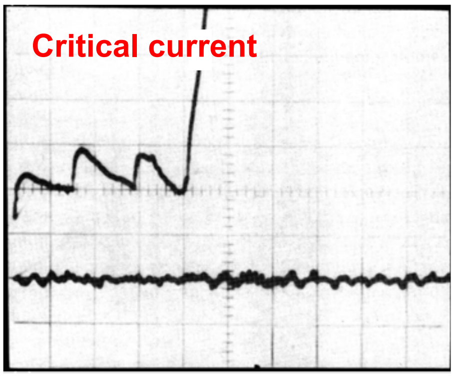
A combination of voltage and AE monitoring permits identification of three distinguished causes of a quench in adiabatic magnets.

- **Conductor motion: a voltage spike and the start of AE signals.**
- **Epoxy fracture: *no* voltage spike but AE signals.**
- **Critical current: *no* voltage spike *nor* AE signals.**

Three Causes of Quench

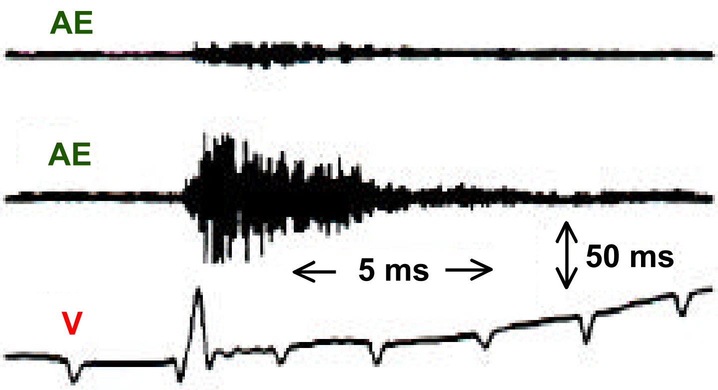






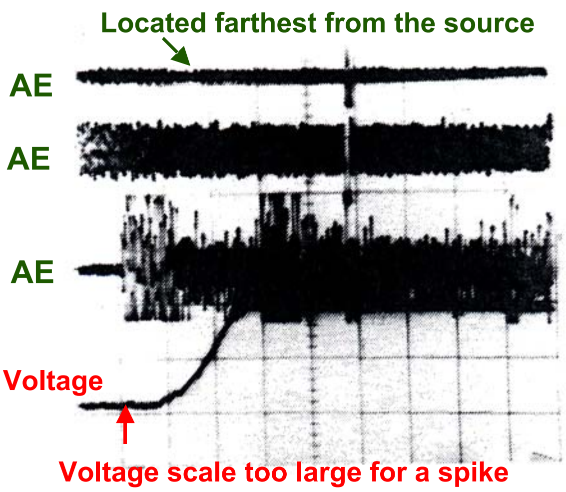
O. Tsukamoto, J.F. Maguire, E.S. Bobrov, Y. Iwasa (*Appl. Phys. Lett.* Vol. 39, 1981)

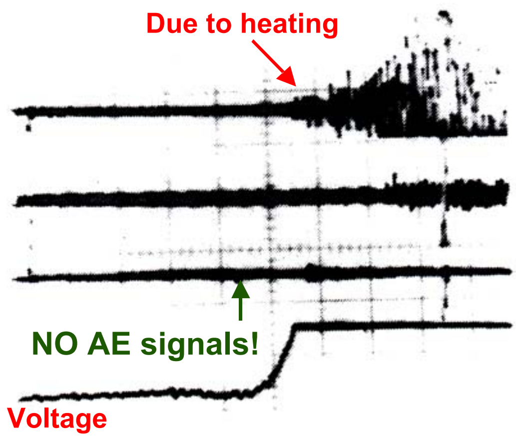
A Conductor-Motion Induced Quench — "Isabella" Dipole



O. Tsukamoto, M.F. Steinhoff, Y. Iwasa (IEEE, 1981)

Conductor-Motion Induced & Critical-Current Quenches — An SSC Dipole





I_q=6510 A

I_c=6828 A

S. Ige (MIT ME Dept. Ph.D. Thesis, 1989)

Conductor Motion

- **Frictional heating energy release, e_f, due to a Lorentz-force induced conductor motion ("slip") of Δr_f:**

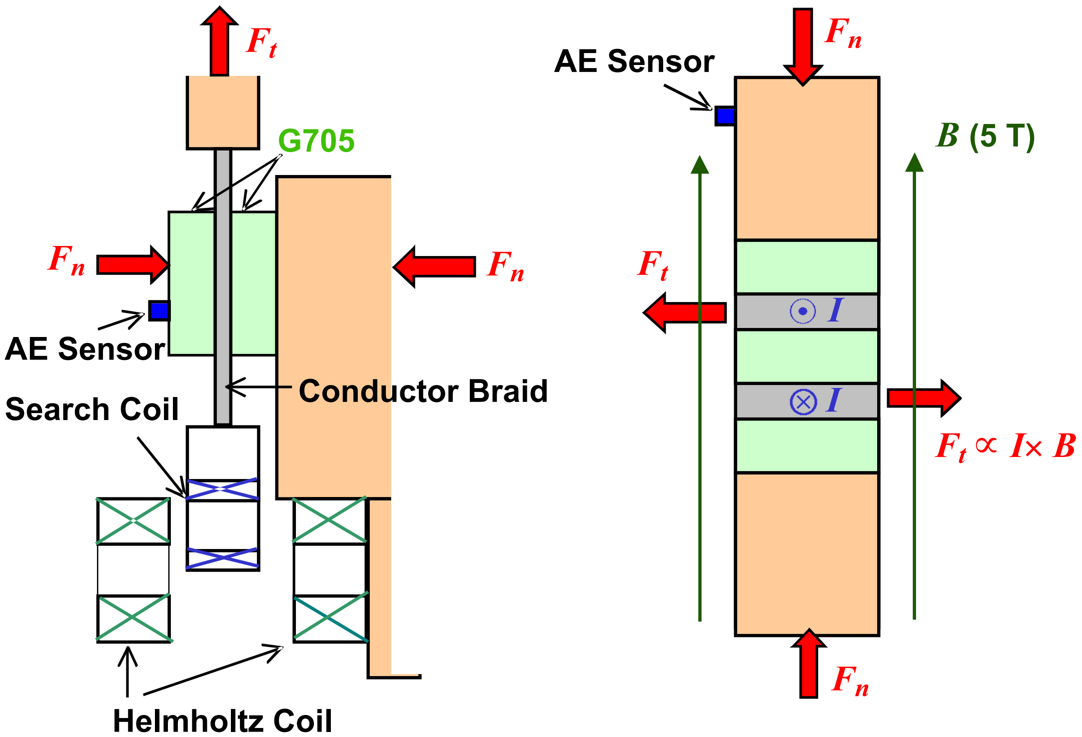
$$e_f = \mu_f f_L \Delta r_f = \mu_f J_\theta B_z \Delta r_f$$

With $\mu_f = 0.3, J_\theta = 200 \times 10^6\ \mathrm{A/m^2}$, $B_z = 5\mathrm{T}$, and $e_f = h_{cu}(5.2\mathrm{K}) - h_{cu}(4.2\mathrm{K})$ $= 1300\mathrm{J/m^3}$:

$$\Delta r_f = \frac{e_f}{\mu_f J_\theta B_z} = \frac{(1300\mathrm{J/m^3})}{(0.3)(200 \times 10^6\ \mathrm{A/m^2})(5\mathrm{T})} \cong 20\ \mu\mathrm{m}$$

- **Actually a conductor slip as small as ~1 μm ("microslip") can drive a short length of the conductor to the normal state, leading to a premature quench.**

Friction/Quench Experiment

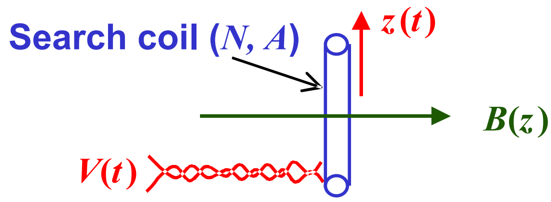


Based on **O. Tsukamoto, H.Maeda, Y. Iwasa (*Appl. Phys. Lett.* Vol. 40, 1982)**

Slip Distance from Extensionmeter

$$V(t) = \frac{d\phi}{dt} = \frac{dNAB}{dt} = NA\frac{dB}{dz}\frac{dz}{dt}$$

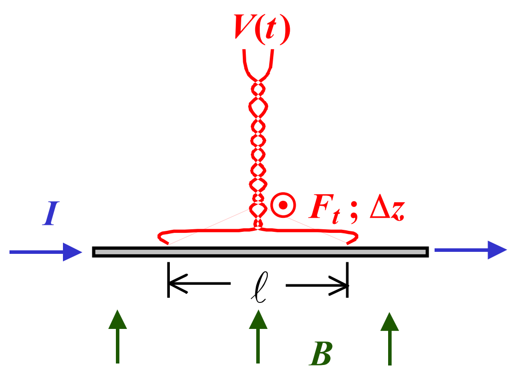
$$\Delta z = \frac{\left[\int V(t)\,dt\right]_{mea}}{\left[NA\dfrac{dB}{dz}\right]_{kwn}}$$



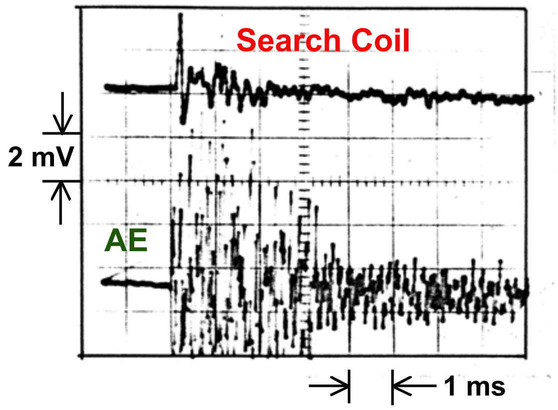
Slip Distance from Voltage Pulse

$$V(t) = \frac{d\phi}{dt} = \frac{d(\ell zB)}{dt} = \ell B\frac{dz}{dt}$$

$$\Delta z = \frac{\left[\int V(t)\,dt\right]_{mea}}{\left[\ell B\right]_{kwn}}$$



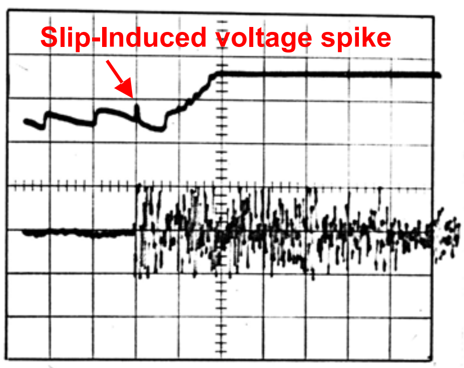
Friction Experiment | *Quench Experiment*



Slip distance: ~1μm

Peak slip velocity: ~ 1 cm/s

F_n=2000 N; F_f~400 N



Slip distance: ~1μm

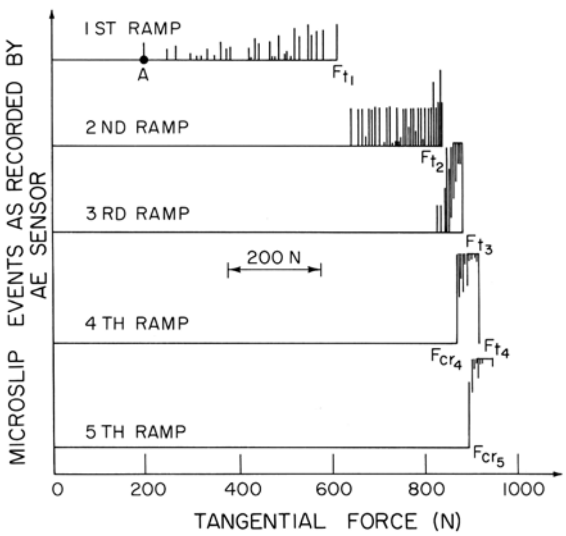
Peak slip velocity: ~ 3 cm/s

F_n=2000 N; F_f=500 N

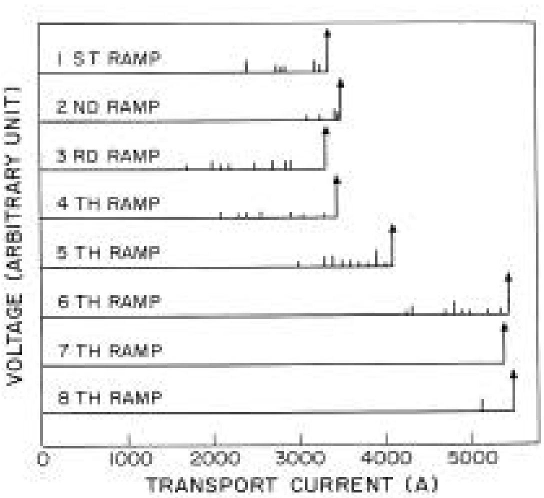
[=(0.025 m)(4000 A)(5 T)]

Observation of Kaiser Effect

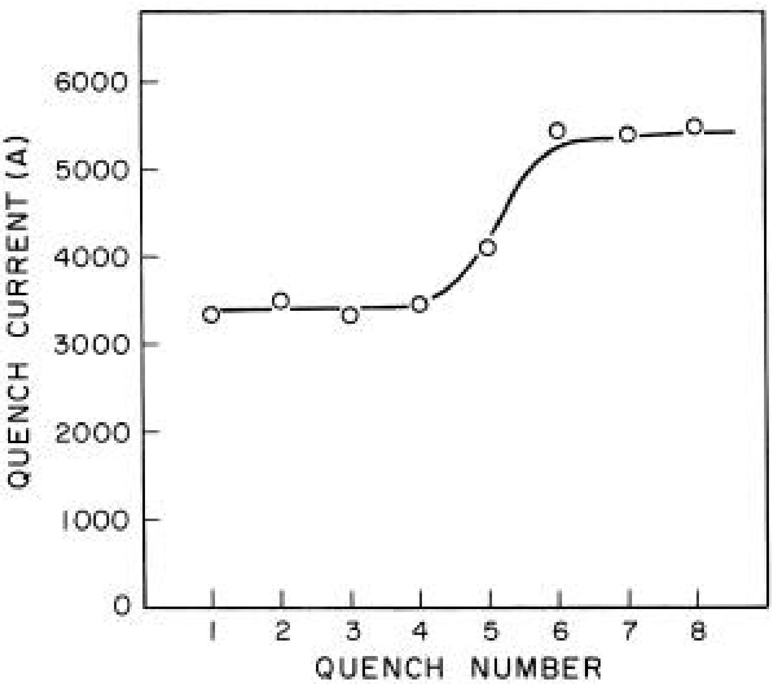
Friction Experiment



Quench Experiment



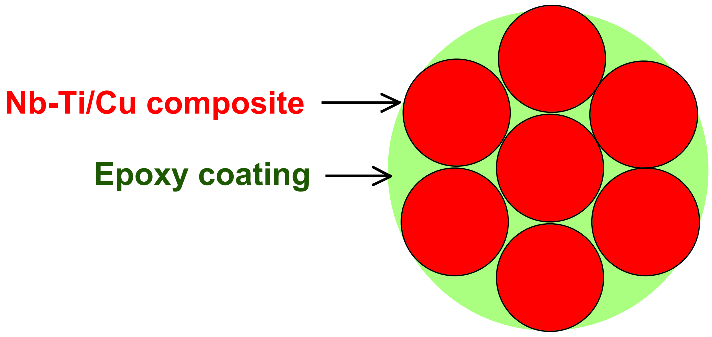
H. Maeda, O. Tsukamoto, Y. Iwasa (*Cryogenics* Vol. 22, 1982)


6000
5000
4000
3000
2000
1000
0
QUENCH CURRENT (A)
1
2
3
4
5
6
7
8
QUENCH NUMBER

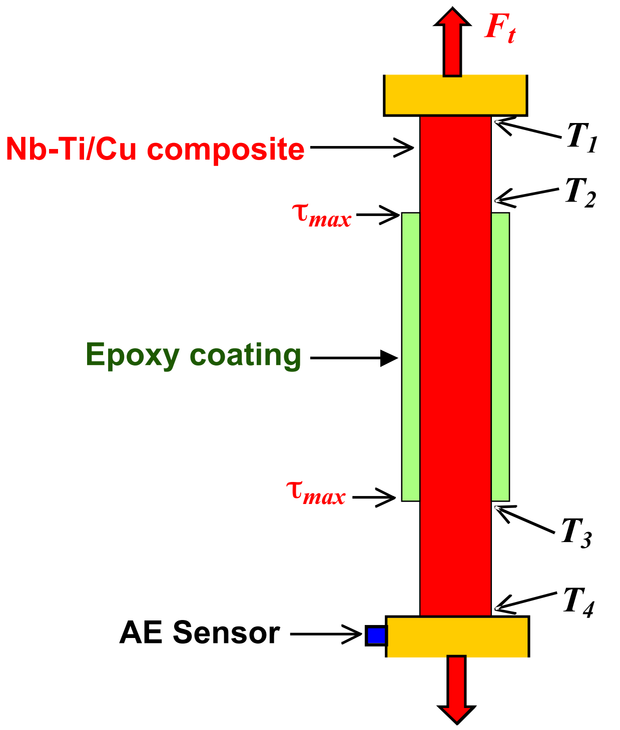
Epoxy Cracking Inducted Heating

Stored elastic energy density, $e_{el,}$ typical epoxy resin:

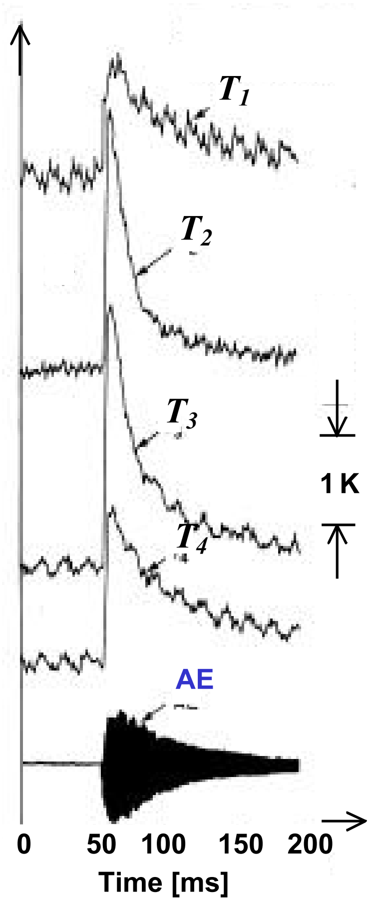
$$e_{el} = \frac{\sigma_{el}^2}{2E_{el}} \approx \frac{(15\times10^6\,\mathrm{Pa})^2}{2(10\mathrm{x}10^9)^2} \cong 1100\,\mathrm{J/m^3} \approx e_f$$



Epoxy Cracking Inducted Heating — Experimental Results

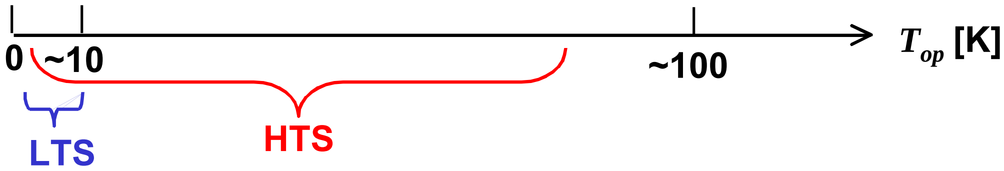


from Y. Yasaka, Y. Iwasa (*Cryogenics* Vol. 24, 1984)



Stability of HTS

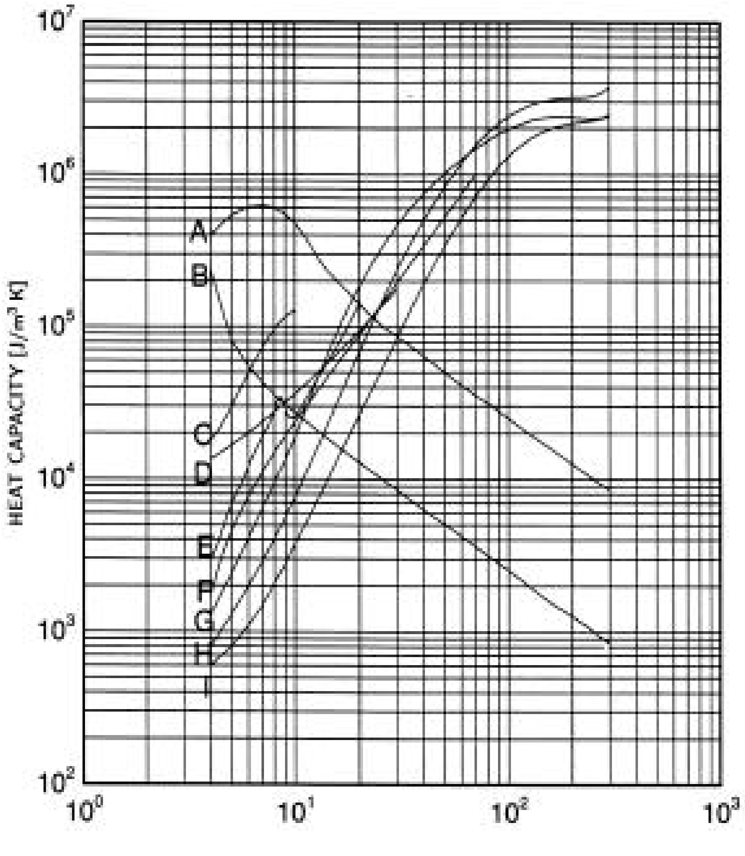
Operating Temperature Ranges



$$[T_{cs} - T_{op}]_{HTS} > \sim 10\times\, [T_{cs} - T_{op}]_{LTS}$$

Heat Capacity Data

$$\frac{[C_p(T_{op}\text{Range})]_{HTS}}{[C_p(T_{op}\text{Range})]_{LTS}} >> 1$$

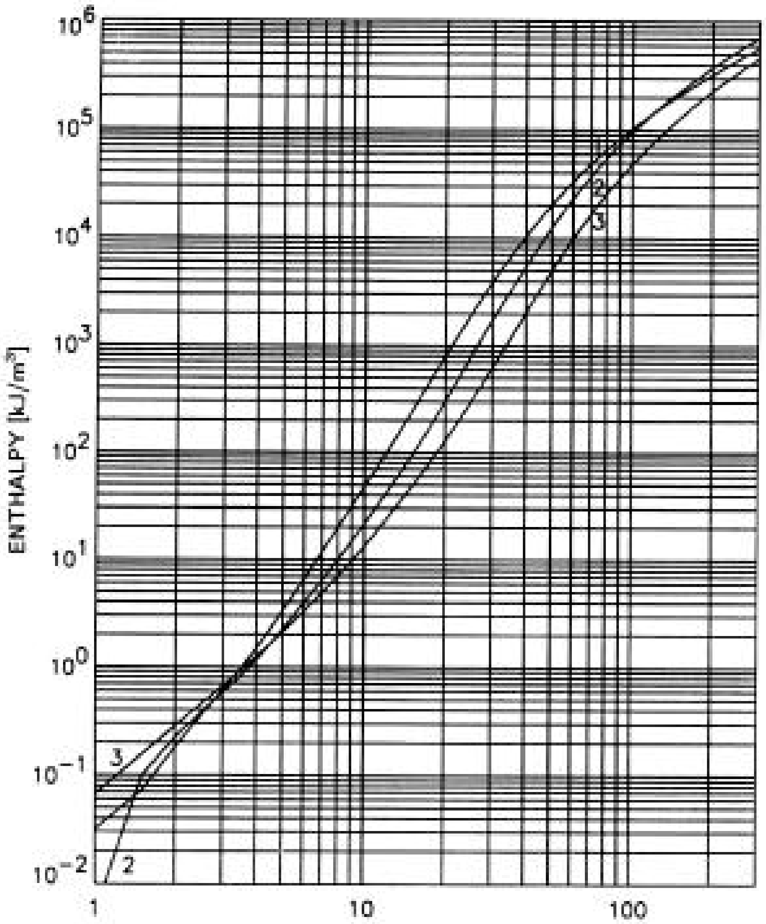


A. **Helium (10 atm)**
B. **Helium (1 atm)**
C. **GE varnish**
D. **Stainless steel**
E. **Nb-Ti**
F. **Epoxy**
G. **Silver**
H. **Copper**
I. **Aluminum**

Enthalpy Data

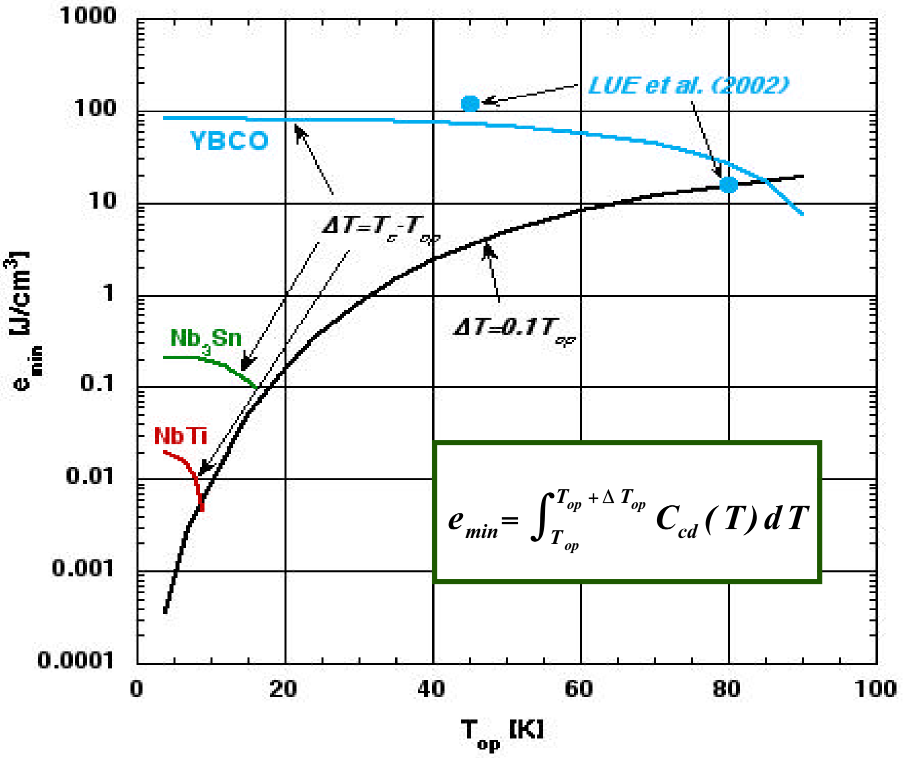
$$\frac{[e_{min}]_{HTS}}{[e_{min}]_{LTS}} >> 1$$

1. Silver
2. Copper
3. Aluminum



Minimum Thermal Energy Density

— $C_p(T)$: copper; T_c: Superconductor—



HTS Stability >> LTS Stability