

Lecture Notes: Alternative Methods for Evaluating Individual Projects1. Introduction

The basic question: Given an individual project, for which the cash flows are known, should we accept or reject it?

There are four methods in common use:

- (a) the present worth (PW) method
- (a) the annual worth (AW) method
- (a) the future worth (FW) method
- (a) the internal rate of return (IRR) method

(a) – (c) are analytically equivalent: each will lead to the same conclusion if properly applied. The IRR method is not equivalent to the others, and can lead to complications and even wrong conclusions. But because it is in widespread use we need to understand it.

2. Assumptions

- a. complete certainty about future cash flows
- a. analysis is in constant dollars
- a. cost of capital is known
- a. capital is always available for profitable projects (i.e., capital is not restricted)

2. PW Method

For any project generating a series of revenue streams, r_n , and expenditures, c_n , the NPV can readily be calculated:

$$NPV = \sum_{n=1}^N \frac{r_n - c_n}{(1+i)^n}$$

Decision criterion: Accept if $NPV > 0$; reject if $NPV < 0$

2. FW Method

$$FV = \sum_{n=0}^N (r_n - c_n)(1+i)^{N-n}$$

Decision criterion (as before): Accept if $FV > 0$, etc.

2. Example (illustrating the applicability of these methods to financial as well as real investments)

Consider a bond for which:

Z = face, or par value

C = redemption or disposal price (usually equal to Z)

r = bond interest rate per period

N = # of periods before redemption

i = bond yield rate per period

$V^{(N)}$ = value (price) of bond N interest rates prior to redemption

Then,

$$\text{PW of bond} = V^{(N)} = C (P/F, i, N) + rZ(P/A, i, N)$$

2. Annual Worth (AW) Method

This is sometimes preferable to the PW or FW method.

Example: An investment company is considering building a 25-unit apartment complex in a growing town. Because of the long-term growth potential of the town, it is felt that the company could average 90% of full occupancy for the complex each year. If the following items are reasonably accurate estimates, use the AW method to determine the minimum monthly rent that should be charged if a 12% rate of return per year is desired.

Land investment cost = \$50,000

Building investment cost = \$225,000

Study period, N = 20 years

Rent per unit per month = ???

Upkeep expense per unit per month = \$35

Property taxes and insurance per year = 10% of total initial investment

Assume: Land cost can be recovered at the end of the 20 year period

Solution: First determine the equivalent AW of all costs at an interest rate of 12%/yr. To earn 12% on this project, the annual rental income must equal the AW of the costs:

Initial investment cost = \$50,000 + \$225,000

Taxes and insurance/yr = $0.1 \times 275000 = \$27,500$

Upkeep/yr = $\$35 (12 \times 25)(0.9) = \9450

Annual worth of capital costs = $\$275,000 (A/P, 0.12, 20) - \$50,000 (A/F, 0.12, 20)$
= \$36, 123

Equivalent annual worth of costs = $-\$27000 - \$9450 - \$36123 = -\73073

Therefore, the minimum annual rental required equals \$73,073 to achieve a 12% rate of return, and with annual compounding, the monthly rental amount is given by:

$$\frac{73,073}{(12 \times 25)(0.9)} = \$270.64$$

2. Internal Rate of Return (IRR) Method

This is one of the most widely used methods in engineering economics.

The IRR is defined as the rate of interest at which the NPV = 0

For a project with net cash flows, F_j the IRR, i^* , is given by

$$PV(i^*) = \sum_{j=0}^N \frac{F_j}{(1+i^*)^j} = 0$$

Decision criterion:

If the minimum required rate of return $< i^*$, accept the project.
If the minimum required rate of return $> i^*$, reject the project.

The equation for the IRR is an N^{th} order polynomial in i^* . There will in general be more than one root. If more than one of the roots is real and positive, how do we interpret the results?

Question: When is there a unique solution to the IRR problem?

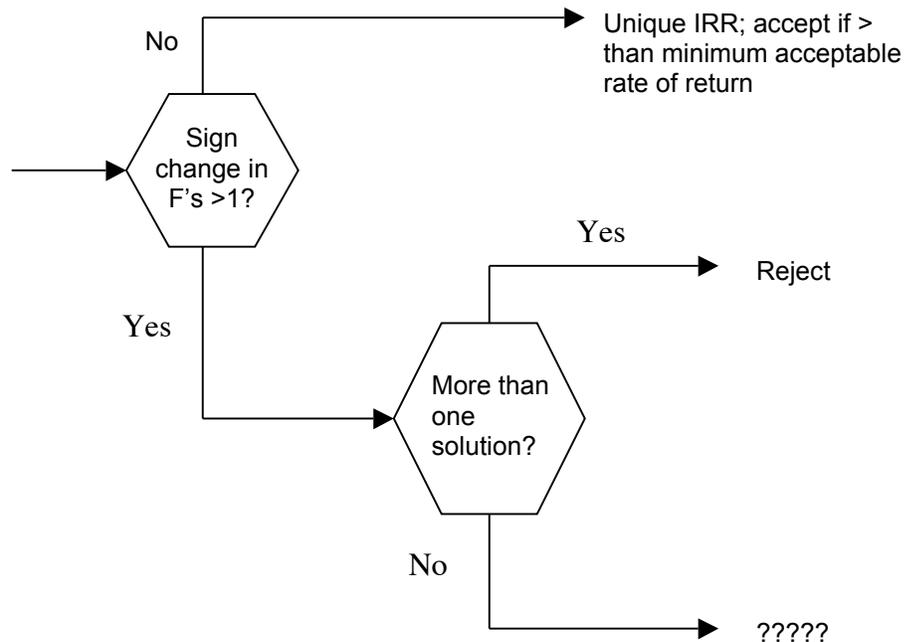
Descartes' Rule of Signs:

For an N-th degree polynomial with real coefficients, the number of real, positive roots is never greater than the number of sign changes in the sequence of coefficients.

If we write $1/(1+i^*) = X$, we can rewrite the IRR equation as

$$F_0 + F_1X + F_2X^2 + \dots + F_N X^N = 0$$

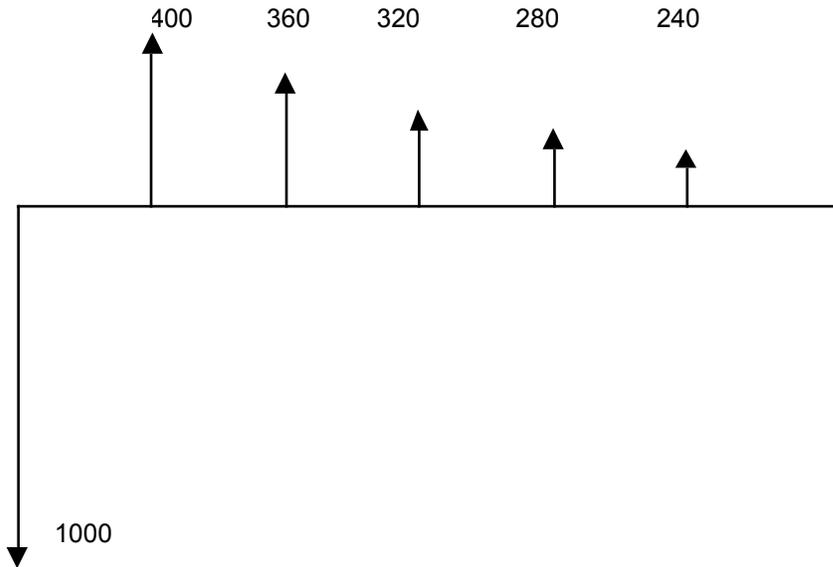
So if the sequence of net cash flows changes sign only once, there will only be one real, positive solution for the IRR. This will be the case for a project that begins with cash outflows, and ends with cash inflows. On the other hand, if there is more than one sign change in the sequence of cash flows, there may be more than one positive root. Moreover, even if there is only one real positive root, the result may not be meaningful.



To understand what lies behind these results, it is necessary to introduce the concept of the project balance, $PB(t)$, the amount of money tied up in, or committed to, the project at a given point in time.

Another way to think about PB is as the amount of money that the owner would have to 'take out' of the project to walk away from it and be indifferent. It is also conceptually equivalent to the quantity V_j , the outstanding or residual investment, that we introduced in the derivation of the modified (after tax) cash flow problem, and to the 'principal remaining' in the levelization spreadsheets.

Example:



Assume MARR = 10%/yr

$$PB(10\%)_0 = -1000$$

$$PB(10\%)_1 = -1000(1 + 0.1) + 400$$

$$PB(10\%)_2 = -1100(1 + 0.1) + 400(1 + 0.1) + 360$$

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In general,

$$PB(i\%)_n = F_0(1+i)^n + F_1(1+i)^{n-1} + \dots + F_n$$

And

$$PB(i\%)_N = FV(i\%)$$

For $i = 10\%/yr$, $PB(10\%)_5 = +241.84$

In general, if $PB(i)_N > 0$, the project recovers its initial investment + the 'interest owed' (or the opportunity cost of invested capital) and makes additional profit.

If $PB(i)_N = 0$, the project makes just enough to pay back the initial investment and the interest owed. The value of i at which this occurs is the IRR.

Next, we can make an important distinction:

Projects for which $PB(i^*)_n \leq 0$ for all $n < N$ □

“PURE INVESTMENT” PROJECTS

Projects for which $PB(i^*)_n > 0$ for some n □

“MIXED INVESTMENT” PROJECTS

For pure investment projects, the owner is always a 'lender' to the project, and the IRR can be interpreted as the interest rate earned on the committed project balance of the investment, i.e., the 'internal earning rate' of the project.

For mixed investment projects, the owner acts as a 'borrower' to the project at certain times (i.e., when $PB > 0$). During these periods, the owner in effect takes a 'loan' out of the project. The overall return on the project will therefore depend on the external interest rate that can be earned on the surplus, and so the 'internal rate of return' cannot be specified in this case.

The IRR method is only meaningful for pure investment projects.

3. Benefit-cost ratio method

The aggregate benefit-cost ratio for a project can be defined as:

$$R = \frac{B}{C} = \frac{\text{present worth of cash inflows}}{\text{present worth of cash outflows}} = \frac{\sum_{n=0}^N r_n (1+i)^{-n}}{\sum_{n=0}^N c_n (1+i)^{-n}}$$

and to accept a project, $R > 1$. It is easy to show that this is equivalent to the PV criterion.

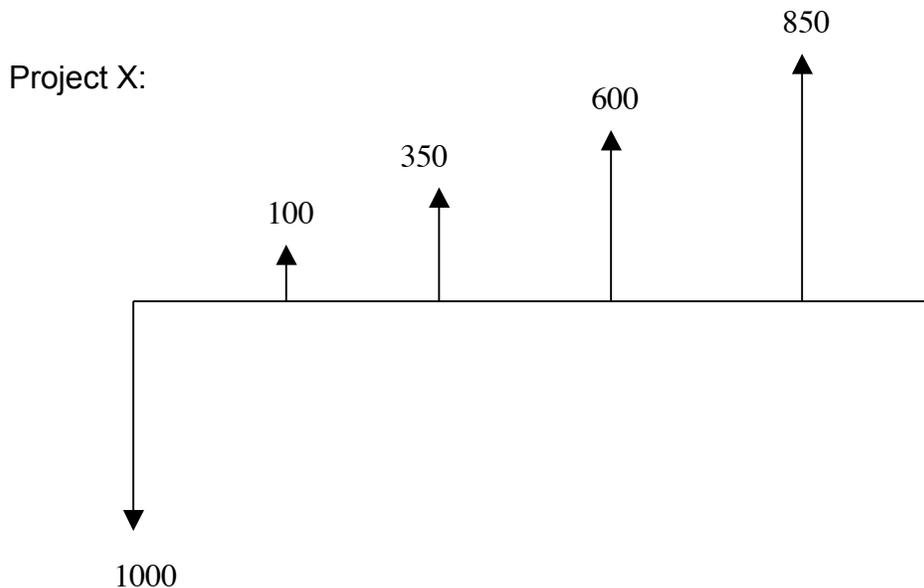
4. Consistency of IRR with other methods

In general, the results of the PV/AW/FW/BC evaluations will be consistent with IRR calculations when the latter aren't complicated by the multiple root problem (i.e., for pure investment problems).

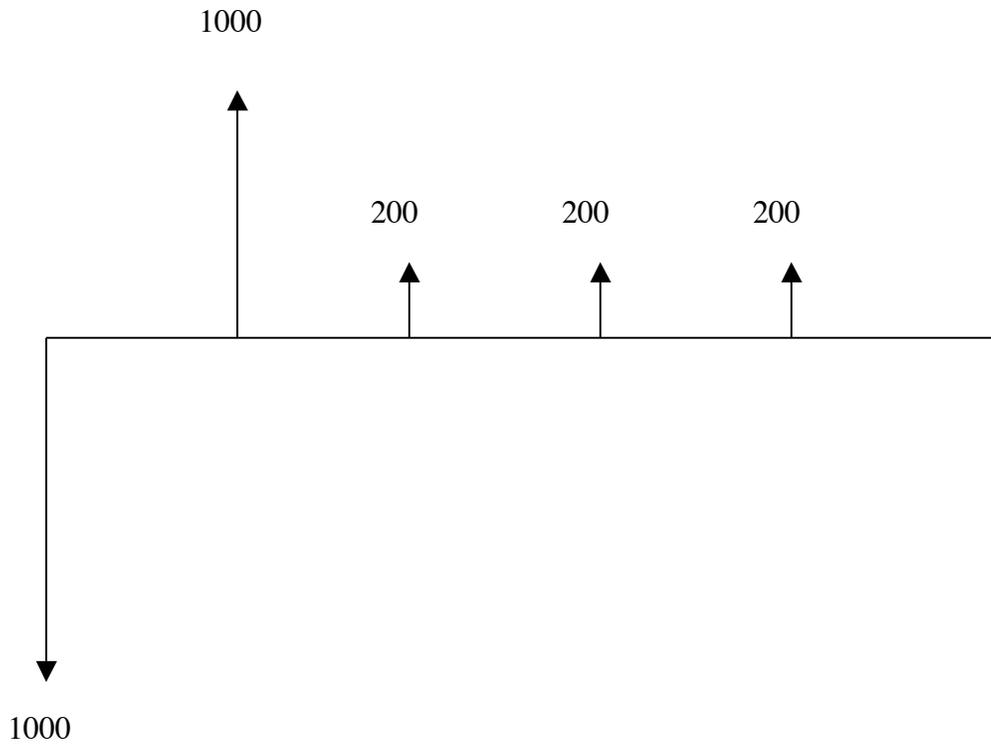
However, there are situations in which, even for pure investment problems, the IRR approach and the PV approach will lead to apparently contradictory results.

Example (from Riggs and West, p. 134):

Suppose we have two projects, X and Y, and we are trying to decide between them:



Project Y:



When MARR = 10%

$$PV(10\%)_X = -1000 + 100 (P/A, 10\%, 4) + 250 (P/G, 10\%, 4) = \$411.56$$

$$PV(10\%)_Y = -1000 + 800 (P/F, 10\%, 1) + 200 (P/A, 10\%, 4) = \$361.27$$

Hence, application of PW method leads to a decision in favor of Project X.

Using IRR method:

$$\text{For Project X: } -1000 + 100 (P/A, i^*, 4) + 250 (P/G, i^*, 4) = 0$$

Solution (obtained by iteration): $i^* = 23.4\%$

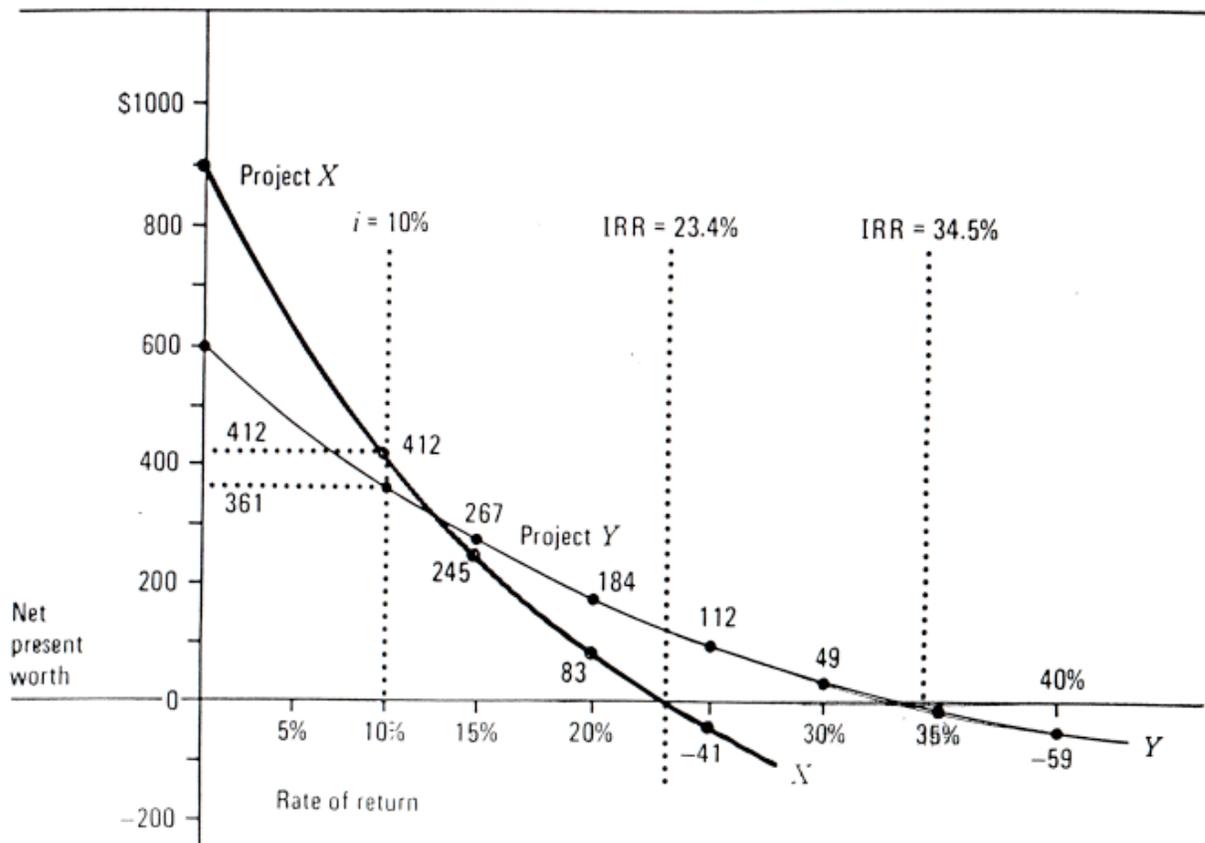
$$\text{For Project Y: } -1000 + 800 (P/F, i^*, 1) + 200 (P/A, i^*, 4) = 0$$

Solution: $i^* = 34.5\%$

Hence, application of IRR method leads to a decision in favor of Project Y!

How to explain this apparent contradiction?

The following chart shows how the NPVs of the two projects vary as a function of the interest rate.



We conclude that:

If $MARR < 13\%$, we should choose Project X

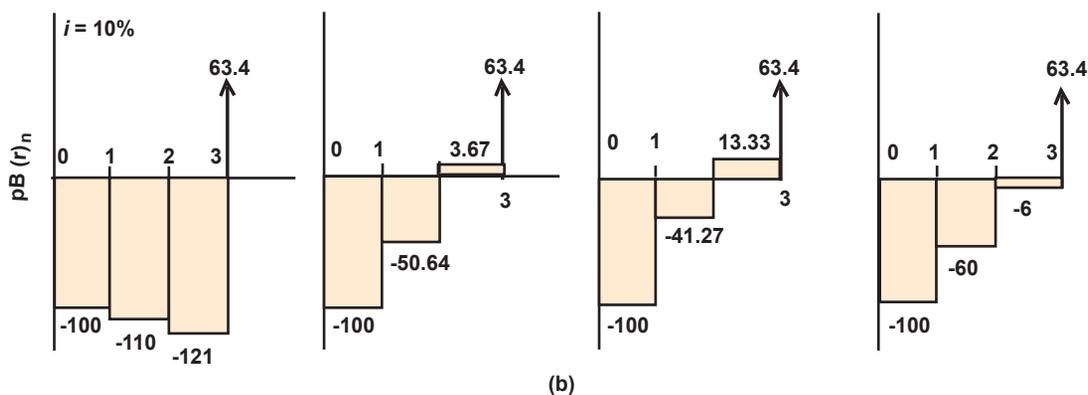
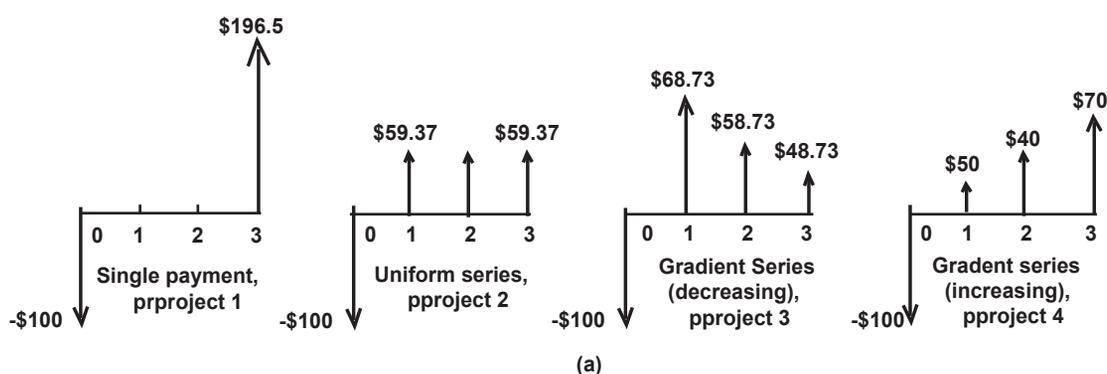
If $13\% < MARR < 35\%$, we should choose Project Y

If $MARR > 35\%$, we should choose neither.

Conclusion: The IRR method is more complex and more easily misinterpreted than the other methods. If the cost of money is known, it isn't necessary to use it. Either the PV or the AW method can be used to reach the right decision in this case, and they are both more straightforward than the IRR method.

5. Investment Flexibility

All of the traditional measures of investment worth (e.g., PW, AW, IRR) fall short in one important respect. To see this, consider the following four projects in the figure (see p. 234 of PS&B):



Project balance for four cash flow patterns.

At an MARR of 10%, each of them has the same PV (\$47.63), implying that none of the projects is preferable to the others.

But when we plot the project balances we obtain some additional information:

- Project 1 takes 3 years to recover the initial investment
- Project 3 recovers its initial investment within 2 years.

Thus, by choosing project 3 over project 1, the investor can be assured of being restored to his initial position within a shorter time span. Thus, if there is any uncertainty about either the projects at hand or about future investment opportunities, project 3 may be preferable.

A possible decision criterion? Minimize the 'payback time' – the time taken to recover the initial investment.

Sometimes this is used on its own to evaluate investment opportunities. For example: "We only undertake investments with payback periods of less than 3 years."

But such a criterion has obvious deficiencies:

1. □ In its crudest form, it fails to consider the time value of money!
2. □ It takes no account of what happens after the payback time.

6. □ Summary

1. □ The PV, FW, and AW criteria always yield the same decision for a project
2. □ Only for pure investment projects is there a true IRR for the project.
3. □ For pure investments, the IRR and PV criteria yield identical acceptance/rejection decisions.
4. □ For mixed investments, the return on invested capital varies with the external cost of capital, and the IRR criterion isn't meaningful. (The phenomenon of multiple IRRs can occur only with mixed investments, but even if there is only a single positive solution, it doesn't necessarily provide useful information.)
5. □ The aggregate B/C ratio criterion will always agree with the PV criterion.
6. □ The payback period is not an acceptable criterion taken on its own. In general it will not agree with the PV criterion. However, it may serve a useful purpose as a supplementary consideration.