

Problem Set 4 Solutions

1. State income taxes are an allowable deduction for the purposes of computing federal taxable income. However, the converse is not always true. In class we derived an expression for the composite (federal and state) income tax rate for the case when federal income taxes are not deductible in computing state taxes. Show that when federal income taxes are deductible, the composite tax rate, τ , is given by

$$\tau = \frac{(\tau_F + \tau_S - 2\tau_F\tau_S)}{1 - \tau_F\tau_S}$$

where τ_F = federal income tax rate and τ_S = state income tax rate.

Solution:

To calculate the taxes, one takes the operating revenue less the operating expenses, depreciation allowance, and interest payments. The federal and state income tax figures are denoted by subscripts F and S respectively. Since the income tax is also deductible, we can write the federal and state income tax payments as follows. The term "A" denotes the taxable income if taxes were not deductible.

$$T_F = \tau_F [OR - OX - D - IP - T_S] = \tau_F [F - D - IP - T_S] = \tau_F [A - T_S]$$

$$T_S = \tau_S [OR - OX - D - IP - T_F] = \tau_S [F - D - IP - T_F] = \tau_S [A - T_F]$$

We can calculate an effective composite tax rate according to the following equation

$$T = T_S + T_F = \tau[A]$$

Substituting terms from the first two expressions

$$T_F = \tau_F [A - \tau_S (A - T_F)] = \tau_F A - \tau_S \tau_F A + \tau_S \tau_F T_F$$

$$T_F (1 - \tau_S \tau_F) = A(\tau_F - \tau_S \tau_F)$$

$$T_F = \frac{\tau_F - \tau_S \tau_F}{1 - \tau_S \tau_F} A$$

Analogously

$$T_S = \tau_S [A - \tau_F (A - T_S)] = \tau_S A - \tau_S \tau_F A + \tau_S \tau_F T_S$$

$$T_S (1 - \tau_S \tau_F) = A(\tau_S - \tau_S \tau_F)$$

$$T_S = \frac{\tau_S - \tau_S \tau_F}{1 - \tau_S \tau_F} A$$

Adding the taxes we find that

$$T_S + T_F = \left[\frac{\tau_S - \tau_S \tau_F}{1 - \tau_S \tau_F} + \frac{\tau_F - \tau_S \tau_F}{1 - \tau_S \tau_F} \right] A$$

Hence, we know that the effective composite tax rate is given by

$$\tau_{eff} = \left[\frac{\tau_S - \tau_S \tau_F}{1 - \tau_S \tau_F} + \frac{\tau_F - \tau_S \tau_F}{1 - \tau_S \tau_F} \right] = \frac{\tau_S + \tau_F - 2\tau_S \tau_F}{1 - \tau_S \tau_F}$$

QED

2. In class we discussed the problem in the classnote, "An Illustration of the Use of Depreciation", for the case in which the owner of the pizza delivery business took out a loan of \$4000 to purchase the car. In this problem, assume that the owner again takes out a \$4000 loan at a 10% interest rate, but that this time the debt is repaid in 4 equal annual installments, and that the SYD depreciation method is used to determine income taxes. Compare the net cash flows of the business in this case with those computed in class.

Solution:

First, we must calculate the four equal annual payments, we can use the A/P factor to compute the equal payments based on the 10% interest rate.

$$A = P \left[\frac{A}{P}, i, N \right] = P \left[\frac{i(1+i)^N}{(1+i)^N - 1} \right] = \$4,000 \left[\frac{0.10(1+0.10)^4}{(1+0.10)^4 - 1} \right] = \$1,261.88$$

The second step to creating the cash flow diagram is to calculate the depreciation allowance in each year.

$$D_n = \frac{N - n + 1}{N(N + 1)/2} (I_0 - I_N)$$

$$D_1 = \frac{4 - 1 + 1}{4(4 + 1)/2} (\$6,000) = \$2,400$$

$$D_2 = \frac{4 - 2 + 1}{4(4 + 1)/2} (\$6,000) = \$1,800$$

$$D_3 = \frac{4 - 3 + 1}{4(4 + 1)/2} (\$6,000) = \$1,200$$

$$D_4 = \frac{4 - 4 + 1}{4(4 + 1)/2} (\$6,000) = \$600$$

We assume that the remaining cost of the car (\$2,000) came out of the owner's pocket at the beginning of the first year.

The results are summarized in the table below

| | Year 1 | Year 2 | Year 3 | Year 4 |
|---------------------------|-------------------|-------------------|-------------------|-------------------|
| Operating Revenue | \$20,000.00 | \$20,000.00 | \$20,000.00 | \$20,000.00 |
| Operating Expenses | (\$10,000.00) | (\$10,000.00) | (\$10,000.00) | (\$10,000.00) |
| Operating Income | \$10,000.00 | \$10,000.00 | \$10,000.00 | \$10,000.00 |
| Depreciation Allowance | \$2,400.00 | \$1,800.00 | \$1,200.00 | \$600.00 |
| Bank Payment | (\$1,261.88) | (\$1,261.88) | (\$1,261.88) | (\$1,261.88) |
| Payment towards interest | (\$400.00) | (\$313.81) | (\$219.01) | (\$114.72) |
| Payment towards principal | (\$861.88) | (\$948.07) | (\$1,042.87) | (\$1,147.16) |
| Remaining Principal | \$3,138.12 | \$2,190.05 | \$1,147.18 | \$0.01 |
| Taxable Income | \$7,200.00 | \$7,886.19 | \$8,580.99 | \$9,285.28 |
| Taxes | (\$2,160.00) | (\$2,365.86) | (\$2,574.30) | (\$2,785.58) |
| Net Income | \$5,040.00 | \$5,520.33 | \$6,006.70 | \$6,499.70 |
| other: | | | | |
| loan | \$4,000.00 | | | |
| car purchase | (\$6,000.00) | | | |
| | | | | |
| <i>Net Cash Flow</i> | <i>\$4,578.12</i> | <i>\$6,372.26</i> | <i>\$6,163.82</i> | <i>\$5,952.54</i> |
| <i>class example</i> | <i>\$4,170.00</i> | <i>\$6,240.00</i> | <i>\$6,310.00</i> | <i>\$6,380.00</i> |

Note: the light shaded boxes indicate cash flows that occur at the beginning of the year.

We can see from using the sum-of-year digits (SYD) depreciation, that the net cash flow is increased in the beginning of the business relative to the SLD method, this is because the depreciation is accelerated, and thus fewer taxes are paid at the beginning of the business. In the first two years the tax payments are substantially reduced compared to the SLD and equal principal repayment approach. However, taxes in the last two years are substantially higher, thus the NCF in the latter years shows an inverted trend. However, the SYD method is advantageous. If one using F/P to calculate the future worth of the NCF for each example, one finds that the total future worth of the NCF is \$29,190.32 for the SYD method and \$29,063.84 for the SLD method for a discount rate of 10%.

3. Consider a corporation for which the marginal income tax rate is 50%. Five years ago the firm acquired a property at a cost of \$100,000. The property was originally estimated to have a life of 10 years and zero net salvage. Tax law requires that the 'gain on disposal' for this property be taxed as ordinary income.
- Suppose the property is sold today for \$58,000. Find the tax on 'gain on disposal' if the depreciation method used was straight-line.
 - Repeat for the case of SYD.
 - Since the tax on 'gain on disposal' was higher when the SYD method is used, does this mean that the method is less advisable for such circumstances?

Solution

(a) First we compute the depreciation allowance for each year using the SLD method, and then after 5 years we are able to calculate the book value of the property

$$D = \frac{I_0 - I_N}{N} = \frac{\$100,000}{10} = \$10,000$$

The book value of the property is therefore

$$BV = I_0 - \sum_{i=1}^5 D_i = \$100,000 - 5(\$10,000) = \$50,000$$

When the property is sold, therefore, the company earns a gain on disposal (GD) of the difference between the sales price (SP) and the book value (BV).

$$GD = SP - BV = \$58,000 - \$50,000 = \$8,000$$

The tax payment (T) on the gain on disposal is therefore

$$T = \tau GD = (\$8,000)0.50 = \$4,000$$

(b) for the case where the SYD method is used, we must calculate the depreciation allowance in each year, and it will be different in each year.

$$D_n = \frac{N - n + 1}{N(N + 1) / 2} (I_0 - I_N)$$

This equation was used to calculate the depreciation allowance in each year and the results are summarized in the following table.

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------|-------------|-------------|-------------|-------------|-------------|------------|------------|------------|------------|------------|
| D | \$18,181.82 | \$16,363.64 | \$14,545.45 | \$12,727.27 | \$10,909.09 | \$9,090.91 | \$7,272.73 | \$5,454.55 | \$3,636.36 | \$1,818.18 |

Now we calculate the book value of the property

$$BV = I_0 - \sum_{i=1}^5 D_i = \$100,000 - \$72,727.27 = \$27,272.73$$

Therefore

$$GD = SP - BV = \$58,000 - \$27,272.73 = \$30,727.27$$

Therefore

$$T = \tau GD = (\$30,727.27)0.50 = \$15,363.64$$

(c) One would initially suspect that the SYD method is not advisable, however, taking the depreciation allowance in each year, and multiplying by the tax rate gives the value in current dollars that the company did not have to pay in taxes in each of those years. Even if we assume a 0% discount rate, and take the worth of these tax savings in the fifth year, we find that the SYD method saves the company, relative to the SLD method approximately \$11,000. This is also the difference between the taxes on gains-on-disposal at the point where the property is sold. However, we all know that the discount rate for calculating the time value of money is never 0%, therefore, if the company were to use SYD depreciation they would be better off. So long as the discount rate is greater than 0%, the SYD method is advisable relative to the SLD method.

4. A firm has an office copying/scanning/fax machine that was accidentally damaged. The machine was purchased a year ago for \$600. The expected operating life was 6 years, and a straight-line tax depreciation scheme was used. The firm is told by the copying machine salesman that the damaged machine will cost \$200 to repair. Alternatively, the salesman offers to buy back the machine for \$100 and to make available a replacement at annual rental rate of \$300. The salesman presents the following cost comparison for the current year and recommends on the basis of this that the firm adopt the rental option.

| | <u>Cost of Owning</u> | | <u>Cost of Renting</u> |
|------------------------------|-----------------------|---------------------|------------------------|
| Depreciation | \$100 | Rental cost | \$300 |
| Repairs | \$200 | Less trade-in value | -\$100 |
| Property taxes and insurance | \$25 | | |
| TOTAL | <u>\$325</u> | | <u>\$200</u> |

Do you think this is an appropriate way for the firm to look at the two options? Should the firm retain ownership of the machine, or trade it in for a rental? (Assume that property tax and insurance are treated as deductible expenses for the purposes of computing income taxes, and that the firm's discount rate is 10%.)

Solution

It is never valid to compare two alternatives based on what is happening in one year, a comprehensive cash flow analysis must be done to see which option is better. Since the property tax and insurance are deductible in each year they affect the income taxes in each year and since the buy-back cash flow occurs only once, this is definitely not a valid basis for comparison.

Since the depreciation is \$100, we can assume that the machine has been in service for a full year, and that there are 5 years left in the operating life of the machine.

Let us create two partial income statements, one for each case for the balance of the operating life of the machine. Assuming that everything else in the business is constant, we must calculate the contribution to the expenses and taxes as a result of pursuing either course of action for the machine.

What we can calculate is the operating expenses in each year and the tax deduction associated with the operating costs of the machine and the depreciation. For the option where will sell back the machine, we must take into account that the resale value is less than the book value, and that this net difference is treated as an operating expense for income tax purposes. The difference (\$600 - \$100 - \$100) is included as part of the tax deduction

The table below summarizes these calculations.

Repair

| | 1 | 2 | 3 | 4 | 5 | | |
|-------------------------------|----------|----------|----------|----------|----------|-------|------------|
| depreciation allowance | \$100.00 | \$100.00 | \$100.00 | \$100.00 | \$100.00 | | |
| cost of repairs | \$200.00 | | | | | | |
| property tax and insurance | \$25.00 | \$25.00 | \$25.00 | \$25.00 | \$25.00 | | |
| total costs | \$225.00 | \$25.00 | \$25.00 | \$25.00 | \$25.00 | | |
| | | | | | | | |
| tax deduction | \$325.00 | \$125.00 | \$125.00 | \$125.00 | \$125.00 | | |
| | | | | | | | |
| future worth of the deduction | \$475.83 | \$166.38 | \$151.25 | \$137.50 | \$125.00 | TOTAL | \$1,055.96 |
| future worth of expenses | \$329.42 | \$33.28 | \$30.25 | \$27.50 | \$25.00 | TOTAL | \$445.45 |

Rent

| Year | 1 | 2 | 3 | 4 | 5 | | |
|-------------------------------|------------|----------|----------|----------|----------|-------|------------|
| Loss on sale (BV-SP) | \$400.00 | | | | | | |
| buy-back | \$100.00 | | | | | | |
| Costs | \$300.00 | \$300.00 | \$300.00 | \$300.00 | \$300.00 | | |
| | | | | | | | |
| tax deduction | \$700.00 | \$300.00 | \$300.00 | \$300.00 | \$300.00 | | |
| | | | | | | | |
| future worth of the deduction | \$1,024.87 | \$399.30 | \$363.00 | \$330.00 | \$300.00 | TOTAL | \$2,417.17 |
| future worth of expenses | \$439.23 | \$399.30 | \$363.00 | \$330.00 | \$300.00 | TOTAL | \$1,831.53 |
| future worth of the sale | \$146.41 | \$0.00 | \$0.00 | \$0.00 | \$0.00 | TOTAL | \$146.41 |

We cannot simply add the deductions and expenses to get an aggregate picture of each alternative, therefore, we must either use present or future worth factors to put the cash flows on equal bases for comparisons. Therefore, the future worth in year 5 of the expenses incurred in year n were calculated in the bottom two rows of the table, and these were summed together to find the total future worth of both the operating costs and the tax deduction.

Assuming a constant tax rate in each year, the value of the tax savings in each year (or avoided tax payment) is the product of the tax rate and the total future worth of the tax deductions.

The total future worth of the cost of the option selected will be the difference between the total future worth of the tax savings and the total future worth of the operating expenses. In the case of the renting, the future worth of the buy-back payment must be included.

We are not certain of the income tax rate, so several rates were compared. The table below summarizes the difference between the future worth of the tax savings less the future worth of the operating costs for each option.

| tax rate | repair | rent |
|----------|------------|--------------|
| 0.1 | (\$339.85) | (\$1,443.40) |
| 0.2 | (\$234.26) | (\$1,201.69) |
| 0.3 | (\$128.66) | (\$959.97) |
| 0.4 | (\$23.06) | (\$718.25) |
| 0.5 | \$82.53 | (\$476.54) |
| 0.6 | \$188.13 | (\$234.82) |
| 0.7 | \$293.72 | \$6.90 |

We can see that in each case for the tax rate, that the repair option is better, this is mostly because of the low operating expenses, even though in the second case much more can be deducted from taxes.

5. An entrepreneur negotiated a 5-year \$100,000 loan from his bank. The loan schedule called for equal annual payments at a rate of 9%/yr. Two years later, the bank offered to refinance the loan at 8%/yr. The refinancing would cover the balance of the original loan, and the loan term would be 3 years. Though no penalty would be incurred for paying off the first loan early, the bank would charge the entrepreneur an 'origination fee' of three 'points' (i.e., 3 percent of the loan amount) for the refinancing, payable at the beginning of the new loan term. The tax code does not permit the origination fee to be treated as an expense for tax purposes. The entrepreneur's marginal tax rate is 30%.

- (a) What is the amount of the second loan?
- (b) Should the entrepreneur accept the bank's offer to refinance? Explain your answer.

Solution

(a)
We first consider the original loan financing option and construct a partial income statement to calculate the interest payments and tax savings. The results are shown in the table below. An A/P factor was used to calculate the annual payments, light shaded boxes indicate cash flows made at the beginning of the year

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
|---------------------|--------------|-------------|-------------|-------------|-------------|
| Total Loan Payment | \$25,709.25 | \$25,709.25 | \$25,709.25 | \$25,709.25 | \$25,709.25 |
| Interest Payment | \$9,000.00 | \$7,496.17 | \$5,856.99 | \$4,070.29 | \$2,122.78 |
| Principal Repayment | \$16,709.25 | \$18,213.08 | \$19,852.25 | \$21,638.96 | \$23,586.46 |
| Principal Remaining | \$83,290.75 | \$65,077.68 | \$45,225.42 | \$23,586.46 | \$0.00 |
| Tax Deduction | \$9,000.00 | \$7,496.17 | \$5,856.99 | \$4,070.29 | \$2,122.78 |
| Tax Savings | \$2,700.00 | \$2,248.85 | \$1,757.10 | \$1,221.09 | \$636.83 |
| Initial Loan Amount | \$100,000.00 | | | | |

One can see from this table that if the entrepreneur refinances the loan after 2 years, that the amount of the loan will be the principal he has left to repay, which is \$65,077.68

(b)
If we construct a partial income statement for the second option, which is to refinance at the end of the second year and pay a fee at the beginning of the third year, the annual payments will be different, the new partial income statement is shown below.

| | Year 3 | Year 4 | Year 5 |
|---------------------|-------------|-------------|-------------|
| Total Loan Payment | \$25,252.32 | \$25,252.32 | \$25,252.32 |
| Interest Payment | \$5,206.21 | \$3,602.53 | \$1,870.54 |
| Principal Repayment | \$20,046.11 | \$21,649.79 | \$23,381.78 |
| Principal Remaining | \$45,031.57 | \$23,381.78 | \$0.00 |
| Tax Deduction | \$5,206.21 | \$3,602.53 | \$1,870.54 |
| Tax Savings | \$1,561.86 | \$1,080.76 | \$561.16 |
| Refinancing Fee | \$1,952.33 | | |

The fee to refinance the loan is $0.03 \times \$65,077.68 = \$1,952.33$.

To see which option is better, one must calculate the value of the options at the same point in time. The Beginning of Year 3 (BY3) is chosen as the reference point. One must be sure to keep track of the interest rates when discounting the amounts, one can check to make sure the method is correct by verifying that the present value of the loan and the present value of the loan payments are the same at the BY3

The present value of the first option will be the amount of the loan plus the present value of the tax savings. We assume that present values are taken at BY3.

In the first case, the BY3Vs are summarized below

| Interest Rate | 9% | 9% | 9% | 9% | 9% | TOTAL |
|-------------------|--------------|-------------|-------------|-------------|-------------|-----------------------|
| Year | 1 | 2 | 3 | 4 | 5 | |
| BY3V tax savings | \$2,943.00 | \$2,248.85 | \$1,612.02 | \$1,027.76 | \$491.75 | \$8,323.38 |
| BY3V loan payment | \$28,023.08 | \$25,709.25 | \$23,586.46 | \$21,638.96 | \$19,852.25 | (\$118,810.00) |
| BY3V principal | \$118,810.00 | | | | | \$118,810.00 |
| | | | | | | TOTAL BY3V \$8,323.38 |

The present value of the second option will be different because the fee incurred in the 3rd year must be included, and the discount rate in the 3rd, 4th, and 5th years is 8%, whereas it was 9% in preceding years. Therefore, the payments made at the end of 3rd, 4th, and 5th years are discounted at 8%, this must be done so that the BY3V of the loan payments are still equal to the principal of the loan (BY3V).

The same calculation was performed including the cost of the refinancing charge and the results are shown below, which is already valued at BY3.

| Interest Rate | 9% | 9% | 8% | 8% | 8% | TOTAL |
|-------------------|--------------|-------------|-------------|-------------|-------------|-----------------------|
| Year | 1 | 2 | 3 | 4 | 5 | |
| BY3V tax savings | \$2,943.00 | \$2,248.85 | \$1,446.17 | \$926.58 | \$445.47 | \$8,010.07 |
| BY3V loan payment | \$28,023.08 | \$25,709.25 | \$23,381.78 | \$21,649.79 | \$20,046.11 | (\$118,810.00) |
| BY3V principal | \$118,810.00 | | | | | \$118,810.00 |
| BY3V fee | | | \$1,952.33 | | | (\$1,952.33) |
| | | | | | | TOTAL BY3V \$6,057.73 |

One can see that the BY3V of the first option is higher, therefore, the entrepreneur should not refinance.

6. (P&SB, problem 4.5) An asset costing \$40,000 was installed in 1989 and depreciated using MACRS percentages for the 5-year property class. In 1991 the asset was sold. Calculate the net cash flow after taxes resulting from the sale of the asset. The company is a large, stable, profitable corporation with a marginal tax rate of 34%. The selling price of the asset was (a) \$25,000; (b) \$10,000.

Solution

The following table summarizes the recovery rates from MACRS and calculates the depreciation allowance based on the initial cost of the asset and the book value.

| Year | 1989 | 1990 | 1991 |
|------------------------|-------------|-------------|-------------|
| Initial Asset Cost | \$40,000.00 | \$40,000.00 | \$40,000.00 |
| Recovery Rate | 0.200 | 0.320 | 0.192 |
| Depreciation Allowance | \$8,000.00 | \$12,800.00 | \$7,680.00 |
| Book Value | \$32,000.00 | \$19,200.00 | \$11,520.00 |

(a)

If the asset sells for \$25,000, the company sells the asset above the book value and must therefore pay taxes on gain on disposal, the taxes the company must pay are

$$T = (\$25,000 - \$11,520)0.34 = \$4,583.20$$

The net cash flow resulting from the sale are given by the sale price less the taxes

$$NCF = \$25,000 - \$4,583.20 = \$20,416.80$$

(b)

If the asset sells for \$10,000, the company sells the asset below the book value and will therefore receive a tax credit.

The tax credit is given by the tax rate times the loss-on-disposal

$$T = (\$10,000 - \$11,520)0.34 = -\$516.80$$

The net cash flow resulting from the sale is given by the same formula as above

$$NCF = \$10,000 - -\$516.80 = \$10,516.80$$

Addendum

Use the spreadsheet we discussed in class this morning to solve the following variant of the levelized cost problem: Instead of holding the debt-to-equity ratio constant over the life of the project, assume that the loan must be retired according to a uniform repayment schedule (i.e., \$300 per year.) Calculate the levelized unit cost for this problem (a) for the case of no taxes, and (b) assuming a tax rate of 38%. (For the latter case, assume straight-line depreciation for tax purposes, as before.)

Compare your results for these two cases with the results we obtained in class for a constant debt-to-equity ratio. Does the comparison conform to your expectations?

The following changes had to be made to the spread sheets to get the right answers

- A tally must be included for the debt to be repaid and the equity to be repaid separately
- The amount that must be paid each year is based on the remaining amount in each category and the respective interest rates, so the required return formula must be corrected.
- The interest payment is based on the remaining debt to be repaid, so this formula must be changed

See attached sheets for more complete information

(a)

\$9.2104 using an iterative method for the uniform repayment schedule

\$9.196 was the result from the classroom example with no taxes

One would expect the levelized cost to increase because it is cheaper to maintain a balance in debt than in equity because of the lower rate of return. Therefore, forcing the company to repay debt earlier means that higher payments must be made on the remaining balance of the initial investment that was financed with equity, thus increasing the levelized cost.

(b)

\$9.6823 using an iterative method for the uniform repayment schedule

it was somewhat lower (\$9.634) in the classroom example

The inclusion of taxes means that the debt repayments are treated differently as the interest payments on the debt are tax deductible. The uniform repayment schedule accelerates the rate at which the debt is repaid. Keeping in mind that the return on equity is higher than the interest rate on debt, it is expected that the levelized cost would increase in this case.

The company would like to repay equity more quickly than debt because the interest rate is higher, and to compound this trend, the interest payments are tax deductible. To recap, maintaining a larger fraction of the investment capital in the form of debt principal to be repaid is beneficial because (1) the equity rate is higher and (2) maintaining a large remaining principal balance in the form of debt is not so bad because the interest payments are tax deductible, further reducing the cost of maintaining a remaining principal. Therefore, the increase in price is larger than before because of the added benefit to keeping unpaid capital in form of debt over equity.

fb= 0.6 fe= 0.4
rb= 8% re= 12.00%

Tax rate = 0%
Initial investment = \$5,000
Annual inflation rate = 3%
Operating expenses in year zero dollars = \$900

Levelized unit price in dollars = **9.2104**

| Current-year dollars | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------------------------|-------|------------|------------|------------|------------|------------|----------|----------|----------|----------|----------|
| Output (# of units) | | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 |
| Levelized unit price (\$/unit) | | 9.2104 | 9.2104 | 9.2104 | 9.2104 | 9.2104 | 9.2104 | 9.2104 | 9.2104 | 9.2104 | 9.2104 |
| Operating income (levelized) | | 1842.08 | 1842.08 | 1842.08 | 1842.08 | 1842.08 | 1842.08 | 1842.08 | 1842.08 | 1842.08 | 1842.08 |
| Operating expenses | | 927.00 | 954.81 | 983.45 | 1012.96 | 1043.35 | 1074.65 | 1106.89 | 1140.09 | 1174.30 | 1209.52 |
| Operating income | | 915.08 | 887.27 | 858.63 | 829.12 | 798.73 | 767.43 | 735.19 | 701.99 | 667.78 | 632.56 |
| Tax depreciation (SL) | | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 |
| Interest payment | | 240.00 | 216.00 | 192.00 | 168.00 | 144.00 | 120.00 | 96.00 | 72.00 | 48.00 | 24.00 |
| Taxable income | | 175.08 | 171.27 | 166.63 | 161.12 | 154.73 | 147.43 | 139.19 | 129.99 | 119.78 | 108.56 |
| Taxes | | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| After-tax cash flow | | 915.08 | 887.27 | 858.63 | 829.12 | 798.73 | 767.43 | 735.19 | 701.99 | 667.78 | 632.56 |
| Required return on debt+equity | | 480.00 | 439.79 | 398.09 | 354.83 | 309.91 | 263.26 | 214.75 | 164.30 | 111.78 | 57.06 |
| Free cash (after paying returns) | | 435.08 | 447.48 | 460.53 | 474.29 | 488.82 | 504.18 | 520.44 | 537.69 | 556.01 | 575.50 |
| Principal remaining | | 4,564.92 | 4,117.44 | 3,656.91 | 3,182.61 | 2,693.79 | 2,189.62 | 1,669.18 | 1,131.49 | 575.49 | (0.01) |
| Debt retired | | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 |
| Equity repaid | | 135.08 | 147.48 | 160.53 | 174.29 | 188.82 | 204.18 | 220.44 | 237.69 | 256.01 | 275.50 |
| Debt remaining | | 2700.00 | 2400.00 | 2100.00 | 1800.00 | 1500.00 | 1200.00 | 900.00 | 600.00 | 300.00 | 0.00 |
| Equity to be repaid | | \$1,864.92 | \$1,717.44 | \$1,556.91 | \$1,382.61 | \$1,193.79 | \$989.62 | \$769.18 | \$531.49 | \$275.49 | (\$0.01) |
| CASH FLOWS | -5000 | 915.08 | 887.27 | 858.63 | 829.12 | 798.73 | 767.43 | 735.19 | 701.99 | 667.78 | 632.56 |

fb= 0.6 fe= 0.4
rb= 8% re= 12.00%

Tax rate = 38%
Initial investment = \$5,000
Annual inflation rate = 3%
Operating expenses in year zero dollars = \$900

Levelized unit price in dollars = 9.6823

| Current-year dollars | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------------------------|-------|------------|------------|------------|------------|------------|------------|----------|----------|----------|----------|
| Output (# of units) | | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 |
| Levelized unit price (\$/unit) | | 9.6823 | 9.6823 | 9.6823 | 9.6823 | 9.6823 | 9.6823 | 9.6823 | 9.6823 | 9.6823 | 9.6823 |
| Operating income (levelized) | | 1936.46 | 1936.46 | 1936.46 | 1936.46 | 1936.46 | 1936.46 | 1936.46 | 1936.46 | 1936.46 | 1936.46 |
| Operating expenses | | 927.00 | 954.81 | 983.45 | 1012.96 | 1043.35 | 1074.65 | 1106.89 | 1140.09 | 1174.30 | 1209.52 |
| Operating income | | 1009.46 | 981.65 | 953.01 | 923.50 | 893.11 | 861.81 | 829.57 | 796.37 | 762.16 | 726.94 |
| Tax depreciation (SL) | | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 | 500.00 |
| Interest payment | | 240.00 | 216.00 | 192.00 | 168.00 | 144.00 | 120.00 | 96.00 | 72.00 | 48.00 | 24.00 |
| Taxable income | | 269.46 | 265.65 | 261.01 | 255.50 | 249.11 | 241.81 | 233.57 | 224.37 | 214.16 | 202.94 |
| Taxes | | 102.39 | 100.95 | 99.18 | 97.09 | 94.66 | 91.89 | 88.76 | 85.26 | 81.38 | 77.12 |
| After-tax cash flow | | 907.07 | 880.70 | 853.82 | 826.41 | 798.45 | 769.92 | 740.82 | 711.11 | 680.78 | 649.82 |
| Required return on debt+equity | | 480.00 | 440.75 | 399.96 | 357.49 | 313.22 | 267.00 | 218.65 | 167.99 | 114.81 | 58.89 |
| Free cash (after paying returns) | | 427.07 | 439.95 | 453.87 | 468.92 | 485.23 | 502.93 | 522.17 | 543.12 | 565.97 | 590.93 |
| Principal remaining | | 4,572.93 | 4,132.98 | 3,679.12 | 3,210.20 | 2,724.98 | 2,222.05 | 1,699.88 | 1,156.76 | 590.79 | (0.14) |
| Debt retired | | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 | 300.00 |
| Equity repaid | | 127.07 | 139.95 | 153.87 | 168.92 | 185.23 | 202.93 | 222.17 | 243.12 | 265.97 | 290.93 |
| Debt remaining | | 2700.00 | 2400.00 | 2100.00 | 1800.00 | 1500.00 | 1200.00 | 900.00 | 600.00 | 300.00 | 0.00 |
| Equity to be repaid | | \$1,872.93 | \$1,732.98 | \$1,579.12 | \$1,410.20 | \$1,224.98 | \$1,022.05 | \$799.88 | \$556.76 | \$290.79 | (\$0.14) |
| CASH FLOWS | -5000 | 907.07 | 880.70 | 853.82 | 826.41 | 798.45 | 769.92 | 740.82 | 711.11 | 680.78 | 649.82 |