# Subcritical Multiplication and Reactor Startup

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# Outline

First Lecture:

A. Subcritical Reactor Behavior

- Neutron Sources
- Source - Detector Geometry
- Neutron Life Cycle
- Subcritical Multiplication

B. Critical Operation w/o Feedback

- Prompt and Delayed Neutrons
- Reactivity
- Power-Period Relation

Second Lecture:

- Dynamic Period Equation
- Step and Ramp Reactivity Transients

C. Critical Operation w/Feedback

- Coolant Temperature (Moderator Coefficient)
- Fuel Temperature (Doppler Effect)
- Xenon
- Estimated Critical Position
- Examples of Reactor Transients
- Laboratory Exercise

# Reactor Physics and Operation

Nuclear fission reactors operate by maintaining a precise neutron balance. The reactor is in a critical condition if the number of neutrons created by the fission process equals the number that are either lost by leakage or captured within the reactor's structural materials. Neutrons produced from fission are called "fast" because they are traveling at very high speeds. Uranium-235, which is the principal fuel in most reactors absorbs very few high-energy or fast neutrons. In contrast, it will absorb, in large quantity, those neutrons that are moving slowly. Such neutrons are referred to as being "thermal." Hence, in order for the fission process to be maintained, it is necessary that the fast neutrons produced from fission be slowed down or thermalized. The need for the efficient thermalization of fast neutrons drives both the design and the operation of nuclear reactors.

# Reactor Operating Regimes

Reactor Operation is traditionally divided into three regimes. These are:

1. Subcritical – Critical

    a) Subcritical Multiplication

2. Critical – Point of Adding Heat

    a) Inhour Equation
    b) Dynamic Period Equation

3. Point of Adding Heat – Hot Operating

    a) Temperature Feedback
    b) Doppler Effect
    c) Xenon Feedback

The same physical relations describe all three regimes. However, because different terms in those relations dominate during each regime, the equations often look different. We will cover certain aspects of all three regimes with emphasis on subcritical multiplication, the dynamic period equation, and feedback mechanisms.

# Subcritical – Critical

- Crucial concepts are:
  - Need for neutron sources
  - Source-detector geometry
  - Neutron thermalization
  - Subcritical multiplication
  - '1/M' plots

# Need for Source Neutrons

- It should be possible to monitor the neutronic condition of a reactor at all times, including when shutdown.

- Nuclear instruments (power level and rate of change of power level) must be on scale prior to initiating a startup. Otherwise, the operator has no means of determining if his actions are having the intended effect.

- If a reactor core is brand new or if a reactor has been shutdown for many months, then the neutron population will be so low as to be undetectable. Small movements of the control devices can therefore result in large rates of changes – rates that are so rapid that power can rise to the level where fuel damage occurs before the nuclear safety system is capable of responding.

- The installation of a neutron source ensures safety by keeping all instruments on scale.

# Neutron Sources

1. Photo-Neutron

$$^{0}_{0}\gamma + ^{2}_{1}D \rightarrow ^{1}_{0}n + ^{1}_{1}H$$

Note: Fission products provide the gamma rays. So, the reactor must have a power history for this source to be effective. The needed fission products decay within 2-3 months.

2. Plutonium-Beryllium

$$^{239}_{94}Pu \rightarrow ^{4}_{2}He + ^{235}_{92}U$$

$$^{4}_{2}He + ^{9}_{4}Be \rightarrow ^{12}_{6}C + ^{1}_{0}n$$

CAUTION: PuBe sources are doubly encapsulated in steel. Hence, heat transfer is poor and PuBe source must not be left in a reactor if power exceeds a few hundred Watts.

# Neutron Sources (cont.)

3. Antimony-Beryllium

$$^{1}_{0}n + ^{123}_{51}Sb \rightarrow ^{124}_{51}Sb$$

$$^{124}_{51}Sb \rightarrow ^{0}_{0}\gamma$$

$$^{0}_{0}\gamma + ^{9}_{4}Be \rightarrow ^{1}_{0}n + 2\left(^{4}_{2}He\right)$$

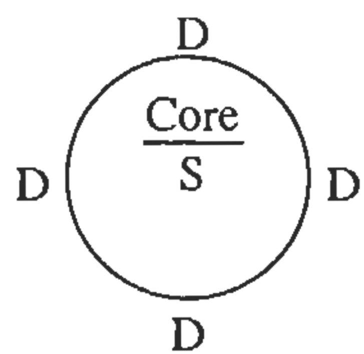
$$^{4}_{2}He + ^{9}_{4}Be \rightarrow ^{1}_{0}n + ^{12}_{6}C$$

Note: Must have radioactive antimony as initial condition.

# Source-Detector Geometry

- Photoneutrons are homogeneous because neutrons are produced throughout the entire volume of the core. The other types of sources are discrete entities and care must be taken to ensure proper source-detector geometry.

- A discrete source should be placed in the center of the reactor core with fuel surrounding it. The detectors should be located beyond the fuel. Thus:

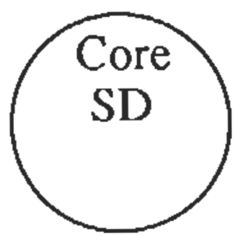



- Power reactors are required to have four detectors, one in each quadrant.

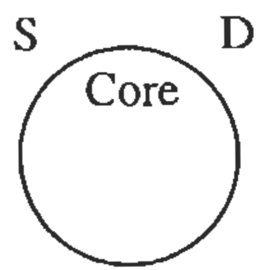
# Examples of Incorrect Source-Detector Geometry

Configuration

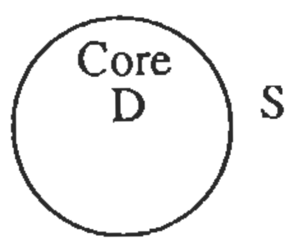
Problem



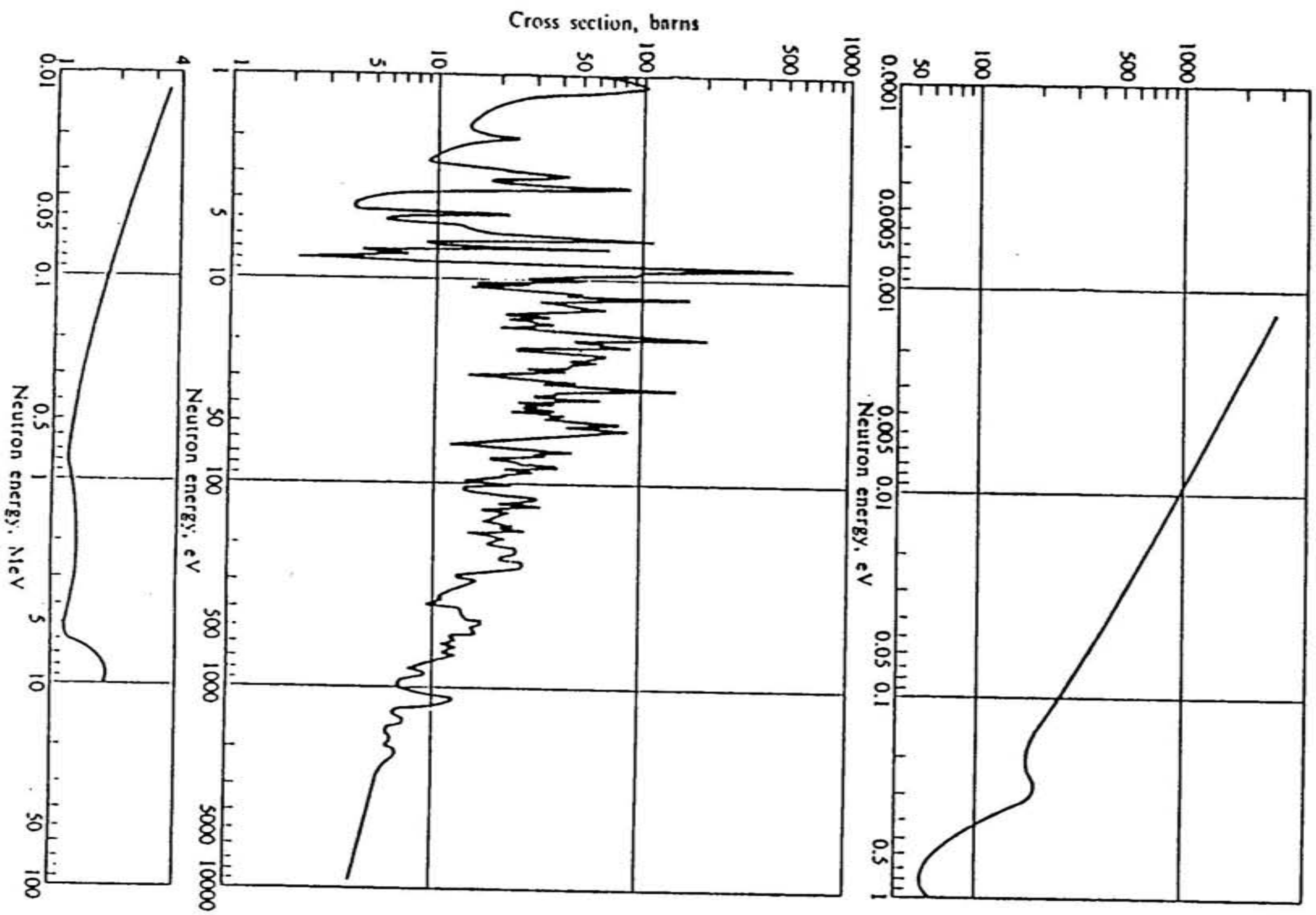

Both source and detector are at core center. Hence, detector registers only source neutrons.



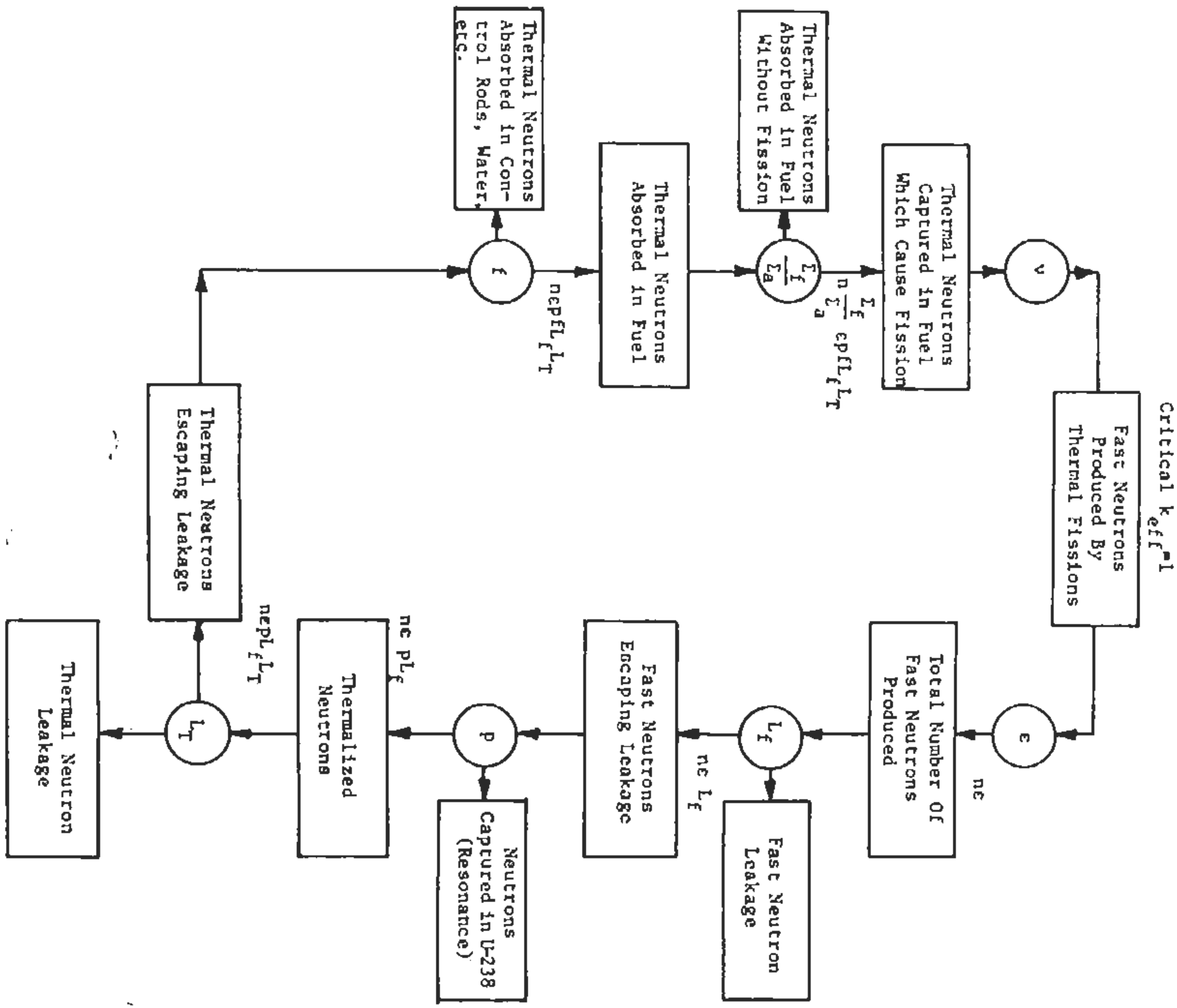

Both source and detector are external to core. Again, the detector only registers source neutrons.




Source is exterior to core. Hence, only a small fraction of the fuel is exposed to the source neutrons.




Courtesy of Brookhaven National Laboratory.




Neutron Life Cycle

# Definitions of Neutron Life Cycle Factors

$$\varepsilon = \frac{\text{Total Number of Fast Neutrons Produced from Fast and Thermal Fission}}{\text{Number of Fast Neutrons Produced from Thermal Fission}}$$

$$L_f = \frac{\text{Total Number of Fast Neutrons Escaping Leakage}}{\text{Total Number of Fast Neutrons Produced from Fast and Thermal Fission}}$$

$$p = \frac{\text{Total Number of Thermalized Neutrons}}{\text{Total Number of Fast Neutrons Escaping Leakage}}$$

$$L_t = \frac{\text{Total Number of Thermal Neutrons Escaping Leakage}}{\text{Total Number of Thermalized Neutrons}}$$

# Definitions of Neutron Life Cycle Factors (cont.)

$$f = \frac{\text{Thermal Neutrons Absorbed in Fuel}}{\text{Total Number of Thermal Neutrons Escaping Leakage}}$$

$$\eta = \frac{\text{Thermal Neutrons Captured in Fuel Which Cause Fission}}{\text{Thermal Neutrons Absorbed in Fuel}} \quad \text{or}$$

$$\eta = \frac{\text{Number of Fast Neutrons Produced from Thermal Fission}}{\text{Thermal Neutrons Absorbed in the Fuel}}$$

Of the above factors, the reactor operator can alter 'f' by changing the control rod position or by adjusting the soluble poison content. The leakage terms also vary during routine operation whenever coolant temperature changes. The other terms are fixed by the fuel type.

# Core Multiplication Factor

1. It is useful to define a 'core multiplication factor' which is denoted by the symbol 'K' and which is the product of the six factors that define the neutron life cycle. Thus,

$$K = \varepsilon L_f p L_t f \eta$$

2. The above expression, which is called the 'six-factor formula, has physical meaning:

$$K = \frac{\text{Neutrons Produced from Fission}}{\text{Neutrons Absorbed} + \text{Neutron Leakage}} \quad \text{or}$$

$$K = \frac{\text{Number Neutrons in Present Generation}}{\text{Number Neutrons in Preceding Generation}}$$

$$K = \frac{n_1}{n_0} = \frac{n_2}{n_1} = \frac{n_3}{n_2}$$ when n is the number of neutrons in each generation.

3. If K is unity, the reactor is critical.

4. If we know the K-value for a reactor core, we can determine the rate of change of its neutron population. This is most useful in reactor startups.

## Buildup of Neutron Population – A Result of Subcritical Multiplication

| Generation | Neutrons From: Source | Neutrons From: Multiplication | Total |
|---|---|---|---|
| 0 | 100 | 0 | 100 |
| 1 | 100 | 60 | 160 |
| 2 | 100 | 60 + 36 | 196 |
| 3 | 100 | 60 + 36 + 22 | 218 |
| 4 | 100 | 60 + 36 + 22 + 13 | 231 |
| 5 | 100 | 60 + 36 + 22 + 13 + 8 | 239 |
| 6 | 100 | 60 + 36 + 22 + 13 + 8 + 5 | 244 |
| 7 | 100 | 60 + 36 + 22 + 13 + 8 + 5 + 3 | 247 |
| 8 | 100 | 60 + 36 + 22 + 13 + 8 + 5 + 3 + 2 | 249 |
| 9 | 100 | 60 + 36 + 22 + 13 + 8 + 5 + 3 + 2 + 1 | 250 |
| 10 | 100 | 60 + 36 + 22 + 13 + 8 + 5 + 3 + 2 + 1 + 0 | 250 |

All neutron quantities have been rounded to the nearest unit. Note that the incremental increase to the total population from the first generation is zero in the tenth generation. Hence, the system has reached equilibrium.

# Mathematics of Subcritical Multiplication

| Generation # | Source Neutrons | Neutrons from Multiplication |
| --- | --- | --- |
| 0 | $S_0$ | 0 |
| 1 | $S_0$ | $KS_0$ |
| 2 | $S_0$ | $K S_0 + K^2S_0$ |
| 3 | $S_0$ | $K S_0 + K^2S_0 + K^3S_0$ |
| 4 | $S_0$ | $K S_0 + K^2S_0 + K^3S_0 + K^4S_0$ |
| ⋮ | ⋮ | |
| n | $S_0$ | $K S_0 + K^2S_0 + K^3S_0 + K^4S_0 + \ldots + K^nS_0$ |

# Mathematics of Subcritical Multiplication (cont.)

So, after n generations, the neutron population would be:

$$\text{Total Neutrons} = S_0 + KS_0 + K^2S_0 + K^3S_0 + K^4S_0 + \ldots + K^nS_0$$

$$= S_0\,(1 + K + K^2 + K^3 + K^4 + \ldots + K^n)$$

$$= \frac{S_0}{1-K}$$

Note: The fact that the series $1 + K + K^2 + K^3 + K^4 + \ldots + K^n$ does equal $1/(1\text{-}K)$ can be proved mathematically for values of K that are less than 1.

# Subcritical Multiplication

- We now have an expression that may be used to calculate the equilibrium neutron level in a subcritical reactor.

- Of extreme importance to reactor startups is that the total neutron population in a subcritical fissile medium exceeds the source level by a factor of 1/(1-K) or 1/M. This process is called *subcritical multiplication.*

- The following should be noted:

  a) The subcritical multiplication formula does <u>NOT</u> allow calculation of the time required for criticality.

  b) As the multiplication factor, K, approaches 1.0, the number of generations and hence time required for the neutron level to stabilize gets longer and longer. This is one of the reasons why it is important to conduct a reactor startup slowly. If it isn't done slowly, the subcritical multiplication level won't have time to attain equilibrium.

  c) The equilibrium neutron level in a subcritical reactor is proportional to the initial neutron source strength. This is why it is important to have neutron count rates above a certain minimum before conducting a startup.

  d) The formula is only valid while subcritical.

# Application of the Subcritical Multiplication Formula

- How can we apply the subcritical multiplication formula? We can measure neutron counts and source strength. The latter is merely the neutron counts with the reactor shutdown. Hence, we can calculate the multiplication factor, K, and thereby estimate how close a reactor is to criticality.

- We derived the relation:

$$\text{Counts} = S_0/(1-K)$$

- Rearrange the above to obtain:

$$(1-K) = S_0/\text{Counts}$$

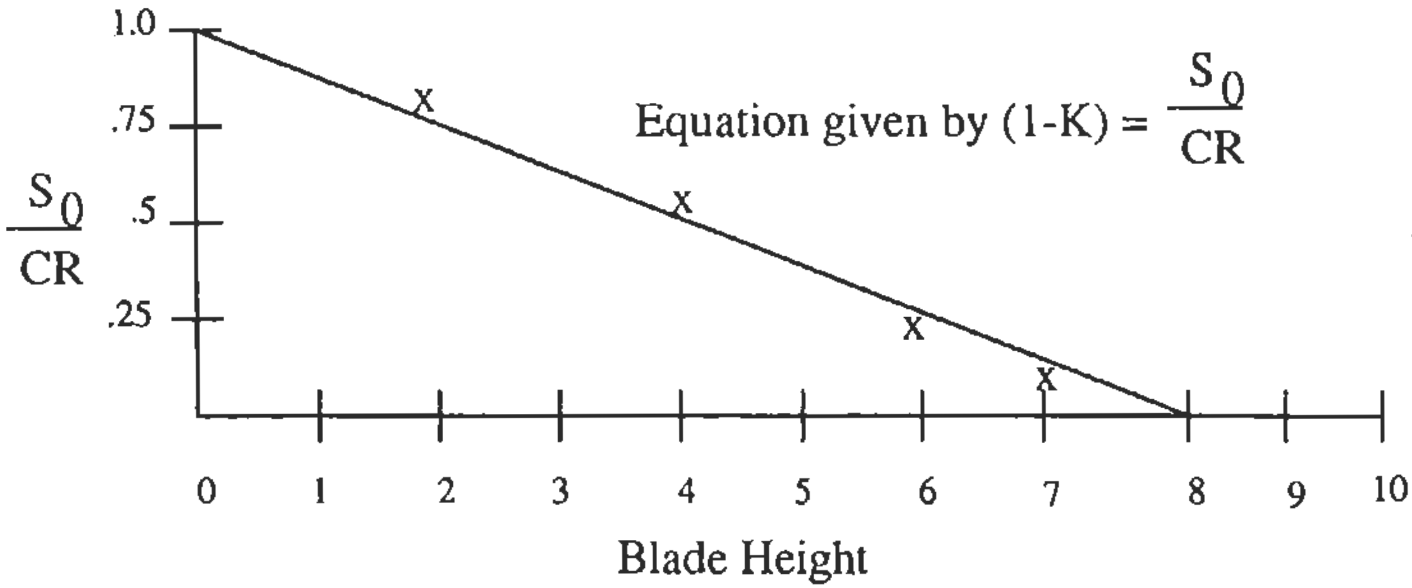
- To make use of this relation, plot inverse counts ($S_0$/Counts) on the vertical axis and control rod position or poison concentration or fuel loading on the horizontal axis. The result is called a '1/M' plot where M stands for multiplication. The point where the plot is extrapolated to cross the horizontal axis is where K equals 1 and the reactor is critical.

# '1/M' Plot

| Blade Height | Source Strength | Count Rate | Source/Count Rate |
|---|---|---|---|
| 0" | $S_0$ | $S_0$ | 1.0 |
| 2" | $S_0$ | 1.30 $S_0$ | 0.77 |
| 4" | $S_0$ | 1.98 $S_0$ | 0.51 |
| 6" | $S_0$ | 4.15 $S_0$ | 0.24 |
| 7" | $S_0$ | 8.00 $S_0$ | 0.125 |

# '1/M' Plot (cont.)

If the data from this example is plotted such that the vertical axis is (Source/Count Rate) or $S_0/CR$, we get:




– Criticality is expected at 8.0". Actual plots are usually not linear. Instrument noise and improper source-detector geometry cause a non-linear response.

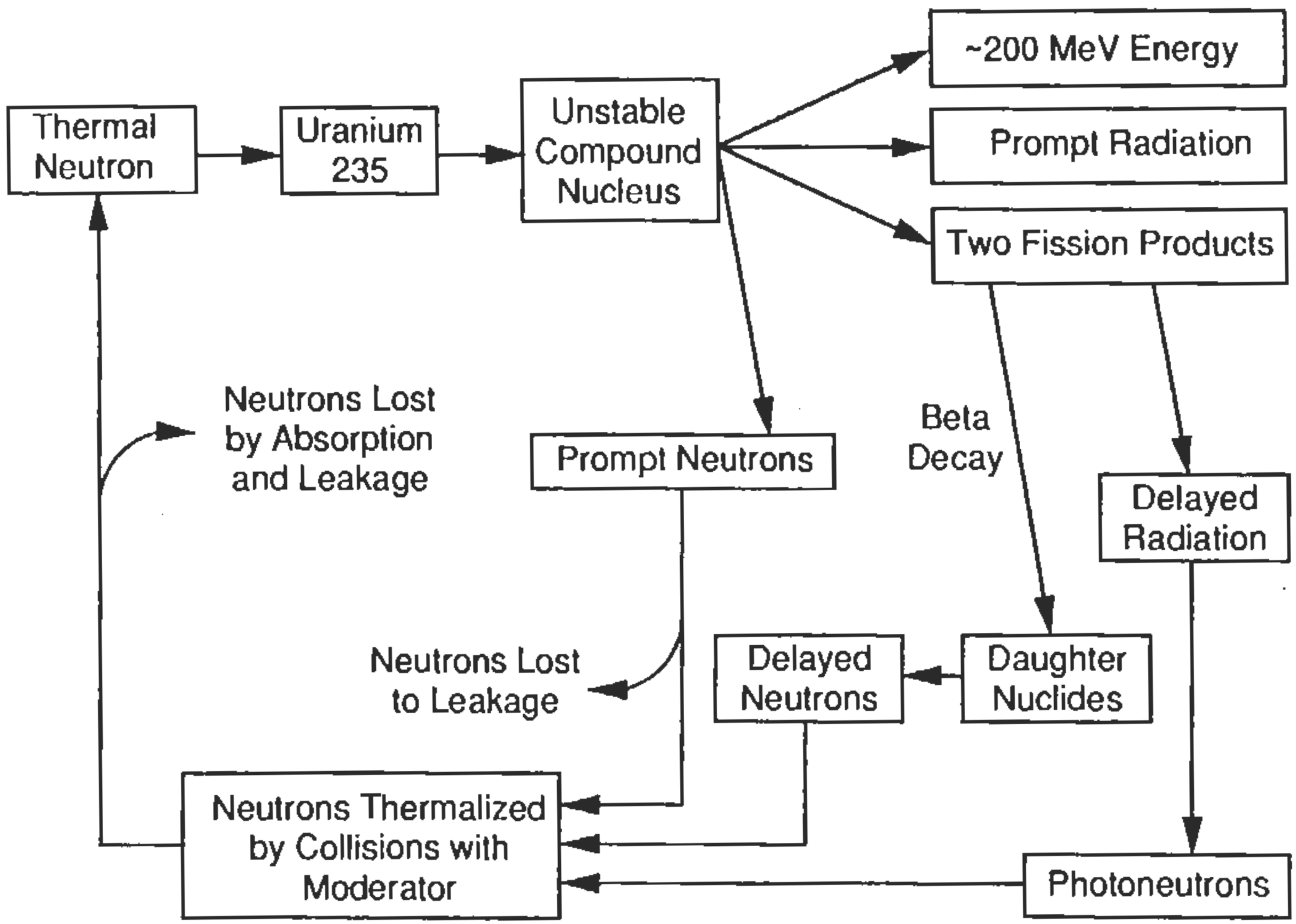
# Critical – Point of Adding Heat

- Crucial concepts are:
  - Prompt and delayed neutrons
  - Importance of delayed neutrons
  - Power-period relation
  - Reactivity
  - Dynamic period equation
  - Step and ramp reactivity transients.

- The 'point of adding heat' is the power level above which a change temperature is observed following a change of power. Reactors may be critical at a few Watts or even less. At such low powers, no change in temperature results because of the core's heat capacity. In general, there are no reactivity feedback effects below the point-of-adding heat.

# Prompt and Delayed Neutrons

The reactor multiplication factor, K, has been defined as the ratio of neutrons in one generation to those in the immediately preceding generation. Combining this definition with a physical understanding of the neutron life cycle allowed us to write an equation that predicted the equilibrium neutron count rate in a subcritical reactor. Unfortunately, that relationship is not valid for a critical or supercritical reactor. We need to develop a means of describing both the neutron level and its rate of change in a supercritical reactor. The first thing that we should do is to examine the fission process and determine what types of neutrons are produced. The fission of a U-235 nucleus normally yields two fission fragments, an average of 2.5 neutrons, and an assortment of beta particles, gamma rays and neutrinos. The neutrons that are produced directly from the fission event are referred to as prompt because they appear almost instantly. Most of the neutrons produced in a reactor are prompt. However, certain fission fragments, which are called precursors, undergo a beta decay to a daughter nuclide that then emits a neutron. Neutrons produced in this manner are referred to as delayed. The delay is the time that must elapse for the precursor to undergo its beta decay. Delayed neutrons constitute an extremely small fraction of a reactor's total neutron population. Nevertheless, they are crucial to the safe operation of a reactor during power transients.


Thermal Neutron
Uranium 235
Unstable Compound Nucleus
~200 MeV Energy
Prompt Radiation
Two Fission Products
Neutrons Lost by Absorption and Leakage
Prompt Neutrons
Beta Decay
Delayed Radiation
Neutrons Lost to Leakage
Delayed Neutrons
Daughter Nuclides
Neutrons Thermalized by Collisions with Moderator
Photoneutrons

# Importance of Delayed Neutrons

- The time required for a prompt neutron to be born, thermalize, and cause a fission is on the order of $1 \cdot 10^{-4}$ s. This is too rapid for human or machine control.
- Delayed neutrons have an average lifetime of 12.2 s.
- The fraction of neutrons that are delayed in a typical light-water reactor is 0.0065. This quantity is denoted by the Greek letter β and is called Beta.
- Assume that a reactor has 100,000 neutrons present. The lifetime of an 'average' neutron is therefore:

$$\frac{(\#\text{ prompt n's})(\text{prompt lifetime}) + (\#\text{ delayed n's})(\text{delayed lifetime})}{\text{total \# neutrons}}$$

or
$$\frac{(99350)(0.0001) + (650)(12.2)}{100{,}000}$$

or 0.079 s

Thus delayed neutrons lengthen the average neutron lifetime and result in a controllable reactor.

# Reactivity

- Reactivity is a measure of the departure of a reactor from criticality. It's mathematical definition is:

$$\rho \equiv \frac{K-1}{K}$$

where K is the reactor's multiplication factor.

- If the reactivity is negative, the reactor is subcritical. Conversely, if it is positive, the reactor is supercritical. If the reactivity is zero, the reactor is exactly critical.

- Reactivity may be thought of a the 'fractional change in the neutron population per neutron generation.'

- Reactivity is a global property of a reactor. Nevertheless, it is common practice to speak of the reactivity worth of a control rod or of the soluble poison. Withdrawing a rod or diluting the poison is said to 'add reactivity.'

# Allowed Magnitude of Reactivity

- We noted earlier that there are two kinds of neutrons: prompt and delayed. The latter are produced on a time scale that is controllable by humans and instruments. Thus, it is essential that reactor transients always be conducted in a manner such that the delayed neutrons are the rate determining factor.

- The fraction of delayed neutrons is 0.0065. Therefore, the amount of positive reactivity present in a reactor should never be allowed to exceed some small percent of the delayed neutron fraction.

- Reactivity, being dimensionless, has no units. But, it is common to measure it relative to the delayed neutron fraction. We say that:

$$1 \text{ Beta} = 0.0065\ \Delta K/K$$

Reactivity is sometime also measured in dollars and cents with 1 Beta = $1 = 100 cents.

# Rationale for Limiting Reactivity

- The following table illustrates the importance of limiting reactivity additions. Shown are three cases, all with the same initial condition: reactor critical with a population of 10,000 neutrons.

- For the first case, no change is made. One generation later there are 9935 prompt neutrons and 65 delayed ones. Criticality can NOT be maintained without the delayed neutrons. Hence, they are the rate-determining step.

- For the second case, we add 0.500 Beta of reactivity. This corresponds to (0.0065 $\Delta K/K$) (0.50) or 0.00325 $\Delta K/K$ or 33 neutrons in the first generation. Thus, after one generation there are 9968 prompt neutrons and 65 delayed ones for a total of 10,033. The delayed neutrons are still controlling because it takes 100,000 neutrons to stay critical and there are only 9968 prompt ones.

- For the third case, we add 1.5 Beta of reactivity. This corresponds to (0.0065)$\Delta K/K$ (1.5) or 0.009 $\Delta K/K$ or 98 neutrons in the first generation. Thus, after one generation there are 10,032 prompt neutrons and 66 delayed ones. There are more than enough prompt neutrons to maintain criticality. The prompt neutrons are controlling with their $10^{-4}$ s life cycle.

# Rationale for Limiting Reactivity (Continued)

| Initial Condition | Initial Population | Reactivity Addition | One Generation Later | Condition |
|---|---|---|---|---|
| Critical | 10,000 neutrons | 0% ΔK/K | 10,000 | Steady-State; Not critical on prompt neutrons alone |
| | 9,935 prompt | or 0.0 Beta | 9,935 | |
| | 65 delayed | or 0 neutrons | 65 | |
| Critical | 10,000 neutrons | 0.325% DK/K | 10,033 | Supercritical; Not critical on prompt neutrons alone |
| | 9,935 prompt | or 0.5 Beta | 9,968 | |
| | 65 delayed | or 33 neutrons | 65 | |
| Critical | 10,000 neutrons | 0.98% DK/K | 10,098 | Power Runaway; Critical on prompt neutrons alone |
| | 9,935 prompt | or 1.5 Beta | 10,032 | |
| | 65 delayed | or 98 neutrons | 66 | |

# Power-Period Relation

- Reactivity is not directly measurable and hence most reactor operating procedures do not refer to it. Instead, they specify a limiting rate of power rise, commonly called a 'reactor period.'

- Reactor period is denoted by the Greek letter, $\tau$, and is defined as:

$$\tau \equiv n(t) / (dn(t) / dt)$$

  where n(t) is the reactor power. Thus, a period of infinity corresponds to the critical condition.

- The relation between power and period is:

$$P(t) = P_o e^{t/\tau}$$

  where P(t) is the power level, $P_o$ is the initial power, e is the exponential, and t is time.

# Examples of Power-Period Relation

1. Suppose the period is 100 s and the initial power is 10% of rated. How long before 100% power is attained?

$$P(t) = P_o e^{t/\tau}$$

$$100\% = 10\%\ e^{t/100}$$

$$\ln(100/10) = t/100$$

$$230\ \text{s} = t$$

2. Suppose the reactor period is equal to the prompt neutron lifetime of $1 \cdot 10^{-4}$ s. By what factor would power rise in 1.0 ms?

$$P(t) = P_o e^{t/\tau}$$

$$P/(t)/P_o = e^{1 \cdot 10^{-3}/1 \cdot 10^{-4}}$$

$$= 22{,}026$$

The reactor is uncontrollable.

3. Repeat problem #2 with a period of 0.079 s. The answer is 1.013. Hence the value of delayed neutrons.