Philosophy of QM 24.111

Ninth lecture, 28 February, 2005



Notice that given this spin state:

 $1/2\sqrt{5} |\text{up}, 0^{\circ} > \otimes |\text{down}, 0^{\circ} >$ $+ 2/\sqrt{5} |\text{down}, 0^{\circ} > \otimes |\text{up}, 0^{\circ} >$ $+ \sqrt{3}/2\sqrt{5} |\text{down}, 0^{\circ} > \otimes |\text{down}, 0^{\circ} >$

we can conclude that when

LEFT = 0^{\circ} and RIGHT = 0^{\circ}

The two particles will never both go up.

We now rewrite the spin state using the following basis:

 $\begin{cases} |up, +120^{\circ} \rangle \otimes |up, 0^{\circ} \rangle, \\ |up, +120^{\circ} \rangle \otimes |down, 0^{\circ} \rangle, \\ |down, +120^{\circ} \rangle \otimes |up, 0^{\circ} \rangle, \\ |down, +120^{\circ} \rangle \otimes |down, 0^{\circ} \rangle \end{cases}$

To do this, we need to know how to write $|up, 0^{\circ}>$ and $|down, 0^{\circ}>$ as linear combinations of $|up, +120^{\circ}>$ and $|down, +120^{\circ}>$.







 $|up, 0^{\circ}\rangle = 1/2 |up, +120^{\circ}\rangle - \sqrt{3}/2 |down, +120^{\circ}\rangle$ $|down, 0^{\circ}\rangle = \sqrt{3}/2 |up, +120^{\circ}\rangle + 1/2 |down, +120^{\circ}\rangle$

 $1/2\sqrt{5} |up, 0^{\circ}\rangle \otimes |down, 0^{\circ}\rangle$

 $1/2\sqrt{5}$ (1/2 |up, +120°> - $\sqrt{3}/2$ |down, +120°>) \otimes |down, 0°>

+ $2/\sqrt{5}$ |down, 0° > \otimes |up, 0° >

 $+2/\sqrt{5} (\sqrt{3/2} | up, +120^{\circ} > +1/2 | down, +120^{\circ} >) \otimes | up, 0^{\circ} >$

+ $\sqrt{3}/2\sqrt{5}$ |down, 0°> \otimes |down, 0°>

 $+\sqrt{3/2}\sqrt{5}$ ($\sqrt{3/2}$ |up, +120°> + 1/2 |down, +120°>) \otimes |down, 0°>

$$1/2\sqrt{5} (1/2 |up, +120^{\circ}) - \sqrt{3}/2 |down, +120^{\circ}) \otimes |down, 0^{\circ} + 2/\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) + 1/2 |down, +120^{\circ}) \otimes |up, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) + 1/2 |down, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) + 1/2 |down, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) + 1/2 |down, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) + 1/2 |down, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) + 1/2 |down, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) + 1/2 |down, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ}) \otimes |down, 0^{\circ} + \sqrt{3}/2\sqrt{5} (\sqrt{3}/2 |up, +120^{\circ}) \otimes |down, 0^{\circ}) \otimes |down, 0^{\circ})$$

Conclusion: when LEFT = +120° and RIGHT = 0° the particles never both go down.

 $\sqrt{3}/\sqrt{5}$ |up, +120°> \otimes |up, 0°> + 1/ $\sqrt{5}$ |up, +120°> \otimes |down, 0°> + 1/ $\sqrt{5}$ |down, +120°> \otimes |up, 0°>

+ 0 |down, +120°> ⊗ |down, 0°>

We now rewrite the spin state using the following basis:

$$|up, +120^{\circ} > \otimes |up, -120^{\circ} >, \\ |up, +120^{\circ} > \otimes |down, -120^{\circ} >, \\ |down, +120^{\circ} > \otimes |up, -120^{\circ} >, \\ |down, +120^{\circ} > \otimes |down, -120^{\circ} >$$

To do this, we need to know how to write $|up, 0^{\circ}>$ and $|down, 0^{\circ}>$ as linear combinations of $|up, -120^{\circ}>$ and $|down, -120^{\circ}>$.



 $|up, 0^{\circ}\rangle = 1/2 |up, -120^{\circ}\rangle + \sqrt{3/2} |down, -120^{\circ}\rangle$

 $|\text{down}, 0^\circ\rangle = -\sqrt{3/2} |\text{up}, -120^\circ\rangle + 1/2 |\text{down}, -120^\circ\rangle$

 $\sqrt{3}/\sqrt{5}$ |up, +120°> \otimes |up, 0°>

 $\sqrt{3}/\sqrt{5}$ |up, +120°> \otimes (1/2 |up, -120°> + $\sqrt{3}/2$ |down, -120°>)

+ $1/\sqrt{5}$ |up, +120°> \otimes |down, 0°>

 $+ 1/\sqrt{5} |up, +120^{\circ} > \otimes (-\sqrt{3}/2 |up, -120^{\circ} > + 1/2 |down, -120^{\circ} >)$

+ $1/\sqrt{5}$ |down, +120°> \otimes |up, 0°>

 $+ 1/\sqrt{5}$ |down, $+120^{\circ} > \otimes (1/2 |up, -120^{\circ} > + \sqrt{3}/2 |down, -120^{\circ} >)$

 $\sqrt{3}/\sqrt{5} |up, +120^{\circ}\rangle \otimes (1/2 |up, -120^{\circ}\rangle + \sqrt{3}/2 |down, -120^{\circ}\rangle)$ + $1/\sqrt{5} |up, +120^{\circ}\rangle \otimes (-\sqrt{3}/2 |up, -120^{\circ}\rangle + 1/2 |down, -120^{\circ}\rangle)$ + $1/\sqrt{5} |down, +120^{\circ}\rangle \otimes (1/2 |up, -120^{\circ}\rangle + \sqrt{3}/2 |down, -120^{\circ}\rangle)$

Conclusion: when LEFT = +120° and RIGHT = -120° the particles never both go up.

 $0 |up, +120^{\circ} > \otimes |up, -120^{\circ} >$ + 2/ $\sqrt{5} |up, +120^{\circ} > \otimes |down, -120^{\circ} >$ + 1/2 $\sqrt{5} |down, +120^{\circ} > \otimes |up, -120^{\circ} >$ + $\sqrt{3}/2\sqrt{5} |down, +120^{\circ} > \otimes |down, -120^{\circ} >$

 $|up, 0^{\circ}\rangle = 1/2 |up, -120^{\circ}\rangle + \sqrt{3}/2 |down, -120^{\circ}\rangle$ $|down, 0^{\circ}\rangle = -\sqrt{3}/2 |up, -120^{\circ}\rangle + 1/2 |down, -120^{\circ}\rangle$

 $1/2\sqrt{5}$ |up, 0°> \otimes |down, 0°>

 $1/2\sqrt{5} |up, 0^{\circ}\rangle \otimes (-\sqrt{3}/2 |up, -120^{\circ}\rangle + 1/2 |down, -120^{\circ}\rangle)$

+ $2/\sqrt{5}$ |down, 0° > \otimes |up, 0° >

 $+2/\sqrt{5}$ |down, 0°> \otimes (1/2 |up, -120°> + $\sqrt{3}/2$ |down, -120°>)

+ $\sqrt{3}/2\sqrt{5}$ |down, 0°> \otimes |down, 0°>

 $+\sqrt{3/2}\sqrt{5}$ |down, 0°> \otimes (- $\sqrt{3/2}$ |up, -120°> + 1/2 |down, -120°>)

 $1/2\sqrt{5} |up, 0^{\circ}\rangle \otimes (-\sqrt{3}/2 |up, -120^{\circ}\rangle + 1/2 |down, -120^{\circ}\rangle)$ + $2/\sqrt{5} |down, 0^{\circ}\rangle \otimes (1/2 |up, -120^{\circ}\rangle + \sqrt{3}/2 |down, -120^{\circ}\rangle)$ + $\sqrt{3}/2\sqrt{5} |down, 0^{\circ}\rangle \otimes (-\sqrt{3}/2 |up, -120^{\circ}\rangle + 1/2 |down, -120^{\circ}\rangle)$

Conclusion: when LEFT = 0° and RIGHT = -120° the particles sometimes both go up (prob = 3/80).

- $-\sqrt{3}/4\sqrt{5}$ |up, 0°> \otimes |up, -120°>
- + $1/4\sqrt{5}$ |up, 0°> \otimes |down, -120°>
- + $1/4\sqrt{5}$ |down, 0°> \otimes |up, -120°>
- + $5\sqrt{3}/4\sqrt{5}$ |down, 0°> \otimes |down, -120°>

The basic principles of qm

(1) The physical state of any system is represented by a vector in some vector space (usually an infinite-dimensional vector space; note that this will be a different vector space for each different system).

(2) If Φ is a vector representing one possible physical state of some system, and Ψ is another vector representing another possible physical state of that system, then any arbitrary linear combination $a\Phi + b\Psi$ also represents a possible physical state of the system. This is called the principle of superposition.

(3) Any experiment that can be performed on a system is represented by an orthonormal basis in the vector space for that system. Each basis element can be thought of as "labeled" with one of the possible outcomes of the experiment.

We will now amend this principle.

Subspaces of vector spaces 1

Imagine a 5-dimensional vector space over the reals—think of it as just the set of all 5-tuples of real numbers $(x_1, x_2, x_3, x_4, x_5)$.

Here is one such vector v₁: (1,2,3,4,5).

Here is another such vector v_2 : (2,3,5,7,11).

Here is a linear combination of v₁ and v₂:



Subspaces of vector spaces 2

Imagine a 5-dimensional vector space over the reals—think of it as just the set of all 5-tuples of real numbers $(x_1, x_2, x_3, x_4, x_5)$.

- Here is one such vector v₁: (1,2,3,4,5).
- Here is another such vector v_2 : (2,3,5,7,11).

Here is another linear combination of v₁ and v₂:



Subspaces of vector spaces 3

Imagine a 5-dimensional vector space over the reals—think of it as just the set of all 5-tuples of real numbers $(x_1, x_2, x_3, x_4, x_5)$.

- Here is one such vector v₁: (1,2,3,4,5).
- Here is another such vector v_2 : (2,3,5,7,11).

Here is yet another linear combination of v₁ and v₂:



- **Subspaces of vector spaces 5**
 - How many dimensions does this subspace have?
 - That's right: TWO.
 - **Observe that it has infinitely many orthonormal bases.**
 - For example, these two vectors are orthogonal:

$$\begin{bmatrix}
 2 \\
 3 \\
 5 \\
 7 \\
 11
 \end{bmatrix}
 \begin{bmatrix}
 -4 \\
 98 \\
 94 \\
 94 \\
 90 \\
 -126
 \end{bmatrix}
 \begin{array}{r}
 -8 \\
 +294 \\
 +470 \\
 +630 \\
 -1386
 \end{array}$$

So we can scale them to get an orthonormal basis.

- **Subspaces of vector spaces 6**
 - How many dimensions does this subspace have?
 - That's right: TWO.
 - **Observe that it has infinitely many orthonormal bases.**
 - Similarly, these two vectors are orthogonal:

$$\begin{bmatrix}
 1 \\
 2 \\
 3 \\
 4
 \end{bmatrix}
 -4 -4 -4 +94 \\
 47 +94 \\
 = +129 \\
 39 +156 \\
 -75 -375 \\
 0$$

So, again, we can scale them to get an orthonormal basis.

Degeneracy 1

Suppose the vector space for our system has five dimensions.

Suppose the experiment ("measurement") we are going to perform on it has only four possible outcomes.

Still, we represent that experiment by an orthonormal basis which must have five elements in it.

So: Two of the elements must correspond to the same outcome:



Degeneracy 2

Suppose our system is in the state ψ .

Then the probability that our "measurement" will yield outcome 4 is:

$$|\langle \phi_4 | \psi \rangle|^2 + |\langle \phi_5 | \psi \rangle|^2$$
.

Let (normalized, orthogonal) vectors γ and ν span the same subspace as ϕ_4 and ϕ_5 . Then:

$$|\langle \phi_4 | \psi \rangle|^2 + |\langle \phi_5 | \psi \rangle|^2 = |\langle \gamma | \psi \rangle|^2 + |\langle \nu | \psi \rangle|^2$$
.

So we could just as easily have used the following basis to represent our experiment:



Degeneracy 3

- To remove this unwanted redundancy:
- Represent outcome 4 not by an orthonormal pair of vectors,
- but rather by the subspace they span.
- So: An experiment will now be represented by a set of pairwise-orthogonal subspaces, each corresponding to a distinct outcome. These subspaces span the entire vector space.
- "Pairwise-orthogonal"?
- **Explanation:** Subspace S_1 is orthogonal to subspace S_2 iff every vector in S_1 is orthogonal to every vector in S_2 .

Restating the statistical algorithm

Suppose system S is in a state represented by the unit vector Ψ .

Suppose experiment **E** is performed on **S**, where **E** is represented by the set of pairwise-orthogonal subspaces $\{S_1, S_2, ...\}$, with subspace S_i corresponding to outcome i.

To calculate Prob(outcome i):

1. Project Ψ onto the subspace S_i .

This can be done by picking an arbitrary orthonormal basis $\{\phi_1, \phi_2, ...\}$ for S_i , and calculating $<\phi_1 | \Psi > \phi_1 + <\phi_2 | \Psi > \phi_2 + ...$

2. Square the length of the resulting vector.

Given our choice of orthonormal basis $\{\phi_1, \phi_2, ...\}$ for S_i , this will equal $|\langle \phi_1 | \Psi \rangle|^2 + |\langle \phi_2 | \Psi \rangle|^2 + ...$

3. The resulting number is Prob(outcome i).

Hermitian operators 1

An operator is simply a function that takes vectors as inputs and yields vectors as outputs: $A\Psi = \Phi$.

A linear operator has this additional nice feature:

 $A(a\Phi + b\Psi) = aA\Phi + bA\Psi.$

A Hermitian operator has yet another nice feature:

 $\langle A\Phi | \Psi \rangle = \langle \Phi | A\Psi \rangle.$

Finally, Φ is an eigenvector of **A** iff, for some number **c**,

 $A\Phi = c\Phi$.

c is called the eigenvalue of Φ (for **A**). Note that **A** will have some set of eigenvalues.

Hermitian operators 2

Some cool results:

1. If Φ and Ψ are eigenvectors of (linear) operator **A**, with the same eigenvalue **c**, then so is $(a\Phi + b\Psi)$.

That means that for each eigenvalue **c** of **A**, there is a subspace consisting of all and only those vectors with eigenvalue **c** for **A**. We call these subspace eigenspaces for **A**.

2. If Φ and Ψ are eigenvectors of Hermitian operator A, with different eigenvalues, then Φ and Ψ are orthogonal.

That means that for a Hermitian operator A, its eigenspaces are pairwise-orthogonal.

So a Hermitian operator automatically "picks out" a set of pairwise-orthogonal subspaces, each labeled with a distinct eigenvalue. That makes these operators particularly well suited to represent experiments.

Hermitian operators 3

Haven't we forgotten something? After all, we can also show:

3. If Φ is an eigenvector of Hermitian operator **A**, then the eigenvalue **c** is a real number.

Supposedly this is a **BIG DEAL**, since "only real numbers could be the values of some measured physical quantity".

Exercise: Explain why this claim is—notwithstanding its prominent position in just about every quantum mechanics textbook—COMPLETE AND UTTER NONSENSE.

Two measurement problems 1

Schrödinger's Equation: the state of the world evolves, at all times, in accordance with Schrödinger's Equation.

Okay, okay: "observable".

Von Neumann's Rule: A system S has value v for physical quantity Q iff S is in an eigenstate with eigenvalue v of the Hermitian operator A that represents Q.

Okay, okay: "measurement".

Born's Rule: If a Q-experiment is performed on system S in state Ψ , then the expected value of the outcome is $\Psi | A\Psi >$. (This turns out to be equivalent to our statistical algorithm.)

Two measurement problems 2

Schrödinger's Equation + von Neumann's Rule gives us one problem:

Systems will sometimes possess no value for any recognizable physical quantity.

Schrödinger's Equation + Born's Rule gives us another:

CONTRADICTION

Illustration: Schrödinger's Cat Orientation = 0° UP Spin-1/2DOWN particle



The nice case







The problem

Nice case: final quantum mechanical state will be

|z-up> |no flash detected> |block suspended> |Fluffy purring> Not-so-nice case: final quantum mechanical state will be

|z-down> |flash detected> |block has fallen> |Fluffy squished>

So, if the initial spin state is

 $|x-up> = 1/\sqrt{2(|z-up> + |z-down>)}$

Then the final quantum mechanical state will be

1/√2(|z-up> |no flash detected> |block suspended> |Fluffy purring> + |z-down> |flash detected> |block has fallen> |Fluffy squished>).

WHAT KIND OF STATE IS THAT?!?

AND WHERE ARE OUR TWO POSSIBLE OUTCOMES?

The standard menu of options

- 1. Add extra variables.
 - Bohmian mechanics, modal interpretations, some versions of Many Minds
- 2. Add non-linear, stochastic dynamics.
 - textbook "collapse" theories, GRW
- 3. Give up on Born's Rule without admitting it.
 - Many Worlds, other versions of Many Minds
- 4. Go soft in the head.
 - decoherence, the "bare theory"