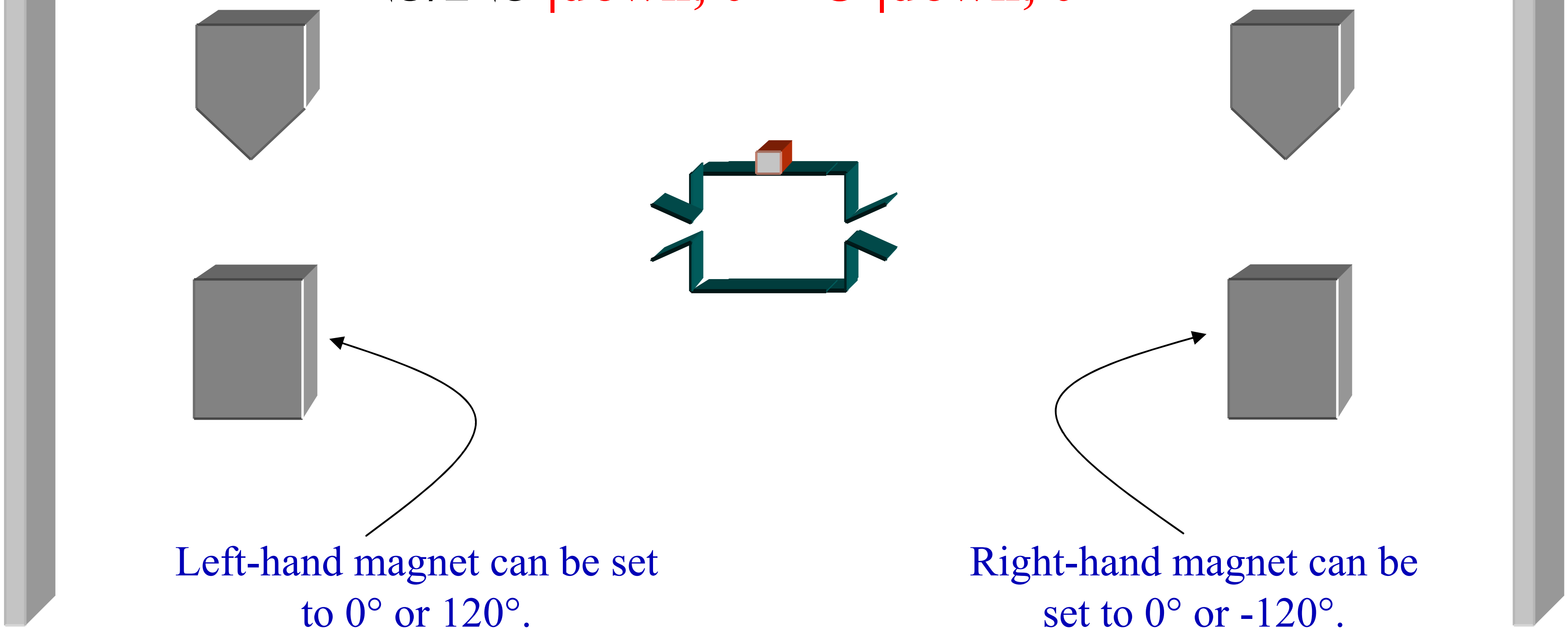
Philosophy of QM 24.111

Eleventh lecture.

Non-locality revisited 1

Spin state of particle pair:

$$\frac{1}{2\sqrt{5}}\,|\text{up}, 0^\circ\rangle \otimes |\text{down}, 0^\circ\rangle + \frac{2}{\sqrt{5}}\,|\text{down}, 0^\circ\rangle \otimes |\text{up}, 0^\circ\rangle + \frac{\sqrt{3}}{2\sqrt{5}}\,|\text{down}, 0^\circ\rangle \otimes |\text{down}, 0^\circ\rangle$$



Non-locality revisited 2

Notice that given this spin state:

$$1/2\sqrt{5}\ |\text{up}, 0^\circ> \otimes |\text{down}, 0^\circ>$$

$$+\ 2/\sqrt{5}\ |\text{down}, 0^\circ> \otimes |\text{up}, 0^\circ>$$

$$+\ \sqrt{3}/2\sqrt{5}\ |\text{down}, 0^\circ> \otimes |\text{down}, 0^\circ>$$

we can conclude that when

LEFT = 0° and RIGHT = 0°

The two particles will never both go up.

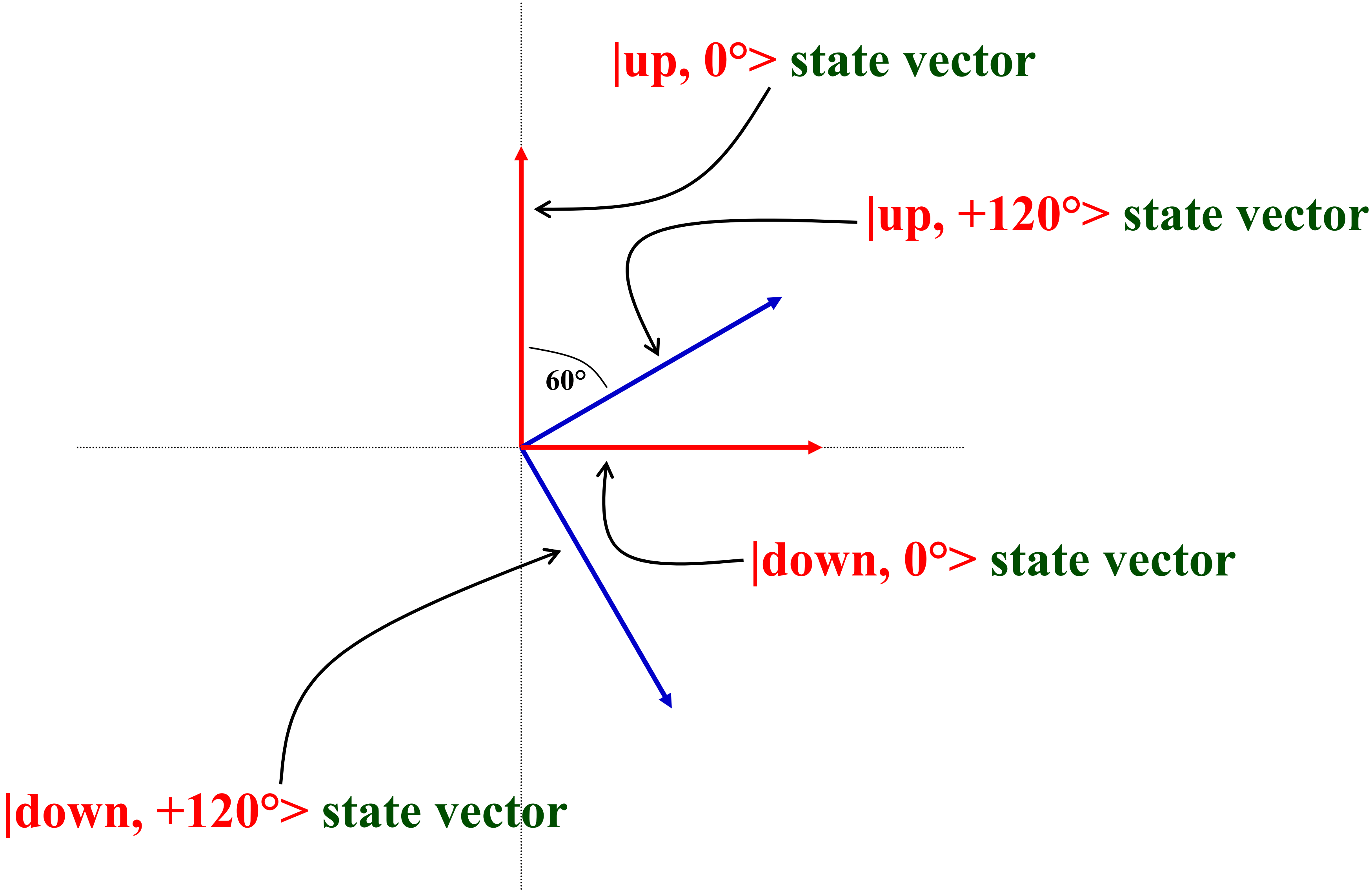
Non-locality revisited 3

We now rewrite the spin state using the following basis:

$$
\left\{
\begin{array}{c}
|up, +120°\rangle \otimes |up, 0°\rangle, \\
|up, +120°\rangle \otimes |down, 0°\rangle, \\
|down, +120°\rangle \otimes |up, 0°\rangle, \\
|down, +120°\rangle \otimes |down, 0°\rangle
\end{array}
\right\}
$$

To do this, we need to know how to write $|up, 0°\rangle$ and $|down, 0°\rangle$ as linear combinations of $|up, +120°\rangle$ and $|down, +120°\rangle$.

Non-locality revisited 4

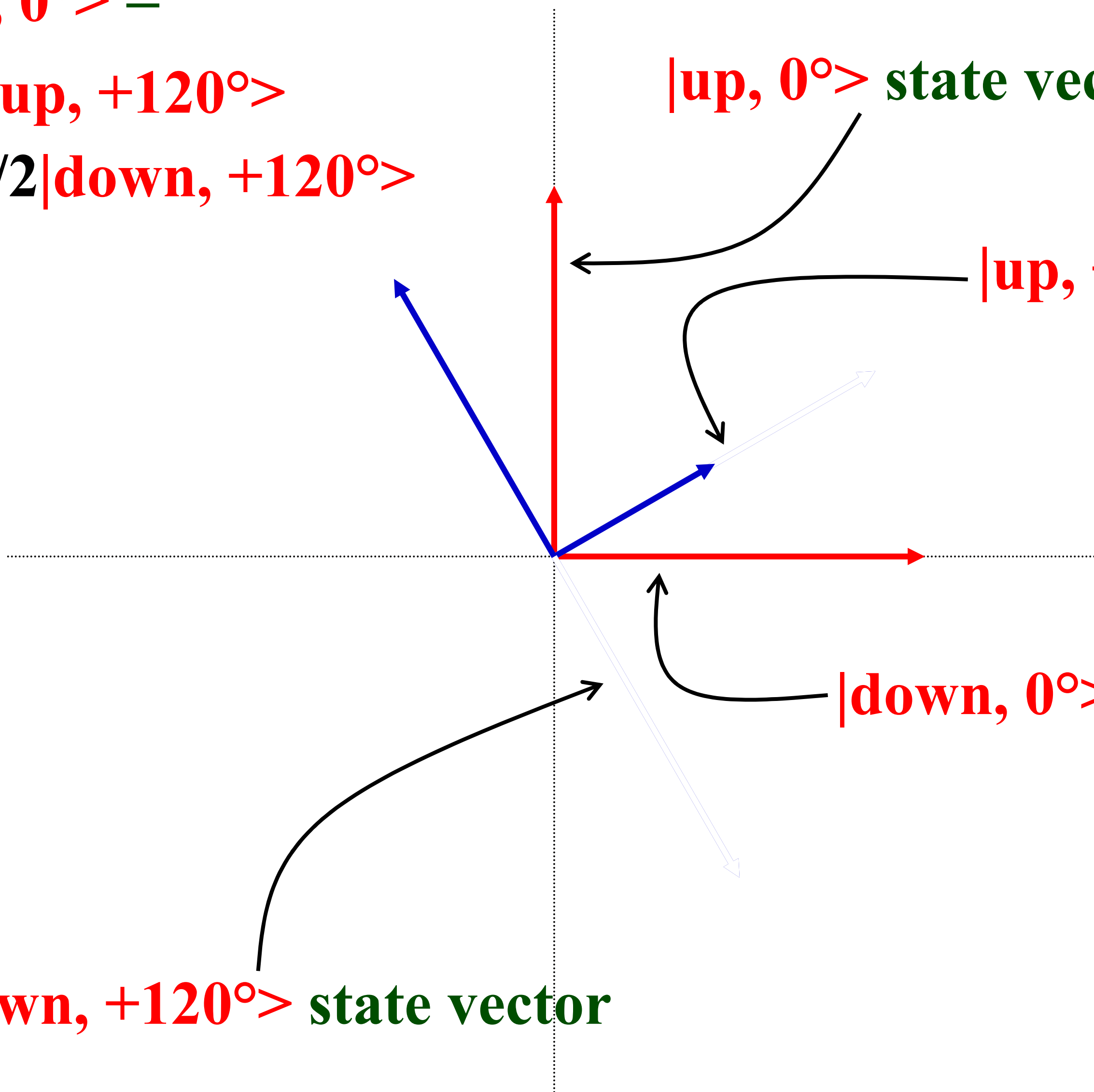


Non-locality revisited 5

$|up, 0°\rangle =$

$1/2|up, +120°\rangle$

$-\sqrt{3}/2|down, +120°\rangle$

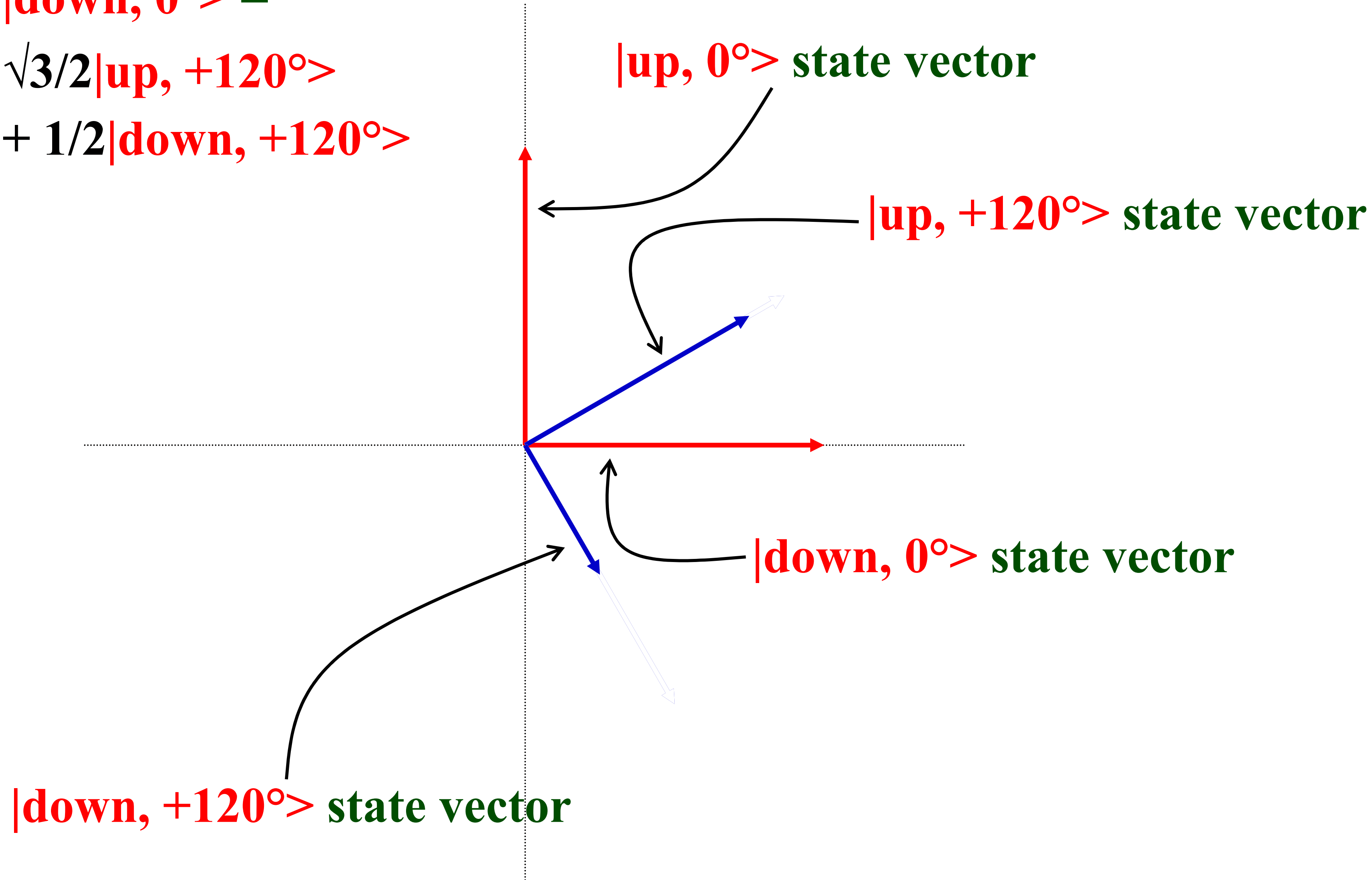


Non-locality revisited 6

|down, 0°> =

√3/2|up, +120°>

+ 1/2|down, +120°>



Non-locality revisited 7

$|\text{up}, 0^\circ> = 1/2\ |\text{up}, +120^\circ> - \sqrt{3}/2\ |\text{down}, +120^\circ>$

$|\text{down}, 0^\circ> = \sqrt{3}/2\ |\text{up}, +120^\circ> + 1/2\ |\text{down}, +120^\circ>$

$$1/2\sqrt{5}\ |\text{up}, 0^\circ> \otimes |\text{down}, 0^\circ>$$

$$1/2\sqrt{5}\ (1/2\ |\text{up}, +120^\circ> - \sqrt{3}/2\ |\text{down}, +120^\circ>) \otimes |\text{down}, 0^\circ>$$

$$+ 2/\sqrt{5}\ |\text{down}, 0^\circ> \otimes |\text{up}, 0^\circ>$$

$$+ 2/\sqrt{5}\ (\sqrt{3}/2\ |\text{up}, +120^\circ> + 1/2\ |\text{down}, +120^\circ>) \otimes |\text{up}, 0^\circ>$$

$$+ \sqrt{3}/2\sqrt{5}\ |\text{down}, 0^\circ> \otimes |\text{down}, 0^\circ>$$

$$+ \sqrt{3}/2\sqrt{5}\ (\sqrt{3}/2\ |\text{up}, +120^\circ> + 1/2\ |\text{down}, +120^\circ>) \otimes |\text{down}, 0^\circ>$$

Non-locality revisited 8

$$\frac{1}{2\sqrt{5}}\left(\frac{1}{2}\,|\text{up}, +120^\circ\rangle - \frac{\sqrt{3}}{2}\,|\text{down}, +120^\circ\rangle\right) \otimes |\text{down}, 0^\circ\rangle$$

$$+ \frac{2}{\sqrt{5}}\left(\frac{\sqrt{3}}{2}\,|\text{up}, +120^\circ\rangle + \frac{1}{2}\,|\text{down}, +120^\circ\rangle\right) \otimes |\text{up}, 0^\circ\rangle$$

$$+ \frac{\sqrt{3}}{2\sqrt{5}}\left(\frac{\sqrt{3}}{2}\,|\text{up}, +120^\circ\rangle + \frac{1}{2}\,|\text{down}, +120^\circ\rangle\right) \otimes |\text{down}, 0^\circ\rangle$$

Conclusion: when
LEFT = +120°
and
RIGHT = 0°
the particles never
both go down.

$$\frac{\sqrt{3}}{\sqrt{5}}\,|\text{up}, +120^\circ\rangle \otimes |\text{up}, 0^\circ\rangle$$

$$+ \frac{1}{\sqrt{5}}\,|\text{up}, +120^\circ\rangle \otimes |\text{down}, 0^\circ\rangle$$

$$+ \frac{1}{\sqrt{5}}\,|\text{down}, +120^\circ\rangle \otimes |\text{up}, 0^\circ\rangle$$

$$+ 0\,|\text{down}, +120^\circ\rangle \otimes |\text{down}, 0^\circ\rangle$$

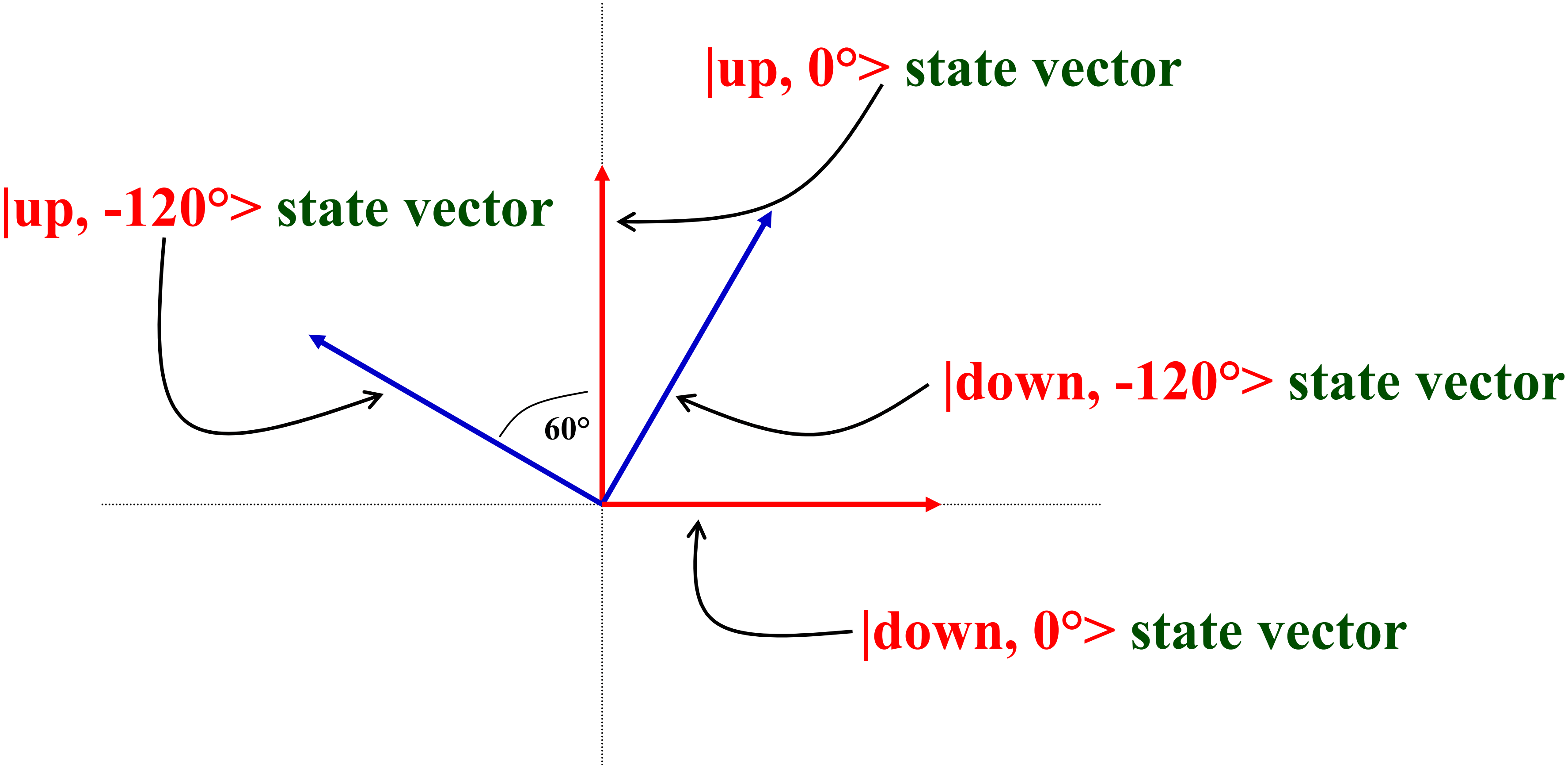
Non-locality revisited 9

We now rewrite the spin state using the following basis:

$$\left\{ \begin{array}{c} |\text{up}, +120^\circ\rangle \otimes |\text{up}, -120^\circ\rangle, \\ |\text{up}, +120^\circ\rangle \otimes |\text{down}, -120^\circ\rangle, \\ |\text{down}, +120^\circ\rangle \otimes |\text{up}, -120^\circ\rangle, \\ |\text{down}, +120^\circ\rangle \otimes |\text{down}, -120^\circ\rangle \end{array} \right\}$$

To do this, we need to know how to write $|\text{up}, 0^\circ\rangle$ and $|\text{down}, 0^\circ\rangle$ as linear combinations of $|\text{up}, -120^\circ\rangle$ and $|\text{down}, -120^\circ\rangle$.

Non-locality revisited 10



$$|\text{up}, 0°\rangle = 1/2|\text{up}, -120°\rangle + \sqrt{3}/2|\text{down}, -120°\rangle$$

$$|\text{down}, 0°\rangle = -\sqrt{3}/2|\text{up}, -120°\rangle + 1/2|\text{down}, -120°\rangle$$

Non-locality revisited 11

$$|\text{up}, 0^\circ\rangle = 1/2\ |\text{up}, -120^\circ\rangle + \sqrt{3}/2\ |\text{down}, -120^\circ\rangle$$

$$|\text{down}, 0^\circ\rangle = -\sqrt{3}/2\ |\text{up}, -120^\circ\rangle + 1/2\ |\text{down}, -120^\circ\rangle$$

$$\sqrt{3}/\sqrt{5}\ |\text{up}, +120^\circ\rangle \otimes |\text{up}, 0^\circ\rangle$$

$$\sqrt{3}/\sqrt{5}\ |\text{up}, +120^\circ\rangle \otimes (1/2\ |\text{up}, -120^\circ\rangle + \sqrt{3}/2\ |\text{down}, -120^\circ\rangle)$$

$$+\ 1/\sqrt{5}\ |\text{up}, +120^\circ\rangle \otimes |\text{down}, 0^\circ\rangle$$

$$+\ 1/\sqrt{5}\ |\text{up}, +120^\circ\rangle \otimes (-\sqrt{3}/2\ |\text{up}, -120^\circ\rangle + 1/2\ |\text{down}, -120^\circ\rangle)$$

$$+\ 1/\sqrt{5}\ |\text{down}, +120^\circ\rangle \otimes |\text{up}, 0^\circ\rangle$$

$$+\ 1/\sqrt{5}\ |\text{down}, +120^\circ\rangle \otimes (1/2\ |\text{up}, -120^\circ\rangle + \sqrt{3}/2\ |\text{down}, -120^\circ\rangle)$$

Non-locality revisited 12

$$\sqrt{3}/\sqrt{5}\ |\text{up}, +120^\circ> \otimes (1/2\ |\text{up}, -120^\circ> + \sqrt{3}/2\ |\text{down}, -120^\circ>)$$

$$+\ 1/\sqrt{5}\ |\text{up}, +120^\circ> \otimes (-\sqrt{3}/2\ |\text{up}, -120^\circ> + 1/2\ |\text{down}, -120^\circ>)$$

$$+\ 1/\sqrt{5}\ |\text{down}, +120^\circ> \otimes (1/2\ |\text{up}, -120^\circ> + \sqrt{3}/2\ |\text{down}, -120^\circ>)$$

Conclusion: when
LEFT = +120°
and
RIGHT = -120°
the particles never
both go up.

$$0\ |\text{up}, +120^\circ> \otimes |\text{up}, -120^\circ>$$

$$+\ 2/\sqrt{5}\ |\text{up}, +120^\circ> \otimes |\text{down}, -120^\circ>$$

$$+\ 1/2\sqrt{5}\ |\text{down}, +120^\circ> \otimes |\text{up}, -120^\circ>$$

$$+\ \sqrt{3}/2\sqrt{5}\ |\text{down}, +120^\circ> \otimes |\text{down}, -120^\circ>$$

Non-locality revisited 13

|up, 0°> = 1/2 |up, -120°> + √3/2 |down, -120°>

|down, 0°> = -√3/2 |up, -120°> + 1/2 |down, -120°>

1/2√5 |up, 0°> ⊗ |down, 0°>

1/2√5 |up, 0°> ⊗ (-√3/2 |up, -120°> + 1/2 |down, -120°>)

+ 2/√5 |down, 0°> ⊗ |up, 0°>

+ 2/√5 |down, 0°> ⊗ (1/2 |up, -120°> + √3/2 |down, -120°>)

+ √3/2√5 |down, 0°> ⊗ |down, 0°>

+ √3/2√5 |down, 0°> ⊗ (-√3/2 |up, -120°> + 1/2 |down, -120°>)

Non-locality revisited 14

$1/2\sqrt{5}\ |\text{up}, 0^\circ> \otimes (-\sqrt{3}/2\ |\text{up}, -120^\circ> + 1/2\ |\text{down}, -120^\circ>)$

$+\ 2/\sqrt{5}\ |\text{down}, 0^\circ> \otimes (1/2\ |\text{up}, -120^\circ> + \sqrt{3}/2\ |\text{down}, -120^\circ>)$

$+\ \sqrt{3}/2\sqrt{5}\ |\text{down}, 0^\circ> \otimes (-\sqrt{3}/2\ |\text{up}, -120^\circ> + 1/2\ |\text{down}, -120^\circ>)$

Conclusion: when LEFT = 0° and RIGHT = -120° the particles sometimes both go up (prob = 3/80).

$-\sqrt{3}/4\sqrt{5}\ |\text{up}, 0^\circ> \otimes |\text{up}, -120^\circ>$

$+\ 1/4\sqrt{5}\ |\text{up}, 0^\circ> \otimes |\text{down}, -120^\circ>$

$+\ 1/4\sqrt{5}\ |\text{down}, 0^\circ> \otimes |\text{up}, -120^\circ>$

$+\ 5\sqrt{3}/4\sqrt{5}\ |\text{down}, 0^\circ> \otimes |\text{down}, -120^\circ>$

The basic principles of qm

(1) The physical state of any system is represented by a **vector** in some vector space (usually an **infinite-dimensional** vector space; note that this will be a **different** vector space for each different system).

(2) If Φ is a vector representing one possible physical state of some system, and Ψ is another vector representing another possible physical state of that system, then any arbitrary **linear combination** $a\Phi + b\Psi$ also represents a possible physical state of the system. This is called the **principle of superposition**.

(3) Any **experiment** that can be performed on a system is represented by an **orthonormal basis** in the vector space for that system. Each basis element can be thought of as "labeled" with one of the possible **outcomes** of the experiment.

↑

We will now amend this principle.

Subspaces of vector spaces 1

Imagine a 5-dimensional vector space over the reals—think of it as just the set of all 5-tuples of real numbers (x_1,x_2,x_3,x_4,x_5).

Here is one such vector v_1: (1,2,3,4,5).

Here is another such vector v_2: (2,3,5,7,11).

Here is a linear combination of v_1 and v_2:

$$\underline{-0.68}\begin{pmatrix}1\\2\\3\\4\\5\end{pmatrix} + \underline{1.21}\begin{pmatrix}2\\3\\5\\7\\11\end{pmatrix} = \begin{pmatrix}1.74\\2.27\\4.01\\5.75\\9.91\end{pmatrix}$$

Subspaces of vector spaces 2

Imagine a 5-dimensional vector space over the reals—think of it as just the set of all 5-tuples of real numbers (x_1,x_2,x_3,x_4,x_5).

Here is one such vector v_1: (1,2,3,4,5).

Here is another such vector v_2: (2,3,5,7,11).

Here is another linear combination of v_1 and v_2:

$$\underline{208}\begin{pmatrix}1\\2\\3\\4\\5\end{pmatrix} + \underline{-106}\begin{pmatrix}2\\3\\5\\7\\11\end{pmatrix} = \begin{pmatrix}-4\\98\\94\\90\\-126\end{pmatrix}$$

Subspaces of vector spaces 3

Imagine a 5-dimensional vector space over the reals—think of it as just the set of all 5-tuples of real numbers (x_1,x_2,x_3,x_4,x_5).

Here is one such vector v_1: (1,2,3,4,5).

Here is another such vector v_2: (2,3,5,7,11).

Here is yet another linear combination of v_1 and v_2:

$$106\begin{pmatrix}1\\2\\3\\4\\5\end{pmatrix} + -55\begin{pmatrix}2\\3\\5\\7\\11\end{pmatrix} = \begin{pmatrix}-4\\47\\43\\39\\-75\end{pmatrix}$$

Subspaces of vector spaces 4

The set of **all** such linear combinations

is the **subspace** "spanned" by $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$\underline{\ A\ }\begin{pmatrix}1\\2\\3\\4\\5\end{pmatrix} + \underline{\ B\ }\begin{pmatrix}2\\3\\5\\7\\11\end{pmatrix} = \begin{pmatrix}A+2B\\2A+3B\\3A+5B\\4A+7B\\5A+11B\end{pmatrix}$$

Subspaces of vector spaces 5

How many dimensions does this subspace have?

That's right: **TWO**.

Observe that it has infinitely many orthonormal bases.

For example, these two vectors are orthogonal:

$$\begin{pmatrix} 2 \\ 3 \\ 5 \\ 7 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} -4 \\ 98 \\ 94 \\ 90 \\ -126 \end{pmatrix} = \begin{array}{r} -8 \\ +294 \\ +470 \\ +630 \\ -1386 \\ \hline 0 \end{array}$$

So we can scale them to get an ortho**normal** basis.

Subspaces of vector spaces 6

How many dimensions does this subspace have?

That's right: TWO.

Observe that it has infinitely many orthonormal bases.

Similarly, these two vectors are orthogonal:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 47 \\ 43 \\ 39 \\ -75 \end{pmatrix} = \begin{array}{r} -4 \\ +94 \\ +129 \\ +156 \\ -375 \\ \hline 0 \end{array}$$

So, again, we can scale them to get an orthonormal basis.

Degeneracy 1

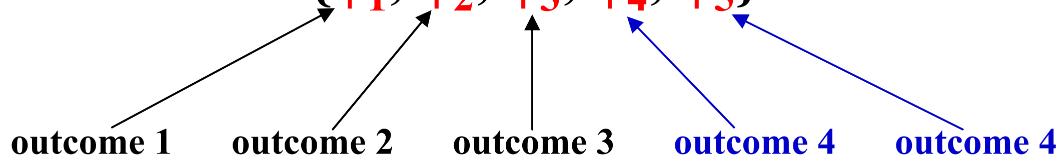
Suppose the vector space for our system has **five** dimensions.

Suppose the experiment (“measurement”) we are going to perform on it has only **four** possible outcomes.

Still, we represent that experiment by an orthonormal basis—which must have **five elements** in it.

So: Two of the elements must correspond to the **same outcome**:

$$\{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5\}$$



Degeneracy 2

Suppose our system is in the state ψ.

Then the probability that our "measurement" will yield outcome 4 is:

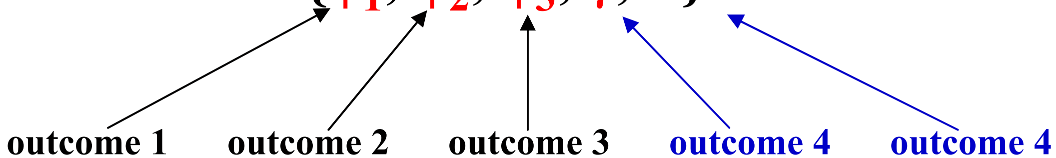
$$|\langle\phi_4 \mid \psi\rangle|^2 + |\langle\phi_5 \mid \psi\rangle|^2.$$

Let (normalized, orthogonal) vectors γ and ν span the same subspace as ϕ_4 and ϕ_5. Then:

$$|\langle\phi_4 \mid \psi\rangle|^2 + |\langle\phi_5 \mid \psi\rangle|^2 = |\langle\gamma \mid \psi\rangle|^2 + |\langle\nu \mid \psi\rangle|^2 .$$

So we could just as easily have used the following basis to represent our experiment:

$$\{\phi_1, \phi_2, \phi_3, \gamma, \nu\}$$



Degeneracy 3

To remove this unwanted redundancy:

Represent outcome 4 not by an orthonormal pair of vectors,

but rather by the subspace they span.

So: An experiment will now be represented by a set of pairwise-orthogonal subspaces, each corresponding to a distinct outcome. These subspaces span the entire vector space.

“Pairwise-orthogonal”?

Explanation: Subspace S_1 is orthogonal to subspace S_2 iff every vector in S_1 is orthogonal to every vector in S_2.

Restating the statistical algorithm

Suppose system S is in a state represented by the unit vector Ψ.

Suppose experiment E is performed on S, where E is represented by the set of pairwise-orthogonal subspaces $\{S_1, S_2, \ldots\}$, with subspace S_i corresponding to outcome i.

To calculate Prob(outcome i):

1. Project Ψ onto the subspace S_i.

> **This can be done by picking an arbitrary orthonormal basis $\{\phi_1, \phi_2, \ldots\}$ for S_i, and calculating**
> $< \phi_1 \mid \Psi > \phi_1 + < \phi_2 \mid \Psi > \phi_2 + \ldots$

2. Square the length of the resulting vector.

> **Given our choice of orthonormal basis $\{\phi_1, \phi_2, \ldots\}$ for S_i, this will equal**
> $|< \phi_1 \mid \Psi >|^2 + |< \phi_2 \mid \Psi >|^2 + \ldots$

3. The resulting number is Prob(outcome i).

Hermitian operators 1

An **operator** is simply a function that takes vectors as inputs and yields vectors as outputs: $A\Psi = \Phi$.

A **linear** operator has this additional nice feature:

$$A(a\Phi + b\Psi) = aA\Phi + bA\Psi.$$

A Hermitian operator has yet another nice feature:

$$\langle A\Phi \mid \Psi \rangle = \langle \Phi \mid A\Psi \rangle.$$

Finally, Φ is an eigenvector of A iff, for some number c,

$$A\Phi = c\Phi.$$

c is called the **eigenvalue** of Φ (for A). Note that A will have some **set** of eigenvalues.

Hermitian operators 2

Some cool results:

1. If Φ and Ψ are eigenvectors of (linear) operator A, with the same eigenvalue c, then so is (aΦ + bΨ).

That means that for each eigenvalue c of A, there is a subspace consisting of all and only those vectors with eigenvalue c for A. We call these subspace eigenspaces for A.

2. If Φ and Ψ are eigenvectors of Hermitian operator A, with different eigenvalues, then Φ and Ψ are orthogonal.

That means that for a Hermitian operator A, its eigenspaces are pairwise-orthogonal.

So a Hermitian operator automatically "picks out" a set of pairwise-orthogonal subspaces, each labeled with a distinct eigenvalue. That makes these operators particularly well suited to represent experiments.

Hermitian operators 3

Haven't we forgotten something? After all, we can also show:

3. If Φ is an eigenvector of Hermitian operator A, then the eigenvalue c is a real number.

Supposedly this is a BIG DEAL, since "only real numbers could be the values of some measured physical quantity".

Exercise: Explain why this claim is—notwithstanding its prominent position in just about every quantum mechanics textbook—COMPLETE AND UTTER NONSENSE.

Two measurement problems 1

Schrödinger's Equation: the state of the world evolves, at all times, in accordance with Schrödinger's Equation.

Okay, okay: "observable".

Von Neumann's Rule: A system **S** has value **v** for physical quantity **Q** iff **S** is in an eigenstate with eigenvalue **v** of the Hermitian operator **A** that represents **Q**.

Okay, okay: "measurement".

Born's Rule: If a **Q**-experiment is performed on system **S** in state Ψ, then the expected value of the outcome is $<\Psi \mid A\Psi>$. (This turns out to be equivalent to our statistical algorithm.)

Two measurement problems 2

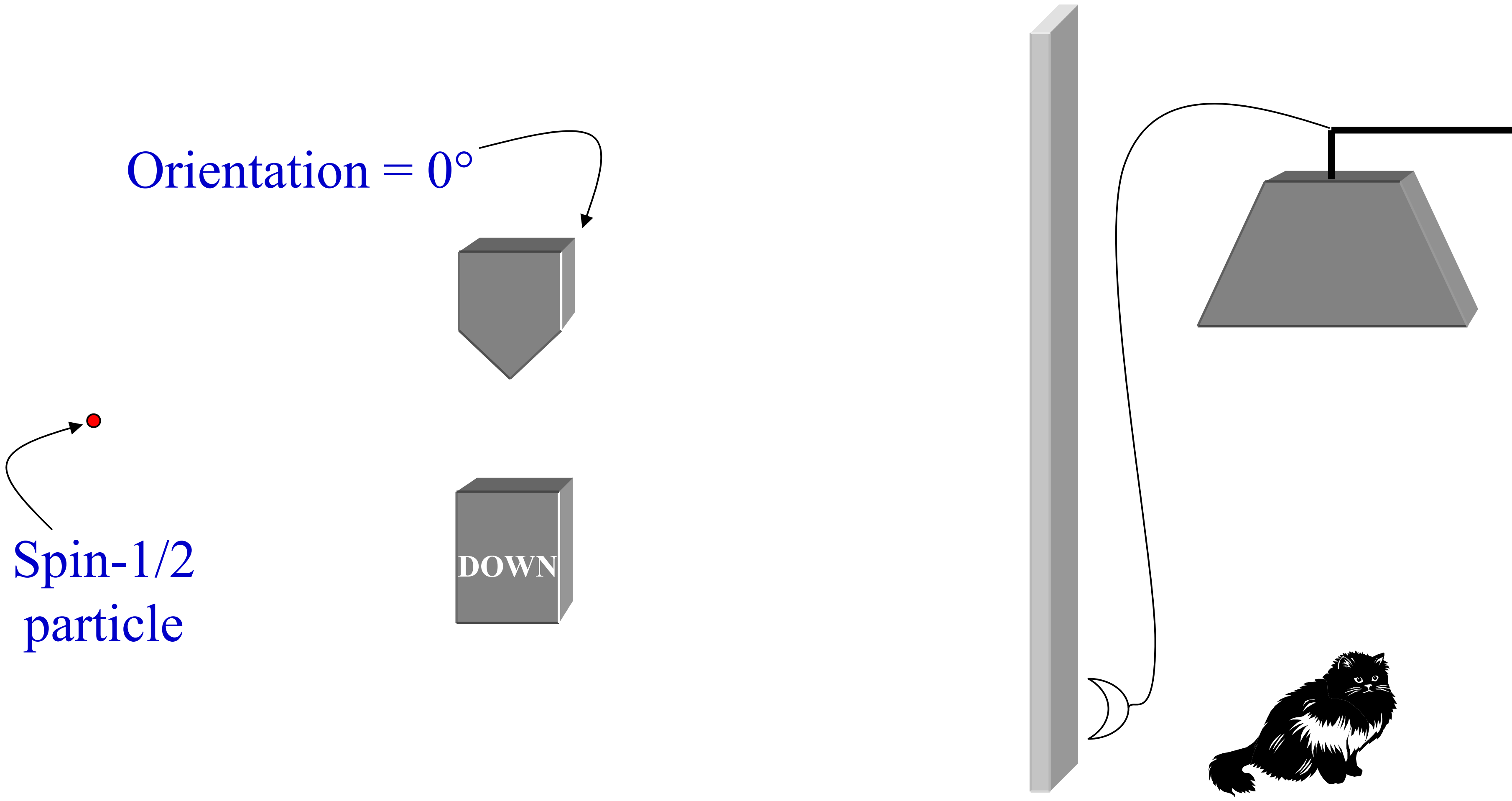
Schrödinger's Equation + von Neumann's Rule gives us one problem:

Systems will sometimes possess no value for any recognizable physical quantity.

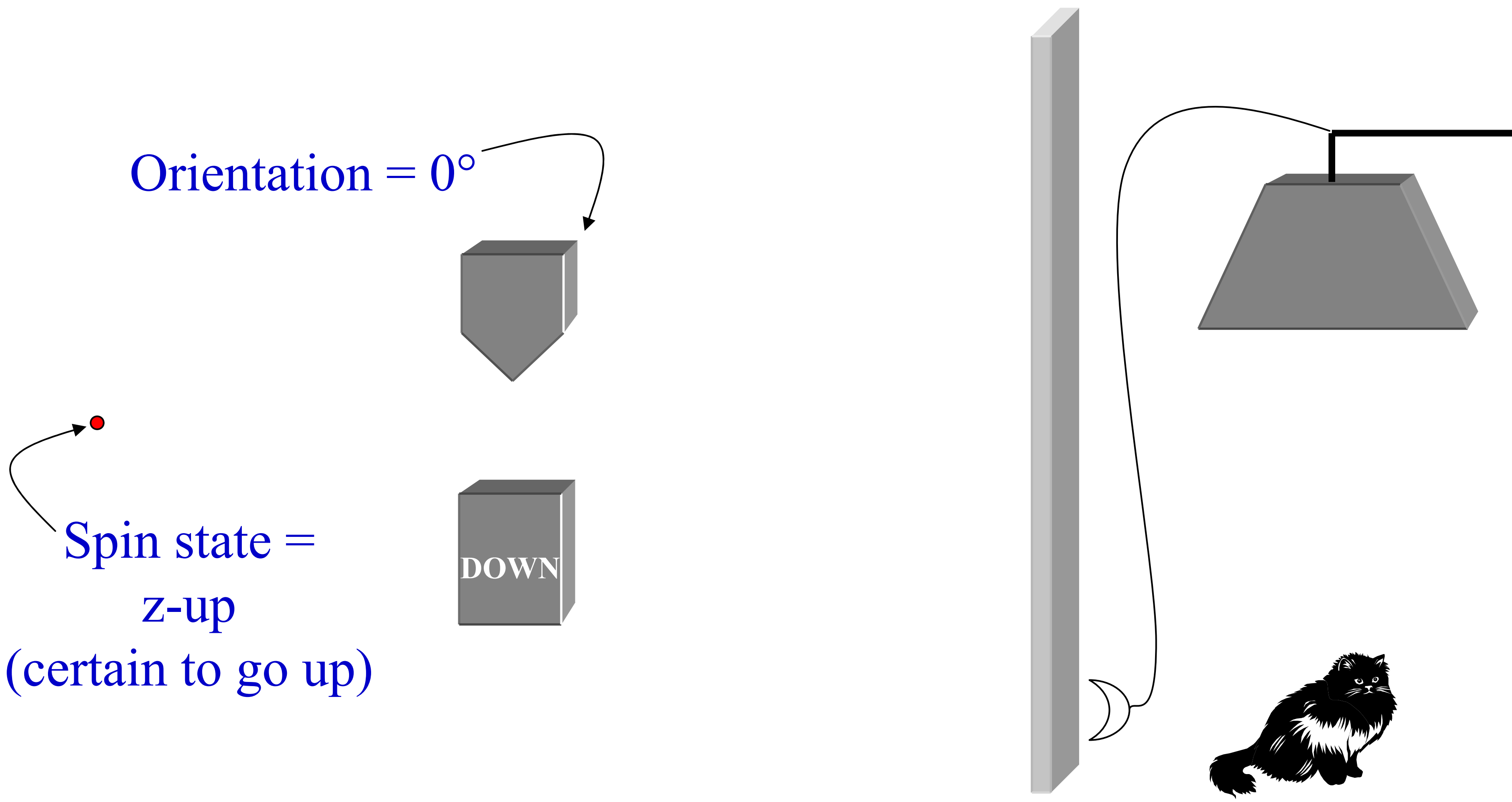
Schrödinger's Equation + Born's Rule gives us another:

CONTRADICTION

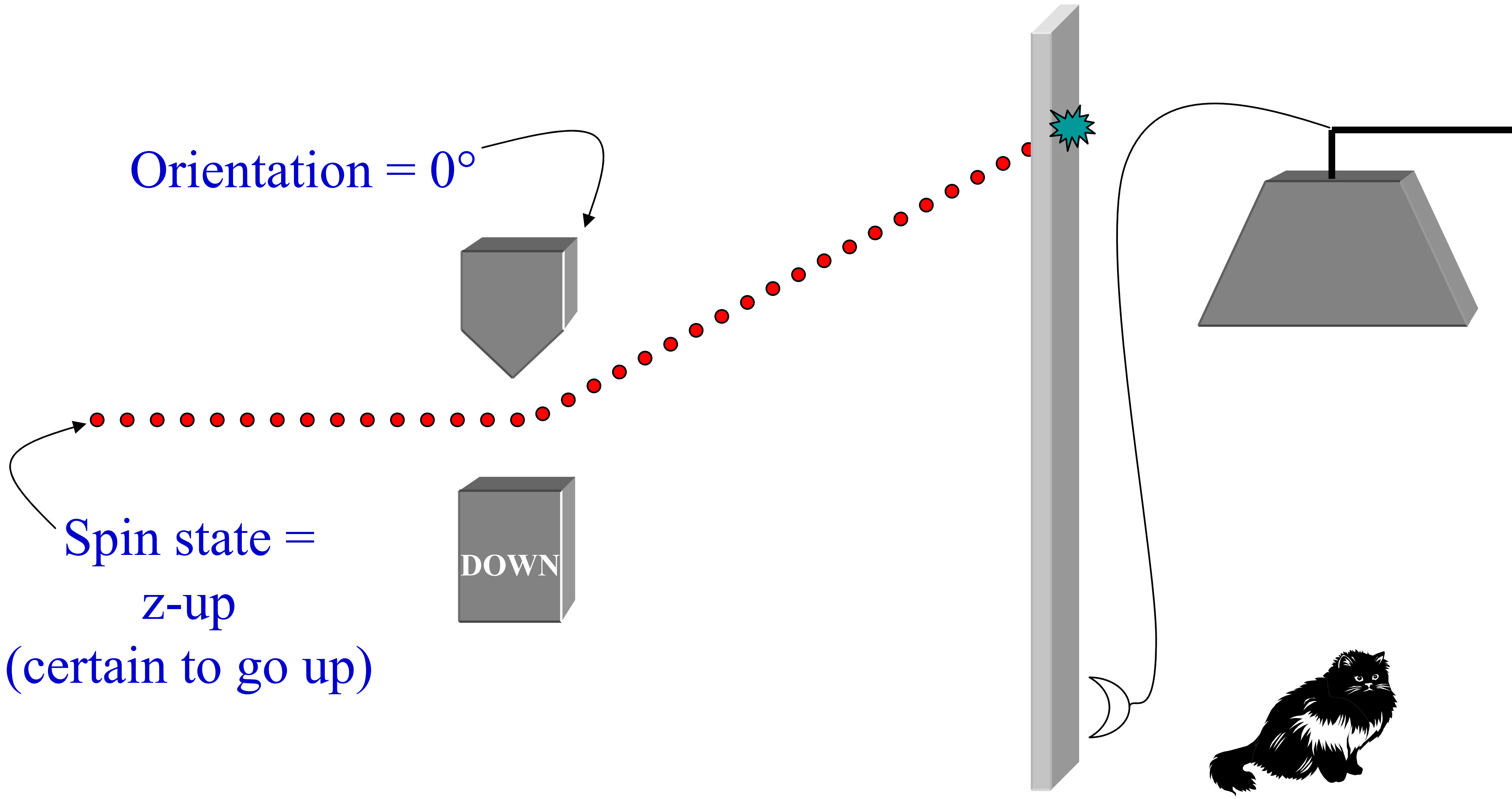
Illustration: Schrödinger's Cat



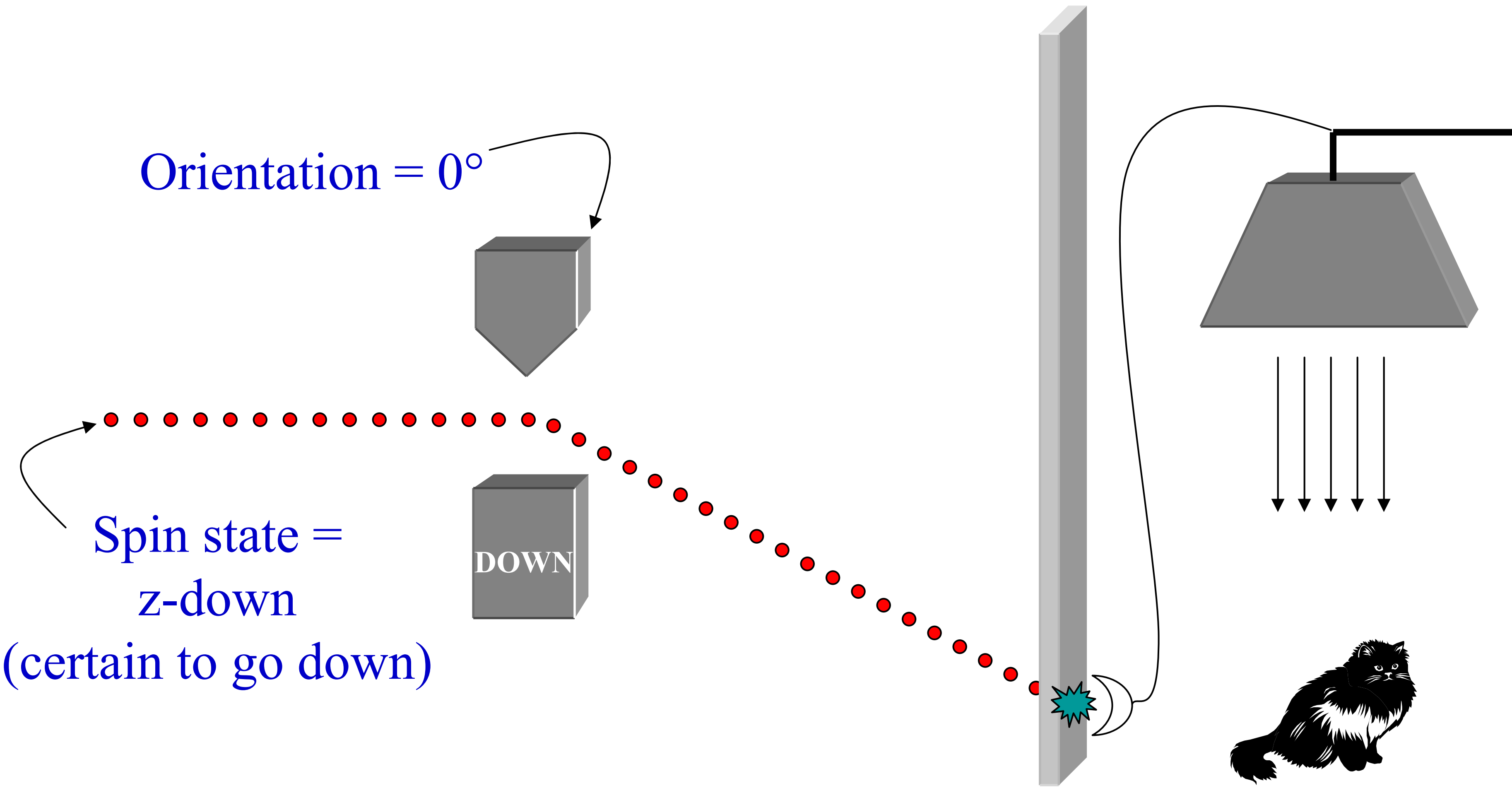
The nice case



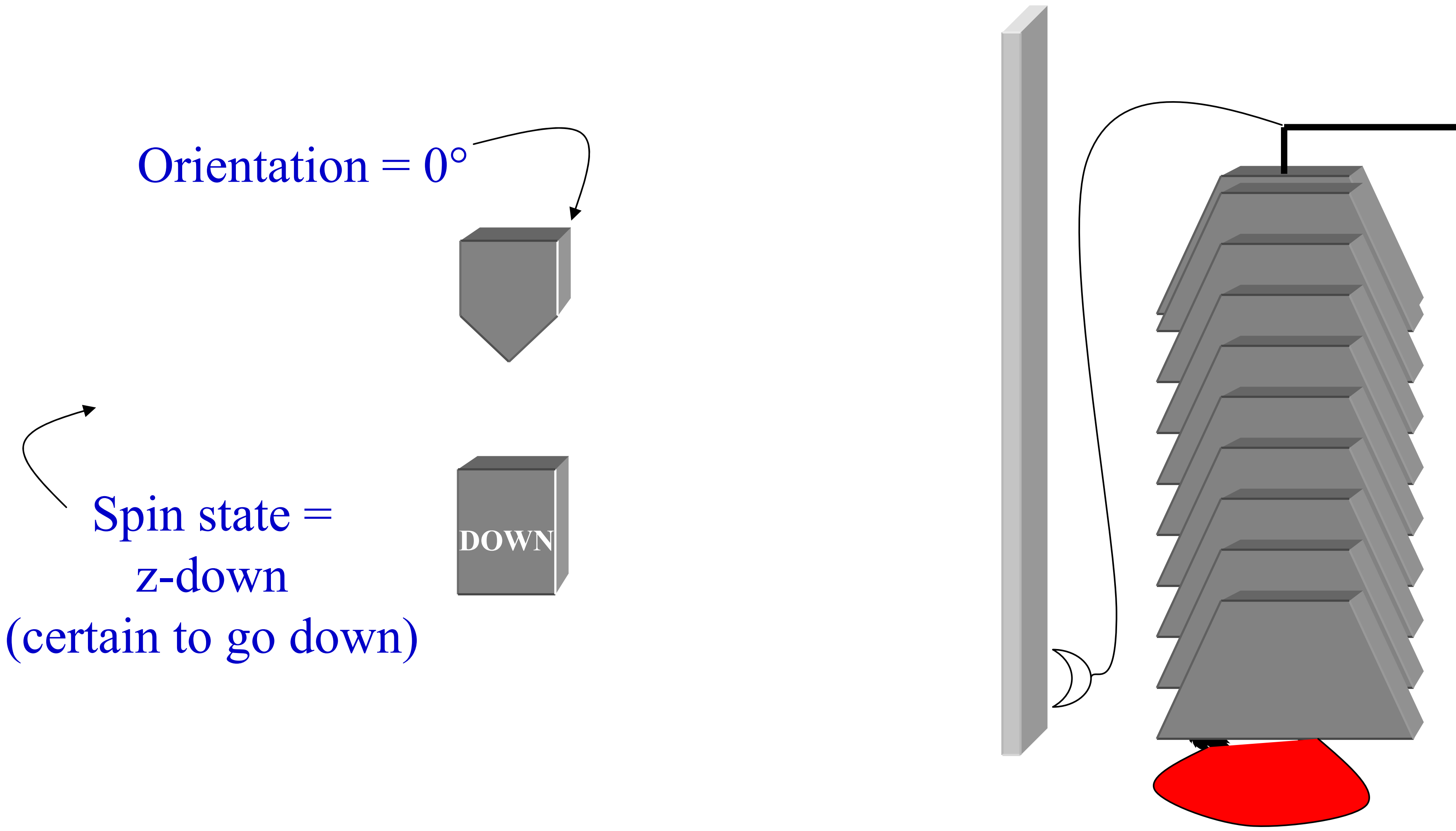
The nice case



The not-so-nice case



The not-so-nice case



The problem

Nice case: final quantum mechanical state will be

|z-up> |no flash detected> |block suspended> |Fluffy purring>

Not-so-nice case: final quantum mechanical state will be

|z-down> |flash detected> |block has fallen> |Fluffy squished>

So, if the initial spin state is

|x-up> = 1/√2(|z-up> + |z-down>)

Then the final quantum mechanical state will be

1/√2(|z-up> |no flash detected> |block suspended> |Fluffy purring>
+ |z-down> |flash detected> |block has fallen> |Fluffy squished>).

<u>WHAT KIND OF STATE IS THAT?!?</u>

<u>AND WHERE ARE OUR TWO POSSIBLE OUTCOMES?</u>

The standard menu of options

1. Add extra variables.
    - Bohmian mechanics, modal interpretations, some versions of Many Minds
2. Add non-linear, stochastic dynamics.
    - textbook "collapse" theories, GRW
3. Give up on Born's Rule without admitting it.
    - Many Worlds, other versions of Many Minds
4. Go soft in the head.
    - decoherence, the "bare theory"