Logic I
Fall 2009
Session 2 Handout

Things to know from last time:

- Validity. A(n) . . . . is valid iff . . .
- Soundness. A(n) . . . . is sound iff . . .
- Connectives. Molecular / compound sentences. Atomic / simple sentences.
- Truth-functionality. A connective is truth-functional iff . . .

Also: a couple things we didn't get to last time:

- Logical truth / falsehood
- Logical indeterminacy
- Logical consistency
- Logical equivalence

The plan for today: Syntax of SL, connectives, truth-tables, translation issues.

- The symbols of SL: A, B, C, . . . , H_7, J_3, . . . , $\sim$, &, $\vee$, $\supset$, $\equiv$, (,).
- A, . . . , G_{34} are *atomic sentences* or *sentence letters*
- $\sim$ (or $\neg$) forms a . . .
  It corresponds roughly to the English phrase . . .
- & (or $\wedge$) forms a . . .
  The two sentences it connects are . . .
  It corresponds roughly to the English term . . .
- $\vee$ forms a . . .
  The two sentences it connects are . . .
  It corresponds roughly to the English term . . .
- $\supset$ (or $\rightarrow$) forms a . . .
  The left-hand side is the . . .
  The right-hand side is the . . .
  It corresponds roughly to the English terms . . .

- $\equiv$ (or $\leftrightarrow$) forms a ...
  It corresponds roughly to the English terms ...

- The set of sentences of SL is the smallest set $\mathscr{F}$ such that:
  - Every atomic sentence is in $\mathscr{F}$.
  - If ϕ and ψ are in $\mathscr{F}$, so are $\sim\phi$, $(\phi \mathbin{\&} \psi)$, $(\phi \vee \psi)$, $(\phi \supset \psi)$, and $(\phi \equiv \psi)$.

- Truth-functional connectives correspond to *functions* from truth-values to truth-values.
  - For a *one-place connective*, $f : \{T, F\} \rightarrow \{T, F\}$.
  - For a *two-place connective*, $f : \{T, F\} \times \{T, F\} \rightarrow \{T, F\}$.

- *Truth-tables* illustrate these functions. Fill in the simple truth-table for $\sim$:

A	$\sim$ A

- A *two-place connective* will have a larger truth-table.

A	B	A ... B

English and SL: Translations and problems

- Not all uses of English connectives are (or at least appear) truth-functional.
- 'And'
- 'Or'
- 'If ... then'. In SL:
  - Contraposition: (A $\supset$ B) implies ($\sim$B $\supset$ $\sim$A).
  - Strengthening: (A $\supset$ B) implies ((A & B) $\supset$ B).
  - (A $\supset$ B) or (B $\supset$ A).
  - $\sim$(A $\supset$ B) iff A and $\sim$B.

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