Subject 24.242. Logic II. Sample problems from the third homework, due Thursday, March 18

For each term τ, we have defined a code number $\ulcorner\tau\urcorner$, according to the following prescription:

$\ulcorner 0\urcorner = \text{Pair}(1,1)$.
$\ulcorner x_i\urcorner = \text{Pair}(2,i)$.
$\ulcorner s\tau\urcorner = \text{Pair}(4, \ulcorner\tau\urcorner)$
$\ulcorner(\tau + \rho)\urcorner = \text{Pair}(5,\text{Pair}(\ulcorner\tau\urcorner, \ulcorner\rho\urcorner))$.
$\ulcorner(\tau \cdot \rho)\urcorner = \text{Pair}(6,\text{Pair}(\ulcorner\tau\urcorner, \ulcorner\rho\urcorner))$.
$\ulcorner(\tau\ E\ \rho)\urcorner = \text{Pair}(7,\text{Pair}(\ulcorner\tau\urcorner, \ulcorner\rho\urcorner))$.

Pair(x,y) is, you will recall, $\frac{1}{2}(x + y)(x + y + 1) + x$.

1. Give the Arabic numeral for $\ulcorner(0 + 0)\urcorner$.

2. Show that a set of natural numbers is decidable if and only if it is either finite or the range of an increasing calculable total function. (A total function f is *increasing* iff, for any x and y, if $x < y$, then $f(x) < f(y)$.)