Subject 24.242. Logic II. Answers to the April 8 homework.

Define the *restricted sentential calculus* (*RSC*) as follows: The atomic RSC sentences are "A_0," "A_1," "A_2,", and so on, and the RSC sentences are the atomic sentences and expressions formed from RSC sentences by one or more applications of the operations of forming conditionals (sentences of the form $(\phi \rightarrow \psi)$) and negations (sentences of the form $\sim\phi$). A *normal truth assignment* (*NTA*) is a functions $\mathfrak{I}$ taking the RSC sentences to {0,1} that meets these conditions, for any ϕ and ψ: $\mathfrak{I}((\phi \rightarrow \psi)) = 1$ iff either $\mathfrak{I}(\phi) = 0$ or $\mathfrak{I}(\psi) = 1$ (or both); $\mathfrak{I}(\sim\phi) = 1$ iff $\mathfrak{A}(\phi) = 0$. An RSC sentence is a *tautology* iff it's assigned the value 1 by every NTA.

An *RSC derivation* is a finite sequence of sentences, each with an associated finite premise set of sentences, conforming to the following rules:

(PI) You may write ϕ with $\{\phi\}$ as its premise set.
(CP) If you've written ψ with Γ as its premise set, you may write $(\phi \rightarrow \psi)$ with $\Gamma \sim \{\phi\}$ as its premise set.
(MP) If you've written $(\phi \rightarrow \psi)$ with Γ as its premise set and you've written ϕ with Δ as its premise set, you may write ψ with $\Gamma \cup \Delta$ as its premise set.
(MT) If you've written $(\sim\phi \rightarrow \sim\psi)$ with Γ as its premise set and you've written ψ with Δ as its premise set, you may write ϕ with $\Gamma \cup \Delta$ as its premise set.

We encode the RSC numerically by associating with each RSC sentence ϕ a code number $\ulcorner\phi\urcorner$, according to the following stipulation:

$\ulcorner A_i \urcorner = \text{Pair}(1,i)$.
$\ulcorner(\phi \rightarrow \psi)\urcorner = \text{Triple}(2, \ulcorner\phi\urcorner, \ulcorner\psi\urcorner) = \text{Pair}(2,\text{Pair}(\ulcorner\phi\urcorner, \ulcorner\psi\urcorner))$
$\ulcorner\sim\phi\urcorner = \text{Pair}(3, \ulcorner\phi\urcorner)$

1. Give a derivation of the following sentences from the empty set of premises:
I indicate the premise set of a line by listing the line numbers.
(a) $((A_0 \rightarrow A_1) \rightarrow ((A_1 \rightarrow A_2) \rightarrow (A_0 \rightarrow A_2)))$

{1)	**1. $(A_0 \rightarrow A_1)$**	**PI**
{2}	**2. $(A_1 \rightarrow A_2)$**	**PI**
{3}	**3. A_0**	**PI**
{1,3}	**4. A_1**	**MP, 1, 3**
{1,2,3}	**5. A_2**	**MP, 2, 4**
{1,2}	**6. $(A_0 \rightarrow A_2)$**	**CP, 3, 5**
{1}	**7. $((A_1 \rightarrow A_2) \rightarrow (A_0 \rightarrow A_2))$**	**CP, 2,6**
∅	**8. $((A_0 \rightarrow A_1) \rightarrow ((A_1 \rightarrow A_2) \rightarrow (A_0 \rightarrow A_2)))$**	**CP, 1, 7**

2. Show that the set of codes of RSC sentences in Σ. (You don't need to go all the way to primitive notation here; you may use reasonable abbreviations.)
x is the code of an RSC sentence iff
$(\exists s)$(s is a finite sequence $\wedge$ x $\in$ s $\wedge$ $(\forall y < s)$(y $\in$ s $\rightarrow$ ((1st(y) = [1] $\vee$ 1st(y) = [2] $\vee$1st(y) = [3] $\wedge$ (1st(y) = [2] $\rightarrow$ (2ndin3(y) $\in$ s $\wedge$ 3rdin3(y) = s)) $\wedge$ (1st(y) = [3] $\rightarrow$ 2nd(y) $\in$ s))))