Subject 24.242. HW Sample answers.

- 1. Write a register program that calculates (x + y).
 - 1. If Register 2 is 0, go to 5.
 - 2. Subtract 1 from Register 2, unless it's 0.
 - 3. Add 1 to register 1.
 - 4. Go to 1.
 - 5. STOP.
- 2. Show that a set is Δ if and only if its characteristic function is Σ . (The *characteristic function* χ_S of a set S is given by stipulating that $\chi_S(n) = 1$ if $n \in S$, and it's equal to 0 if $n \notin S$.
 - (⇒) If the set S is Δ , then there are bounded formulas $\varphi(x,y)$ and $\psi(x,y)$ such that $S = \{x: (\exists y)\varphi(x,y), \text{ and its complement is } \{x: (\exists y)\psi(x,y)\}.$ Then χ_S is equal to $\{\langle x,z\rangle: (\exists y)((\varphi([x],y) \land z = s0) \lor (\psi(x,y) \land z = 0)\}.$
 - (\Leftarrow) Suppose χ_S is Δ ; say it's $\{\langle x,y\rangle: (\exists z)\theta([x],[y],z)$. The S is equal to $\{x: (\exists z)\theta([x],s0,z), \text{ and its complement is } \{x: (\exists z)\theta([x],0,z)\}.$