

Subject 24-242. Logic II. Answers to the Seventh Homework

1. Take a sentence  $\alpha$  so that  $\alpha$  is provably equivalent to  $(Bew_{PA}([\ulcorner\alpha\urcorner]) \vee Bew_{PA}([\ulcorner\neg\alpha\urcorner]))$ . Is  $\alpha$  decidable in PA? Is it true?

**PA  $\vdash (Bew_{PA}([\ulcorner\alpha\urcorner]) \rightarrow \alpha)$ , and so, by Löb's Theorem,  $\alpha$  is true and provable.**

2. Show that, for each  $n$ , one can find an arithmetical formula  $\tau_n$  such that, for each sentence  $\phi$ , PA  $\vdash ([\ulcorner\phi\urcorner] < [n] \rightarrow (\tau_n([\ulcorner\phi\urcorner]) \leftrightarrow \phi))$ .

**Let's say the sentences less than  $n$  are  $\phi_1, \phi_2, \dots, \phi_m$ . We can take  $\tau_n(x)$  to be the formula  $((x = [\ulcorner\phi_1\urcorner] \wedge \phi_1) \vee (x = [\ulcorner\phi_2\urcorner] \wedge \phi_2) \vee \dots \vee (x = [\ulcorner\phi_m\urcorner] \wedge \phi_m))$ .**