## Schlenker 2008 (only the basic cases)

For Schlenker (like Heim, Karttunen and Stalnaker), the problem of projection is the following:

Given a minimal sentence, $S$, with presupposition $p$ (henceforth $S_{p}$ ), ${ }^{1}$ define constraints on the CG for the assertion of various sentences that dominate $S$.

Strategy: let's deal with another problem, namely that of explaining various conditions on the assertability of conjunctive sentences or sentences that dominate them.

## 1. On the distribution of conjunctions

(1) Let C be a context of utterance in which the participants - speaker and addressee(s) - share the belief that a given sentence, $S_{1}$, is true. In C, an utterance of the conjunction $S_{1}$ and $S_{2}$ is odd.

Example of a relevant Context:
Mary just announced that she is pregnant.
Mary continues:
\#I am pregnant and I plan to buy many toys for the child I hope to have.
(2) \#If Mary is pregnant, she is pregnant and is planning to buy many toys for the child she's gonna have.

### 1.1. Global Redunandancy

## (3) Global Redundancy Condition

a. A sentence that has the conjunction $p$ and $q$ as a sub-constituent, $\varphi(p \wedge q)$, is not assertable given a context-set, C , if either p or q is globally redundant in $\varphi$ given C .
b. A conjunct $p(r e s p . q$ ) is globally redundant in $\varphi(p \wedge q)$ given C, if $\varphi(p \wedge q)$ conveys exactly the same information given C as $\varphi(\mathrm{q})$ (resp. $\varphi(\mathrm{p})$ )
c. The information that a sentence $\varphi$ conveys in $C:=C \cap\left\{\mathrm{w}: \llbracket \varphi \rrbracket^{\mathrm{w}}=1\right\}$

While this condition might seem to have the ring of truth, it can't be right as is:
(4) a. \#Mary is expecting a daughter, and she is pregnant.
b. Mary is pregnant, and she is expecting a daughter.

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### 1.2. Dynamic Triviality

-There is an account of this paradigm based on Heim's CCPs.
-The denotation of a sentence is a context change potential, a function from a context set to a subset thereof.
-This update is dynamic. Instead of assuming that a sentence updates the common ground C by simply adding the proposition it expresses to C, the update procedure for a complex sentence $S$ consists of a sequence of intermediate updates determined by the constituent structure of $S$.
-As we've seen, each expression in the language is associated by its lexical entry with a particular update rule. For conjunction the rule states that $S_{1}$ and $S_{2}$ updates C in the following fashion: first C is updated by $S_{1}$ and then the result of this update is updated by $S_{2}$. For other lexical entries there are other lexical rules which I will not repeat here.
(5) Dynamic Triviality Condition: at every step of dynamic update, the context set C cannot entail the constituent that updates C.
(van der Sandt 1992, with obvious roots in Stalnaker 1978).

### 1.3. Incremental Redundancy

As we've seen in the case of presuppositions, the achievement here depends on the stipulative nature of the CCPs.

An alternative:

## (6) Incremental Redundancy

a. A sentence, $\varphi(p \wedge q)$, is not assertable in $C$ if either $p$ or $q$ is incrementally redundant in $\varphi$ given C.
b. p is incrementally redundant in $\varphi(\mathrm{p} \wedge q)$ if it is globally redundant given C in all $\varphi^{\prime} \in \operatorname{GOOD}-\operatorname{FINAL}(\wedge, \varphi)$.
$\mathrm{b}^{\prime}$. q is incrementally redundant in $\varphi(\mathrm{p} \wedge \mathrm{q})$ if it is globally redundant given C in all $\varphi^{\prime} \in \operatorname{GOOD}-\operatorname{FINAL}(q, \varphi)$.
c. A conjunct $p$ (resp. $q$ ) is globally redundant in $\varphi(p \wedge q)$, if $\varphi(p \wedge q)$ conveys exactly the same information given C as $\varphi(\mathrm{q})$ (resp. $\varphi(\mathrm{p})$ ).
d. $\varphi^{\prime} \in$ GOOD-FINAL $(\alpha, \varphi)$ iff it is obtained from $\varphi$ by replacing any number of $\varphi$-constituents pronounced after alpha.
(6) is clearly preferable to (5) as the theory of the assertability of conjunctive senteces. Schlenker's point is that it can become a theory of presupposition projection once we realize a connection between the assertability of sentences that dominate a presupposition trigger and the assertability of certain conjunctive sentences.

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## 3. Core Generalization

(7) Schlenker's Generalization: A sentence, $\varphi$, that dominates $S_{p}, \varphi\left(S_{p}\right)$ is assertable in a context $C$ only if $\varphi\left(p \wedge S_{p}\right)$ is not assertable in $C$.

## Evidence:

(8) Context: Mary just announced that she is pregnant. Mary's Husband:
a. \#She is pregnant and she is happy about the fact that she is pregnant.
b. She is happy about the fact that she is pregnant.
(9) a. \#If Mary is pregnant, she is pregnant and she knows that she is pregnant.
b. If Mary is pregnant, she knows that she is pregnant.
(10) a. \#Mary is pregnant and she is pregnant and she knows that she is pregnant.
b. Mary is pregnant and she knows that she is pregnant.

Schlenker's Generalization is entirely expected in Heim's framework (as we already mentioned, probably should be called van der Sandt's generalization).

However, Schlenker observes (as we already mentioned) that there is a predictive statement of the environments in which $\varphi\left(p \wedge S_{p}\right)$ is not assertable, and subsequently a predictive statement of the projection properties.
3. Basic setup (restricting attention to cases in which $S_{p}$ is final)
(11) Assumption about the representation of $\mathrm{S}_{\mathrm{p}}$ :
$S_{p}$ has a classical semantics (non-partial), and $S_{p}$ entails p.
E.g., The king of France is bald receives the Russellian semantics.

Assuming no independent factors are involved, we get the following for a sentence $\varphi$ which contains only one presupposition trigger (where $S_{p}$ is the minimal sentence dominating the trigger)

$$
\begin{equation*}
\text { Assertable }\left(\mathrm{C}, \varphi\left(\mathrm{~S}_{\mathrm{p}}\right)\right) \text { iff } \neg \text { Assertable }\left(\mathrm{C}, \varphi\left(\mathrm{p} \wedge \mathrm{~S}_{\mathrm{p}}\right)\right) \tag{12}
\end{equation*}
$$

Restricting ourselves to cases in which $\mathrm{S}_{\mathrm{p}}$ is the last constituent in $\varphi$ we get the following:
(13) If no items in $\varphi$ follow $S_{p}$, we get:

$$
\text { Assertable }\left(\mathrm{C}, \varphi\left(\mathrm{~S}_{\mathrm{p}}\right)\right) \text { iff } \neg \text { Assertable }\left(\mathrm{C}, \varphi\left(\mathrm{p} \wedge \mathrm{~S}_{\mathrm{p}}\right)\right) \text { iff } \forall \mathrm{r}\left[\varphi(\mathrm{p} \wedge \mathrm{r}) \Leftrightarrow_{\mathrm{C}} \varphi(\mathrm{r})\right]
$$

Or, we could skip the middle man:

$$
\begin{equation*}
\text { If no items in } \varphi \text { follow } S_{p} \text {, we get: } \tag{14}
\end{equation*}
$$

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$$
\text { Assertable }\left(\mathrm{C}, \varphi\left(\mathrm{~S}_{\mathrm{p}}\right)\right) \text { iff } \forall \mathrm{r}\left[\varphi(\mathrm{p} \wedge \mathrm{r}) \Leftrightarrow_{\mathrm{C}} \varphi(\mathrm{r})\right]
$$

### 3.1. Unembedded $S_{p}$

$S_{p}$ is assertable in $C$ iff $p \wedge S_{p}$ is unassertable in C.
$p \wedge S_{p}$ is unassertable in $C$ (given (6)), iff $[p \wedge r] \Leftrightarrow_{C} r$, for any choice of $r$, i.e., iff $C \Rightarrow p$.
Hence $S_{p}$ will presuppose $p$.

## 3.2. $\neg S_{p}$

$\neg S_{p}$ is assertable in C iff $\neg\left[\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right]$ is unassertable in C.
$\neg\left[\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right]$ is unassertable in C (given (6)), iff $\neg[\mathrm{p} \wedge \mathrm{r}] \Leftrightarrow_{\mathrm{C}} \neg \mathrm{r}$, for any choice of r , i.e., iff $\mathrm{C} \Rightarrow \mathrm{p}$. Hence $\neg \mathrm{S}_{\mathrm{p}}$ will presuppose p .

## 3.3. $\mathrm{S}_{1} \wedge \mathrm{~S}_{\mathrm{p}}$

$\mathrm{S}_{1} \wedge \mathrm{~S}_{\mathrm{p}}$ is assertable in C iff $\mathrm{S}_{1} \wedge\left(\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right)$ is unassertable in C .
$S_{1} \wedge\left(p \wedge S_{p}\right)$ is unassertable in $C$ (given (6)), iff $\left[S_{1} \wedge(p \wedge r)\right] \Leftrightarrow S_{C} \wedge r$, for any choice of $r$, i.e., iff $\mathrm{C} \wedge \mathrm{S}_{1} \Rightarrow$ p.

Hence $\mathrm{S}_{1} \wedge \mathrm{~S}_{\mathrm{p}}$ will presuppose $\mathrm{S}_{1} \rightarrow \mathrm{p}$.

## 3.4. $\mathrm{S}_{1} \rightarrow \mathrm{~S}_{\mathrm{p}}$

$\mathrm{S}_{1} \rightarrow \mathrm{~S}_{\mathrm{p}}$ is assertable in C iff $\mathrm{S}_{1} \rightarrow\left(\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right)$ is unassertable in C.
$S_{1} \rightarrow\left(p \wedge S_{p}\right)$ is unassert. in $C$ (given (6)), iff $\left[S_{1} \rightarrow(p \wedge r)\right] \Leftrightarrow_{C} S_{1} \rightarrow r$, for any choice of $r$, i.e., iff $\mathrm{C} \wedge \mathrm{S}_{1} \Rightarrow$ p.

Hence $\mathrm{S}_{1} \rightarrow \mathrm{~S}_{\mathrm{p}}$ will presuppose $\mathrm{S}_{1} \rightarrow \mathrm{p}$.
3.5. $\mathrm{S}_{1} \vee \mathrm{~S}_{\mathrm{p}}$
$S_{1} \vee S_{p}$ is assertable in C iff $S_{1} \vee\left(p \wedge S_{p}\right)$ is unassertable in C.
$S_{1} \vee\left(p \wedge S_{p}\right)$ is unassertable in $C$ (given (6)), iff $S_{1} \vee(p \wedge r) \Leftrightarrow_{C} S_{1} \vee r$, for any choice of $r$, i.e., iff $\mathrm{C} \wedge \neg \mathrm{S}_{1} \Rightarrow \mathrm{p}$.

Hence $S_{1} \vee S_{p}$ will presuppose $\neg S_{1} \rightarrow p$.

## 4. Cases in which $S_{p}$ is final

## 4.1. $\mathrm{S}_{\mathrm{p}} \wedge \mathrm{S}_{1}$

$\mathrm{S}_{\mathrm{p}} \wedge \mathrm{S}_{1}$ is assertable in C iff $\left(\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right) \wedge \mathrm{S}_{1}$ is unassertable in C .
$\left(p \wedge S_{p}\right) \wedge S_{1}$ is unassertable in C (given (6)), iff $[(p \wedge r) \& r] \Leftrightarrow_{C}[(r \& r]$, for any choice of sentences $r$, $r^{\prime}$, and connective $\boldsymbol{\&}$, i.e., iff $C \Rightarrow p$.
Hence $S_{p} \wedge S_{1}$ will presuppose $p$.

## 4.2. $\mathrm{S}_{\mathrm{p}} \rightarrow \mathrm{S}_{1}$

$\mathrm{S}_{\mathrm{p}} \rightarrow \mathrm{S}_{1}$ is assertable in C iff $\left(\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right) \rightarrow \mathrm{S}_{1}$ is unassertable in C.
$\left(p \wedge S_{p}\right) \rightarrow S_{1}$ is unassertable in C (given (6)), iff $[(p \wedge r) \neq r] \Leftrightarrow C[(r * r]$, for any choice of sentences r , $\mathrm{r}^{\prime}$, and connective $\boldsymbol{*}$, i.e., iff $\mathrm{C} \Rightarrow \mathrm{p}$.
Hence $S_{p} \wedge S_{1}$ will presuppose $p$.

## 4.3. $\mathrm{S}_{\mathrm{p}} \vee \mathrm{S}_{1}$

$S_{p} \vee S_{1}$ is assertable in $C$ iff $\left(p \wedge S_{p}\right) \vee S_{1}$ is unassertable in C.
$\left(p \wedge S_{p}\right) \vee S_{1}$ is unassertable in $C$ (given (6)), iff $[(p \wedge r) \& r] \Leftrightarrow_{C}[(r \& r]$, for any choice of sentences $r$, $r^{\prime}$, and connective $\AA$, i.e., iff $C \Rightarrow p$.

## 5. Is disjunction Symmetric?

Evidence for symmetry:
(15) a. There is no bathroom here or the bathroom is on the second floor.
b. The bathroom is on the second floor or there is no bathroom here.

If it's symmetric, we need a different theory, and Schlenker considers a non-incremental version of his theory.

However, he considers also another possibility (based on a proposal made to him by Spector).

There is another proposal hinted at in Heim (1990), based on ideas by Soames and by Gazdar, namely that presuppositions can get cancelled in certain environments by a cancelation process (local accommodation in Heim's framework.

Cancelation in Schlenker's framework amounts to exemption of $S_{p}$ from its assertability condition (from competition with $p \wedge S_{p}$ )

Cancelation (as suggested by Heim) is only allowed if its avoidance leads to a fatal result.
In (15)b, if we avoid cancelation, the second disjunct contradicts the common ground and that is generally not allowed. (Homework: relate to Hurford's constraint)

Spector's judgment (which I am sympathetic with): (16)a doesn't have a presupposition (associated with the definite her illness) and (16)b does.
(16) a. Either she has no disease with detectable symptoms, or her illness will be evident to the doctor.
b. Either her illness will be evident to the doctor, or she has no disease with detectable symptoms.

## Incremental-Presupposition-Transparency (IPT)

Constraint on Conjunction (version 2)
A sentence, $X$, which dominates $\mathrm{p} \wedge \mathrm{q}, \mathrm{X}(\mathrm{p} \wedge \mathrm{q})$, is not assertable in C if the first conjunct is idle no matter what comes after the first conjunct:
$\forall \mathrm{X}^{\prime} \in \operatorname{Good}-$ final $(\mathrm{X}, \wedge)\left[\mathrm{X}^{\prime}(\mathrm{p} \wedge \mathrm{r}) \Leftrightarrow_{\mathrm{C}} \mathrm{X}(\mathrm{r})\right]$
$\mathrm{X}^{\prime} \in \operatorname{Good}-\mathrm{final}(\mathrm{X}, \mathrm{a})$ if $\mathrm{X}^{\prime}$ can be derived from X by replacing constituents that follow a in X .

Assuming no independent factors are involved, we get the following for a sentence X which contains only one presupposition trigger (where $S_{p}$ is the minimal sentence dominating the trigger)
(18) Assertable(C, $\left.\mathrm{X}\left(\mathrm{S}_{\mathrm{p}}\right)\right)$ iff $\neg$ Assertable(C, $\left.\mathrm{X}\left(\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right)\right)$ iff $\forall \mathrm{X}^{\prime} \in \operatorname{Good}-$ final $(\mathrm{X}, \wedge)\left[\mathrm{X}(\mathrm{p} \wedge \mathrm{r}) \Leftrightarrow_{\mathrm{C}} \mathrm{X}(\mathrm{r})\right]$

Or, we could skip the middle man:

$$
\begin{equation*}
\text { Assertable }\left(\mathrm{C}, \mathrm{X}\left(\mathrm{~S}_{\mathrm{p}}\right)\right) \text { iff } \forall \mathrm{X}^{\prime} \in \operatorname{Good}-\operatorname{final}(\mathrm{X}, \wedge)\left[\mathrm{X}(\mathrm{p} \wedge \mathrm{r}) \Leftrightarrow_{\mathrm{C}} \mathrm{X}(\mathrm{r})\right] \tag{19}
\end{equation*}
$$

## 3.5. $\mathrm{S}_{1} \vee \mathrm{~S}_{\mathrm{p}}$

$S_{1} \vee S_{p}$ is assertable in C iff $S_{1} \vee\left(p \wedge S_{p}\right)$ is unassertable in C.
$S_{1} \vee\left(p \wedge S_{p}\right)$ is unassertable in C (given (Error! Reference source not found.), iff $S_{1} \vee$ $(p \wedge r) \Leftrightarrow_{C} S_{1} \vee r$, for any choice of $r$, i.e., iff $C \wedge \neg S_{1} \Rightarrow p$.
Hence $S_{1} \vee S_{p}$ will presuppose $\neg S_{1} \rightarrow p$.
This seems to be a good result:
(20) Either this house has no bathroom, or the bathroom is well hidden.

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4.2. $\mathrm{S}_{\mathrm{p}} \vee \mathrm{S}_{1}$
$S_{p} \vee S_{1}$ is assertable in C iff $\left(p \wedge S_{p}\right) \vee S_{1}$ is unassertable in C.
$\left(p \wedge S_{p}\right) \vee S_{1}$ is unassertable in C (given (Error! Reference source not found.), iff ( $\left.p \wedge r\right) \vee$
$S_{1} \Leftrightarrow_{C} r \vee S_{1}$, for any choice of $r$, i.e., iff $C \wedge \neg S_{1} \Rightarrow p$.
Hence $S_{p} \vee S_{1}$ will presuppose $\neg S_{1} \rightarrow p$.
This also seems to be good result:
(21) Either the bathroom is well hidden, or there is no bathroom.
5. $\left[\neg S_{p}\right] \rightarrow S_{1}$
$\left[\neg S_{p}\right] \rightarrow S_{1}$ is assertable in C iff $\left[\neg\left(\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right)\right] \rightarrow \mathrm{S}_{1}$ is unassertable in C.
$\left[\neg\left(p \wedge S_{p}\right)\right] \rightarrow S_{1}$ is unassertable in C (given (Error! Reference source not found.), iff
$[\neg(p \wedge r)] \rightarrow S_{1} \Leftrightarrow_{C}[\neg r] \rightarrow S_{1}$, for any choice of $r$, i.e., iff $\left[\neg S_{1} \rightarrow(p \wedge r)\right] \rightarrow \Leftrightarrow_{C}\left[\neg S_{1} \rightarrow r\right]$, i.e., iff $\mathrm{C} \wedge \neg \mathrm{S}_{1} \Rightarrow \mathrm{p}$.

Hence $\left[\neg S_{p}\right] \rightarrow S_{1}$ will presuppose $\neg S_{1} \rightarrow p$ (i.e, the same presupposition as that of a disjunction of $S_{p}$ or $S_{1}$ ).

This presupposition is radically different from that of Heim, and Philippe presents evidence in its support:
a. If Mary is pregnant, her doctor knows that she is pregnant.
b. If Mary's doctor doesn't know that she is pregnant, she isn't pregnant.

We want to generalize these results. Schlenker discusses this in a separate paper that I didn't read, for the incremental version...A naïve first step:
6. $\neg A$ when $A$ presupposes $p$ and dominates a single atomic $S_{q}$.

A presupposes p. Hence $A\left[S_{q} / q \wedge S_{q}\right]$ is unassertable in C iff $C \Rightarrow$ p. Given (Error!
Reference source not found., $C \Rightarrow p$ iff for any choice of $r A\left[S_{q} / q \wedge r\right] \Leftrightarrow_{C} A\left[S_{q} / r\right]$. Since $\mathrm{X} \Leftrightarrow_{\mathrm{C}} \mathrm{Y}$ iff $\neg \mathrm{X} \Leftrightarrow_{\mathrm{C}} \neg \mathrm{Y}$, we derive
$\mathrm{C} \Rightarrow \mathrm{p}$ iff for any choice of $\mathrm{r} \neg \mathrm{A}\left[\mathrm{S}_{\mathrm{q}} / \mathrm{q} \wedge \mathrm{r}\right] \Leftrightarrow_{\mathrm{C}} \neg \mathrm{A}\left[\mathrm{S}_{\mathrm{q}} / \mathrm{r}\right]$. I.e., we derive that $\neg \mathrm{A}$ presupposes p.

Homework (optional): see what happens with the other connectives, and when you allow for more than one presupposition trigger ()
7. Symmetry in Conjunction?
$\mathrm{S}_{\mathrm{p}} \wedge \mathrm{S}_{1}: \mathrm{S}_{\mathrm{p}} \wedge \mathrm{S}_{1}$ is assertable in C iff $\left(\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}\right) \wedge \mathrm{S}_{1}$ is unassertable in C .
( $\mathrm{p} \wedge \mathrm{S}_{\mathrm{p}}$ ) $\wedge \mathrm{S}_{1}$ is unassertable in C (given (Error! Reference source not found.), iff $(p \wedge r) \wedge S_{1} \Leftrightarrow_{C} r \wedge S_{1}$, for any choice of $r$, i.e., iff $C \wedge S_{1} \Rightarrow p$.
This doesn't seems to be good result: $\# \mathrm{~S}_{\mathrm{p}} \wedge \mathrm{p}$
(23) \#The king of France is bald and France has a king.
8. Proposal
(24) Constraint on Conjunction (version 2)

A sentence, X , that dominates $\mathrm{p} \wedge \mathrm{q}$ is not assertable in C if one of the following holds:
a. $\forall \mathrm{r}\left[\mathrm{X}[\mathrm{p} \wedge \mathrm{q} / \mathrm{p} \wedge \mathrm{r}] \Leftrightarrow_{\mathrm{C}} \mathrm{X}[\mathrm{p} \wedge \mathrm{q} / \mathrm{r}]\right] \quad$ (the first conjunct is idle no matter what the second conjunct is)
b. $\left[\mathrm{X}[\mathrm{p} \wedge \mathrm{q}] \Leftrightarrow_{\mathrm{C}} \mathrm{X}[\mathrm{p} \wedge \mathrm{q} / \mathrm{p}]\right] \quad$ (the second conjunct is idle given the first conjunct)
9. Further Evidence
9.1. No presupposition when $p \Rightarrow S_{p}$ (i.e., when $p \Leftrightarrow S_{p}$ )
(25) a. The king of France exists.
b. ?France has a king and the king of France exists
a. The king of France doesn't exist.
b. ?It's not the case that France has a king and the king of France exists

Note: the argument would be more compelling if we had a semantics, I.e. if we knew why $\mathrm{p} \Leftrightarrow \mathrm{S}_{\mathrm{p}}$.
9.2. $S_{p} \wedge S_{1}$ where $S_{1}$ is more informative than $p$.
(27) a. I can tell you that John knows he is sick and that he has cancer.
b. Is it true that John knows he is sick and that he has cancer?
c. It's not the case that John knows he is sick and that he has cancer.
10. Disjunction (potential problem)
(28) Either this house has no bathroom, or it has a bathroom and the bathroom is well hidden. (Schlenker pc, attributing to Heim pc)

We get the right projection for disjunction based on our constraints on conjunction, but the constraints on conjunction seem to give the wrong results.

Basic fact
$(p \vee q) \Leftrightarrow(p \vee([\neg p] \wedge q))$

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Could we capitalize on the following fact?
$\neg[(\mathrm{p} \nabla \mathrm{q}) \Leftrightarrow(\mathrm{p} \nabla([\neg \mathrm{p}] \wedge \mathrm{q}))]$
See our discussion of Hurford's constraint.
10. Disjunction (potential problem)
11. Quantification

### 11.1 Q A $\left[\lambda \mathrm{x} .[\mathrm{B}(\mathrm{x})]_{\mathrm{p}(\mathrm{x})}\right]$.

## Universal Quantifiers

(29) a. Every one of these ten boys drives his car to school.
b. At least one of these ten boys drives his car to school.
(30) Every A $\left[\lambda \mathrm{x} .[\mathrm{B}(\mathrm{x})]_{\mathrm{p}(\mathrm{x})}\right]$.

To get rid of clutter, we will write
$\forall \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$
$\forall \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is assertable in C iff $\forall \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \rightarrow p(x) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is unassertable in C .
$\forall \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{p}(\mathrm{x}) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is unassertable in C (given (Error! Reference source not
found.), iff
$\forall \mathrm{x}[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{p}(\mathrm{x}) \wedge \mathrm{R}(\mathrm{x})] \Leftrightarrow_{\mathrm{C}} \forall \mathrm{x}[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{R}(\mathrm{x})]$, for any choice of R, i.e., iff
$C \Rightarrow \forall \mathrm{x}[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{p}(\mathrm{x})]$.
Proof (of the last iff statement):

1. Assume $C \Rightarrow \forall x[A(x) \rightarrow p(x)]$, then
2. Assume $\forall x[A(x) \rightarrow R(x)]$. It automatically follows that

$$
\forall \mathrm{x}[\mathrm{~A}(\mathrm{x}) \rightarrow \mathrm{p}(\mathrm{x}) \wedge \mathrm{R}(\mathrm{x})]
$$

1. Assume $\forall \mathrm{x}[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{R}(\mathrm{x})] \Leftrightarrow_{\mathrm{C}} \forall \mathrm{x}[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{p}(\mathrm{x}) \wedge \mathrm{R}(\mathrm{x})]$
2. Choose for $R$ the tautological preidicate ( $\lambda \mathrm{x} . \mathrm{x}=\mathrm{x}$ ).

We now get in C
3. $\forall \mathrm{x}[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{x}=\mathrm{x}] \quad$ (tautology)
4. $\forall x[A(x) \rightarrow p(x) \wedge x=x] \quad(b y 1)$
5. $\forall \mathrm{x}[\mathrm{A}(\mathrm{x}) \rightarrow \mathrm{p}(\mathrm{x})]$

## Existential Quantifiers

$$
\begin{equation*}
\exists \mathrm{x}\left[\mathrm{~A}(\mathrm{x}) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right] . \tag{31}
\end{equation*}
$$

$\exists \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is assertable in C iff $\exists \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \wedge p(x) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is unassertable in C.
$\exists \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \wedge p(x) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is unassertable in C (given (Error! Reference source not found.), iff
$\exists x[A(x) \wedge p(x) \wedge R(x)] \Leftrightarrow_{C} \exists x[A(x) \wedge R(x)]$, for any choice of $R$, i.e., iff $C \Rightarrow \forall x[A(x) \rightarrow p(x)]$.

Proof (of the last iff statement):

1. Assume $C \Rightarrow \forall x[A(x) \rightarrow p(x)]$, then
2. Assume $\exists x[A(x) \wedge R(x)]$, call this $x$, a. By $1, p(a)$, hence

$$
\exists \mathrm{x}[\mathrm{~A}(\mathrm{x}) \wedge \mathrm{p}(\mathrm{x}) \wedge \mathrm{R}(\mathrm{x})]
$$

1. Assume $\exists x[A(x) \wedge p(x) \wedge R(x)] \Leftrightarrow c \exists x[A(x) \wedge R(x)]$
2. Let $\mathrm{a} \in \mathrm{A}$, and choose for R the predicate $\lambda \mathbf{x} . \mathbf{x}=\mathbf{a}$ (if A is empty, we're done)
3. Since $\exists x[A(x) \wedge x=a]$, we conclude by $1, \exists x[A(x) \wedge p(x) \wedge x=a]$.
4. This $x$ can only be a, hence $p(a)$.
5. This hold for any $a \in A$, hence $\forall x[A(x) \rightarrow p(x)]$

Note: As pointed out by Heim (1983), and later by Beaver, this prediction seems too strong.

A few of these 10 women are pregnant. (?) Furthermore, at least one of these 10 women is pregnant and happy to be pregnant.

Each of these 10 women is pregnant. (?) Furthermore, at least one of these 10 women is pregnant and happy to be pregnant.
(32) Constraint on Conjunction (speculation, will be too weak for what follows)

A sentence, $X(p \wedge q)$ is not assertable in $C$ if one of the following holds:
a. $\quad\left[\mathrm{X}(\mathrm{p}) \Leftrightarrow_{\mathrm{C}} \mathrm{X}(\mathrm{T})\right]$ where T is a tautology
(the first conjunct is idle given a tautological second conjunct)
b. $\quad\left[\mathrm{X}(\mathrm{p} \wedge q) \Leftrightarrow_{\mathrm{C}} \mathrm{X}(\mathrm{p}) \quad\right.$ (the second conjunct is idle given the first conjunct)
$\exists \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$.
$\exists \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is assertable in C iff $\exists \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \wedge p(x) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is unassertable in C .
$\exists \mathrm{x}\left[\mathrm{A}(\mathrm{x}) \wedge p(x) \wedge \mathrm{B}_{\mathrm{p}}(\mathrm{x})\right]$ is unassertable in C (given (32), iff
$\exists x[A(x) \wedge p(x) \wedge T(x)] \Leftrightarrow_{C} \exists x[A(x) \wedge T(x)]$, for any choice of R, i.e., iff ?
$C \Rightarrow \exists x A(x) \rightarrow \exists x[A(x) \wedge p(x)]$.

Proof (of the last iff statement):

1. Assume $C \Rightarrow \exists x A(x) \rightarrow \exists x[A(x) \wedge p(x)]$ then
2. Clearly $\exists \mathrm{x}[\mathrm{A}(\mathrm{x})] \Rightarrow{ }_{\mathrm{C}} \exists \mathrm{x}[\mathrm{A}(\mathrm{x}) \wedge \mathrm{p}(\mathrm{x})]$
3. Assume $\exists x[A(x) \wedge p(x)] \Leftrightarrow_{C} \exists x[A(x)]$
4. $C \wedge \exists x[A(x)] \Rightarrow \mathrm{C} \exists \mathrm{x}[A(x) \wedge p(x)]$, Hence
5. $C \Rightarrow \exists x A(x) \rightarrow \exists x[A(x) \wedge p(x)]$.

## 11.2 $\mathrm{Q}\left(\mathrm{NP}\left[\lambda \mathrm{x} .[\operatorname{RC}(\mathrm{x})]_{\mathrm{p}(\mathrm{x})}\right]\right)(\mathrm{VP})$

## Universal Quantifiers

(34) Among these ten boys
a. Every one who likes his car bought this policy.
b. At least one person who likes his car bought this policy.

$$
\begin{equation*}
\forall \mathrm{x}\left[\mathrm{NP}(\mathrm{x}) \wedge \mathrm{RC}_{\mathrm{p}}(\mathrm{x}) \rightarrow \mathrm{B}(\mathrm{x})\right] \tag{35}
\end{equation*}
$$

$\forall x\left[N P(x) \wedge R C_{p}(x) \rightarrow B(x)\right]$ is assertable in C iff $\forall x\left[N P(x) \wedge p(x) \wedge R C_{p}(x) \rightarrow B(x)\right]$ is unassertable in C.
$\forall \mathrm{x}\left[\mathrm{NP}(\mathrm{x}) \wedge \mathrm{p}(\mathrm{x}) \wedge \mathrm{RC}_{\mathrm{p}}(\mathrm{x}) \rightarrow \mathrm{B}(\mathrm{x})\right]$ is unassertable in C (given (Error! Reference source not found.), iff
$\forall x[N P(x) \wedge p(x) \wedge R(x) \rightarrow B(x)] \Leftrightarrow_{C} \forall x[N P(x) \wedge R(x) \rightarrow B(x)]$ for any choice of R, i.e., iff $\mathrm{C} \Rightarrow \forall \mathrm{x}(\mathrm{NP}(\mathrm{x}) \rightarrow[\mathrm{p}(\mathrm{x}) \vee \mathrm{B}(\mathrm{x})])$

Proof (of the last iff statement):

1. Assume $C \Rightarrow \forall x(N P(x) \rightarrow[p(x) \vee B(x)])$, then
2. Assume $\forall x[N P(x) \wedge p(x) \wedge R(x) \rightarrow B(x)]$
3. Let $x \in N P \cap R$. Given $1, x \in p$ or $x \in B$. If $x \in p$ (by 2 ) $x \in B$, hence in either case $x \in B$. Hence $\forall x[N P(x) \wedge R(x) \rightarrow B(x)]$
4. Assume $\forall \mathrm{x}[\mathrm{NP}(\mathrm{x}) \wedge \mathrm{p}(\mathrm{x}) \wedge \mathrm{R}(\mathrm{x}) \rightarrow \mathrm{B}(\mathrm{x})] \Leftrightarrow_{\mathrm{C}} \forall \mathrm{x}[\mathrm{NP}(\mathrm{x}) \wedge \mathrm{R}(\mathrm{x}) \rightarrow \mathrm{B}(\mathrm{x})]$
5. Let R be the complement of p , we derive:

$$
\left.\forall \mathrm{x}[\mathrm{NP}(\mathrm{x}) \wedge \mathrm{p}(\mathrm{x}) \wedge \neg \mathrm{p}(\mathrm{x}) \rightarrow \mathrm{B}(\mathrm{x})] \Leftrightarrow_{\mathrm{C}} \forall \mathrm{x}[\mathrm{NP}(\mathrm{x})) \wedge \neg \mathrm{p}(\mathrm{x}) \rightarrow \mathrm{B}(\mathrm{x})\right]
$$

3. Since the left hand side is a tautology, $C \Rightarrow \forall x[N P(x) \wedge \neg p(x) \rightarrow B(x)]$ $\Rightarrow \forall \mathrm{x}(\mathrm{NP}(\mathrm{x}) \rightarrow[\mathrm{p}(\mathrm{x}) \vee \mathrm{B}(\mathrm{x})])$

This is very different from what is commonly assumed, but is it necessarily a bad result? Not obvious to me:
(34)a suggests that every boy has a car, but that might be an artifact of the particular example (of our particular choice for the predicate B): buying the (relevant) policy suggests owning a car. Things don't seem very different in (36).
(36) Among these ten boys everyone who didn't buy this policy doesn't like his car.

Consider the following:
(37) Among these ten boys
a. Everyone who is sick knows that he is sick.
b. Everyone who doesn't know he is sick isn't sick.
(38) Among these ten boys, everyone who doesn't have a car is a friend of mine. Also everyone who hates his car is a friend of mine.

## Existential Quantifiers

$$
\begin{equation*}
\exists \mathrm{x}\left[\mathrm{NP}(\mathrm{x}) \wedge \mathrm{RC}_{\mathrm{p}}(\mathrm{x}) \wedge \mathrm{B}(\mathrm{x})\right] \tag{39}
\end{equation*}
$$

$\exists x\left[N P(x) \wedge R C_{p}(x) \wedge B(x)\right]$ is assertable in C iff $\exists x\left[N P(x) \wedge p(x) \wedge R C_{p}(x) \wedge B(x)\right]$ is unassertable in C .
$\exists x\left[N P(x) \wedge p(x) \wedge R C_{p}(x) \wedge B(x)\right]$ is unassertable in C (given (Error! Reference source not found.), iff
$\exists x\left[N P(x) \wedge p(x) \wedge R C_{p}(x) \wedge B(x)\right] \Leftrightarrow c \exists x\left[N P(x) \wedge p(x) \wedge R C_{p}(x) \wedge B(x)\right]$, for any choice of R, i.e., iff
$\exists x\left[N P(x) \wedge B(x) \wedge p(x) \wedge R C_{p}(x)\right] \Leftrightarrow_{C} \exists x\left[N P(x) \wedge B(x) \wedge R_{p}(x)\right]$,
i.e., iff
$\mathrm{C} \Rightarrow \forall \mathrm{x}(\mathrm{NP}(\mathrm{x}) \wedge \mathrm{B}(\mathrm{x}) \rightarrow \mathrm{p}(\mathrm{x}))$
This is again very non-traditional, but is it obviously bad?
(40) a. Among these ten boys, at least one person who likes his car is a friend of mine.
b. Among these ten boys, at least one person who is a friend of mine likes his car.

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### 24.954 Pragmatics in Linguistic Theory

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[^0]:    ${ }^{1}$ where $S$ is a minimal sentence with presupposition $p$ if $S$ is the minimal sentence dominating a presupposition trigger and the computed presupposition is p .

