

No plagiarism allowed: see the handbook for students at <http://web.mit.edu/3.012/handbook.pdf> . On the other hand, you should feel free to consult as much as possible books and other educational material. Remember that the homework is primarily designed to give you a chance to gauge your understanding of the material – if you struggle with a concept, go back to the lecture notes, the textbook, the TAs, the instructor.

BONDING

1. Venus Williams has the fastest serve in women's tennis – 205 km/h. Given that the tennis ball weighs 57 g, what is its initial de Broglie wavelength ? How light should be the tennis ball to have a wavelength identical to its diameter (65 mm) ?
2. The solar spectrum has a peak in intensity in the near-infrared wavelengths – around 884 nm. The peak sensitivities in the cone cells in our eyes range from 420 to 560 nm – we have one cone type for red, with a peak at 560 nm, one for green, peaked at 530 nm, and one for blue, at 420 nm (this probably came very handy when it was a matter of picking fruit that was ripe). Cones absorb light with linear, rhodopsin-like molecules. What is your estimate of the size of these molecules, for the three kinds of cone cells ?
3. Katharine Blodgett discovered in 1938 that one could significantly cut glare in glass by spreading a thin film of organic molecules on its surface – this invention was hailed in the popular press as that of “invisible glass”. Explain, in words, the basic principle at hand. Aleksander Smakula made a similar discovery at Carl Zeiss in 1935 (the T* coating, with fluoride compounds. Interestingly, after the war, he continued his research in an exceedingly prestigious academic institution in Cambridge, MA.)
4. Suppose that we have a plane wave of 1 nm wavelength. What is the periodicity of the square modulus of that plane wave ?
5. What are the general solutions to the linear differential equation $\frac{d^2z}{dt^2} = \pm\omega^2 z$? (I.e. discuss both the + and the – cases)

We'll also use some Applets for the problem set – they come from a very good website that has won several educational prizes: <http://www.quantum-physics.polytechnique.fr/en/index.html> We'll use problem 2.3:

6. **Eigenstates in a potential well:** Here our potential is represented by a green line, 0 on the left and on the right side of the picture, and constant and negative in the middle (let's call that value $-V_0$). With the mouse, you can slide the trial value of the energy E (in red) that appears in the Schrödinger equation from below $-V_0$ to above zero. The applet instantaneously tries to calculate a function (in yellow) that solves the Schrödinger equation for the chosen energy E. It turns out that for any energy E greater than 0 the applet finds a well-behaved eigenfunction – so we have a continuous spectrum of eigenvalues (i.e. set of solutions), each

of them with its corresponding eigenfunction. Can you describe what is happening for E lesser than zero ? Do we still have eigenstates ? What happens when E is not an eigenvalue ? Note that the applet starts its numerical integration from left to the right.

7. In a parallel universe, the kinetic energy of a particle is $p^3/(3m)$. What would be the corresponding quantum mechanical operator ?
8. Sketch the graph of the three lowest energy eigenfunctions of a particle in a 2-dimensional box. Show graphically and analytically that they are orthogonal (Use the method of separation of variables to write the ground-state wavefunction of this 2-dimensional problem using the 1-dimensional solutions). What are the three lowest-energy states ?

THERMODYNAMICS

9. **Thermodynamic Variables.** We have described two classes of thermodynamic variables: intensive (such as temperature and pressure) and extensive (such as volume).
 - a. What class would the variable mass (let's use the symbol M for mass) fall into- intensive or extensive?
 - b. What about variables that are derived from other variables? Consider density: We define density as the mass of a material per unit volume:

$$\rho = \frac{M}{V}$$

Can density be classified simply as an intensive or extensive variable? Explain your answer. (Hint: Look carefully at the definitions!)

10. **Thinking about systems and boundaries.** Read the descriptions below of some thermodynamic systems (any macroscopic material can be a thermodynamic system!)

undergoing various changes and identify the type of system (open, closed, etc.). Explain in one sentence for each case why your choice is appropriate- state your assumptions to defend your choice.

(a) The system is a liter of hot coffee placed inside a high-quality, sealed thermos that has an evacuated space between an inner rigid cylinder where the liquid is placed and the outside walls of the container.

(b) If the thermos described above had flexible rather than perfectly rigid walls, what kind of system would we have?

(c) The system is a puddle of water on the sidewalk.

(d) You have a glass jar of strawberry jam and the metal screw-top lid is stuck (the jam is your system). You really want a peanut butter and jelly sandwich, so you place the jar under a stream of hot water to loosen the lid.

(e) *You* are the system. You're on a ski trip at Killington and you are wearing a high-tech insulated parka, hood, and snow pants to keep you warm. It's -10°C on the ski slope, but as you trudge up to the ski lift, you are getting warm.

11. **Using the First Law.** We introduced the theoretical concept of an ideal gas in lecture- 'ideal' because real gases only approach ideal behavior under select conditions. Recall that the two key thermodynamic characteristics of an ideal gas are an equation of state $PV = nRT$ and that the internal energy is only a function of temperature ($U = 1.5nRT$). Suppose we have 1 mole of an ideal gas confined in a cylinder with a movable piston top, which allows the volume of the gas to be controlled. The gas starts at an initial volume $V_0 = 10$ L at a pressure of 1 atm. We want to consider the effect of two different two-step processes, each of which starts with the gas in the same initial state and ends with the gas in the same final state:

Process A: (i) The gas is compressed to a volume of 1 L at constant pressure ($P = 1$ atm). (ii) The pressure is then slowly increased from 1 atm to 10 atm at constant volume.

Process B: (i) The pressure is slowly increased from 1 atm to 10 atm at constant volume ($V = V_0$). (ii) The gas is then compressed from 10 L to 1 L at constant pressure ($P = 10$ atm).

Assume the changes occur slowly enough for the gas to remain in equilibrium at all times (reversible processes).

- a. By which composite process, **A** or **B**, is more work done by the gas? Show why using a P vs. V diagram and also quantitatively by calculation.

- b. Is the change in internal energy of the gas the same for each composite process? Why?
- c. With the given information, can you calculate the heat transferred into/out of the gas during process A? Why or why not? If yes, show your calculation.

Suppose now we carry out a four-step process: Starting in the initial state ($V = V_0$, $P = 1$ atm), we perform composite process A, followed by composite process B in reverse.

- d. What is the total internal energy change for the gas after all 4 steps?
- e. Can you determine the total heat transferred into/out of the gas for this 4-step composite process? If yes, show your calculation.