3.014 Materials Laboratory
Sept. 26th – 30th, 2005

Lab Week 1 – Module α_3

Understanding Electronic Conductivity: Conductivity and Electron Populations

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OBJECTIVES

- ✓ Understand Ohm's law and how to measure conductivity in a solid
- ✓ Know the three classifications of solids based on electronic conductivity
- ✓ Understand the temperature-conductivity behavior of metals and semiconductors
- ✓ Understand the relative importance of number of charge carriers and charge mobility on the conductivity of a solid

Questions

At the end of this laboratory experience you should be able to answer the following questions:

1) What is Ohm's Law and how is it used to measure electrical conductivity?

2) What are two important parameters that determine the electrical conductivity of a material? How are these affected by temperature?

3) How can one distinguish materials through conductivity measurements?

INTRODUCTION

This module is designed to demonstrate the concept of how free electrons play a role in the electronic conductivity of materials. It is meant to be an extension of what was learned in 3.012 for the free electron to the case of electron transport in a solid material.

The goal is to show how electronic conductivity, a fundamental material property, is related to electron populations. To do so, we will measure the resistance to electronic transport of various materials at different temperatures. Included in the materials to be studied are:

- Copper, a metal
- Pure germanium, an intrinsic semiconductor
- Doped silicon, an extrinsic semiconductor at room temperature
- Graphite, a zero-gap semiconductor

BACKGROUND

Electronic conductivity (σ) measures the ease with which electrons travel across a material under the effect of an electric field. For a fixed potential difference it is proportional to the inverse of electronic resistivity (ρ), which measures a materials resistance to electronic transport under an electric field.

These properties may be measured by measuring the current (I) response to an applied potential (V), since the current is directly related to the number of charges that is passed through the material. The resistance (R) may be calculated by Ohm's Law [1], which gives a linear relation between current and voltage in an ideal conductor, as is the case with the materials of interest in this laboratory.

$$V = IR \qquad (1)$$

Resistance is geometry-dependent and may be normalized through the following equation relating resistance with conductivity:

$$\sigma = \frac{l}{RA} \tag{2}$$

where, A is the cross-sectional area and l is the length of the sample.

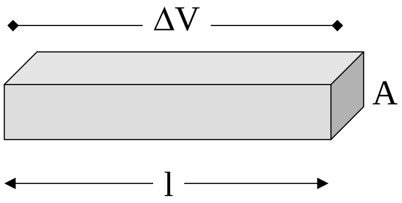

Figure 1: Schematic of conductivity sample and parameters used to calculate conductivity.

Electronic conductivity is related to mobile charge carrier density (n) and mobility of the carriers (μ). e is the value of the electrical charge on an electron (1.6 x 10^{-19}C)

$$\sigma = ne\mu \tag{3}$$

Charge includes ions and electrons; in solids they are most frequently electrons, and we will limit our discussion to the study of electronic conductivity (a material that only allows transport of ionic charge is called a solid-state electrolyte).

Charge Carrier Mobility

Charge mobility can affect a material's temperature-conductivity relationship. Quantum mechanics predicts the continuous acceleration of the electron under the influence of a given electric field, resulting in a continuously increasing current. In real solids, the current response is constant. Electron scattering from thermal vibrations in the crystal lattice, impurity atoms, vacancies, and other imperfections limits the velocity of the electrons to a drift velocity (v_e). The mobility is the proportionality constant that relates the electric field (ε) to the drift velocity.

$$v_d = \mu_e \varepsilon \tag{4}$$

From the above description of mobility, what would be the expected relationship between mobility and temperature?

Charge Carrier Density

The charge carrier density may be used to characterize a solid into three categories:

- metal
- semiconductor
- insulator

To understand the differences between these classes of materials, we can connect the particle-in-a-box model discussed in 3.012 to electronic states observed in a solid, by envisioning electrons as trapped in a box of finite width and depth that might be associated with the lattice spacing and nuclear charge, respectively. The finite potential causes the wave functions to “leak” out the sides of the box [2]. The resulting overlap between wave functions for electrons on adjacent atoms in the solid gives rise to the formation of energy “bands” rather than single values from a given atomic orbital. For lower energy states (low n values corresponding to core electronic states), electrons are still very localized within the box, as illustrated in **Figure 2** below. For the outermost occupied atomic orbitals (the valence band), substantial overlap of electron wave functions gives rise to chemical bonds.

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See http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/pbox.html#c1

Figure 2: Comparison of wavefunctions within a box of infinite (left) and finite (right) potential (from [3]).

The valence band is thus defined as the highest energy band with occupied orbitals. The conduction band is the lowest energy band with unoccupied orbitals. The Fermi level, E_F, is the top of the occupied energy level at absolute zero.

When the valence and conduction bands overlap, electrons are free to move, even at absolute zero. Conduction occurs by excitation from an electric field. These materials are classified as metals and E_F resides in the conduction band. The number of electrons in the conduction band is not closely related to the temperature of the sample (temperature = thermal energy).

When there is a gap (energy gap E_g) between the two bands, the electrons need energy, generally in the form of heat or light, to excite electrons from the valence band to the conduction band. If the energy gap is small, these materials are classified as semiconductors. The number of electrons in the conduction band is related to the temperature of the sample. If the energy gap is very large and cannot be breached even at high temperatures, these materials are classified as insulators. In semiconductors and insulators, the Fermi level, $E_F = E_V + E_g/2$, where E_V is the energy of the valence band [4].

A schematic of the positions of the energy gaps may be seen in Figure 3.

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See http://hyperphysics.phy-astr.gsu.edu/hbase/solids/band.html

Figure 3: Comparison of energy bands for electronic (a) insulators, (b) semiconductors (E_g generally <2eV) and (c) conductors (from [5]).

From the above explanation of the energy bands, what would be the expected behavior of the charge carrier density with respect to temperature for metals? For semiconductors?

Conductivity and Temperature in Metals

In metals, the charge carrier density is high, even at low temperatures. The change of conductivity with temperature is mainly affected by the change of mobility with temperature.

The relationship between resistivity and temperature in metals is approximately linear except at very low temperatures.

$$\rho_T = aT + \rho_o. \qquad (5)$$

where *a* is a constant that is particular for each metal and ρ_o are constant that depends on the type of metal and the imperfections in the metal (impurity atoms, dislocations,…).

Conductivity and Temperature in Semiconductors

In semiconductors, the net effect of temperature on the conductivity is related to the changes in charge carrier density. The concept of holes and electrons is essential to the understanding of charge carriers in semiconductors.

In an intrinsic semiconductor when an electron obtains sufficient energy to jump to the conduction band, a hole is left in the valence band. The electrons and holes are both charge carriers, with the electron carrying a negative charge and the hole representing a positive charge. When a potential difference is applied to a semiconductor, a movement of the hole occurs when an adjacent electron fills the hole, creating a new hole adjacent to the original one.

In an extrinsic semiconductor, impurity atoms are added to provide excess holes or electrons. The excess charge carrier density and the conductivity is related to the impurity concentration. The mobility of holes and electrons may be found for different materials and temperatures in literature or in the CRC handbook [6].

$$\sigma = n|e|\mu_e + p|e|\mu_h \qquad (6)$$

where n is the electron density and p is the hole density, and n=p for intrinsic semiconductors.

Using equation 7, you can see how temperature-conductivity relationships in semiconductors can be used to determine the energy gap in intrinsic semiconductors.

$$\ln\sigma \cong Const - \frac{E_g}{2kT} \qquad (7)$$

where k is Boltzmann's constant and T is temperature in Kelvin.

The mobility of charge carriers is temperature-dependent [1], but it is not as significant to the conductivity/temperature behavior of semiconductors.

Cited References:

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6. CRC Handbook of Chemistry and Physics, 85th ed. online, CRC Press, Cleveland, Ohio, 2004.