[SQUEAKING]

[RUSTLING]

[CLICKING]

RAFAEL JARAMILLO: Hi. Today, we're going to review the lever rule. The lever rule is a tool that we use to answer the following question. Imagine we have a binary phase diagram. And two phases, alpha and beta, which are separated by a two-phase region.

We're going to consider a system with an overall composition and temperature right here. So the overall mole fraction of component two is there along the abscissa, and the temperature's, of course, there at that isotherm. Now, we know that in the two different phases, the mole fractions for component two are going to be this value for phase alpha and this value for phase beta.

So we have overall composition x sub 2. We have some number of moles super alpha. We have some number of moles super beta. What the lever rule does is allow us to calculate for those values, the mole fractions of the alpha and the beta phase.

The way we find the mole fractions is by writing down constitutive relationships which express the conservation of mass. So for example, we have to conserve the total mass. The total number of moles of alpha plus the total number of moles of beta is the total number of moles of the system, so n super alpha and n super beta are related in that way.

We also need to conserve the moles of the individual components. Because the two components, they can't transmutate from one into the other. So the way we do that is we say n super alpha times the mole fraction, component two n phase alpha, plus n super beta times the mole fraction of component two n phase beta should be equal to the overall mole fraction of component 2 times the total number of moles. So this is conservation of mass. So we put these together, we get the following expression.

Now that we've eliminated n super beta, we can simply solve for super alpha. I'm actually going to solve for something called f super alpha, which is the phase fraction, which is the moles of phase alpha divided by the total moles in the system. From the equation on the previous board, simply becomes x2 beta minus x2, x2 beta minus x2 alpha.

And likewise, you could solve for the phase fraction of phase beta, which is the moles of beta divided by the total moles in the system. This is going to be equal to x2 minus x2 alpha, x2 beta minus x2 alpha. These are phase fractions.

And the neat point and the reason why it's called the lever rule is that these expressions correspond to line segments along the timeline. So I want to illustrate that. Let's start with the denominator.

The denominator for both expressions is x2 and beta minus x2 and alpha. So I'll circle that in green. If we come over here to the phase diagram, we see that that is the total length of the timeline.

Now, we'll look at the numerators. Let's start with the numerator here. x2 super beta minus x2. So that's the distance between this point and the overall system composition. It's that length.

And I'll use blue for the remaining numerator, which is x sub 2 minus x sub 2 super alpha. So that's the shorter distance from the overall composition to the composition in phase alpha. Which brings us to the final point, which is why it's called the lever rule.

If we visualize the timeline as a lever or a seesaw, and we put the overall system composition at the fulcrum, so this is at x sub 2, this is x sub 2 super alpha, this is x sub 2 super beta, the phase reactions are just the amount of stuff you would need to put on this lever to balance it so it's perfectly horizontal. In this case, the fulcrum is closer to the alpha phase. So I'm going to need a lot more stuff in the alpha phase than in the beta phase to balance this lever.

f super alpha goes x2 beta minus x2, x2 beta minus x2 alpha. And f super beta goes x2 minus x2 alpha, x2 beta minus x2 alpha. And that's why it's called the lever rule.