3.020 Lecture 4

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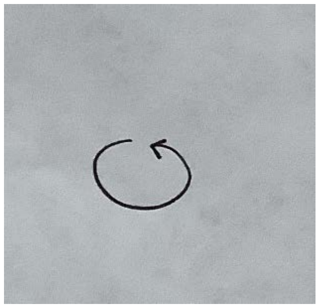
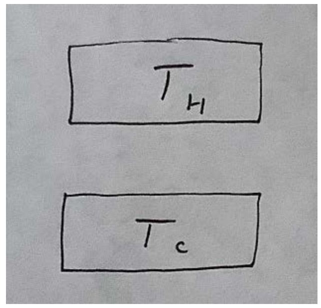
1 Heat engines, abstracted

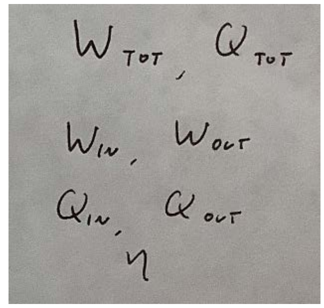
after slides



- Cyclic machine
- Returns to same state after each cycle

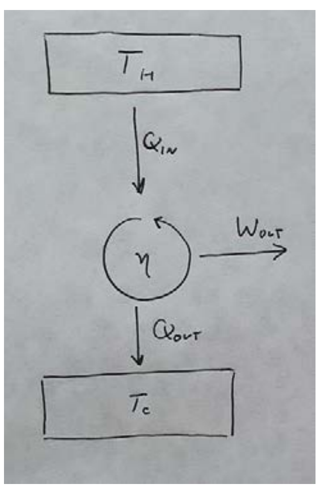


- Thermal reservoirs
- Maintained at hot (T_H) and cold (T_c) throughout



- Total work and heat
- Work in and out
- Heat in and out
- Efficiency

  $\eta \equiv \frac{W_{NET}}{Q_{IN}}$



- Typical representation:

  " Heat engine with efficiency η operating between T_H and T_c"

- Each cycle:

  Q_{IN} absorbed from T_H
  Q_{OUT} "rejected" to T_c
  W_{OUT} performed

slides: identifying T_H, T_C for real engines

Question: What is $Q_{IN} - W_{Out} - Q_{OUT}$?

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2 Calculating a cyclic process

- Work and heat are process variables
- Thermodynamics doesn't describe real-world process

What to do ??

$\Rightarrow$ Describe hypothetical process for which system remains in equilibrium at all times.

- can use state variables and equations of state (if available)
- in practice, such a cycle would take infinite time.

$$\textbf{Power} = \frac{\textbf{Work}}{\textbf{Cyclic tiem period}} \longrightarrow 0$$

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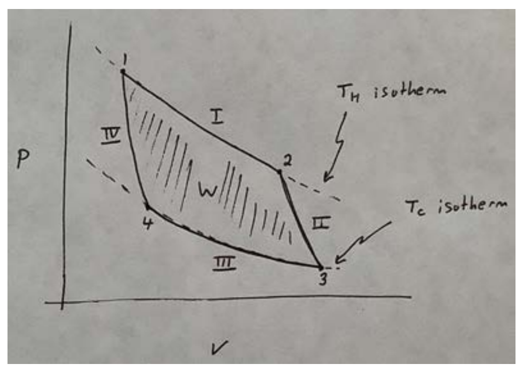
3 Carnot cycle with an ideal gas

(I) Isothermal expansion at T_H

(II) Adiabatic expansion to T_c

(III) Isothermal compression at T_c

(IV) Adiabatic compresion to T_H



Note: W_{out} is the area enclosed by the cycle
(true of any cycle, not just Carnot)

3.1 Isotherms

$$PV = nRT$$
$$PdV + VdP = 0$$
$$dV = -\frac{V}{P}dP = -\frac{nRT}{P^2}dP$$
$$PdV = -\frac{nRT}{P}dP$$
$$\Rightarrow \int \delta W = +\int dP\frac{nRT}{P} = +nRT\ln\left(\frac{P_{final}}{P_{initial}}\right) = +nRT\ln\left(\frac{V_{initial}}{V_{final}}\right)$$

Sanity Check: expansion does work on surroundings, so $\int \delta W < 0$

recall that $\delta W = -PdV$

$$\frac{V_i}{V_f} < 1, \quad \ln\left(\frac{V_i}{V_f}\right) < 0 \quad \checkmark$$

3.2 Isotherms, continued

$\Rightarrow$ For ideal gas, internal energy U is a function of T only, $dU = nC_v dT$

single variable calculus!

$$dU = 0 \text{for isothermal process}$$
$$\delta Q = -\delta W$$
$$Q = -W$$

3.3 Adiabats

$$\delta Q = 0,\ \delta W = -PdW \qquad - \text{how to calculate ?}$$

- For ideal gas:

$$dU = \delta W + \underbrace{\delta Q}_{0} = -PdV = nC_v dT$$
$$W = nC_v(T_{final} - T_{initial})$$

- Adiabatic curves are described by:

$$TV^\gamma = const, \qquad \frac{P_{final}}{P_{initial}} = \left(\frac{V_{initial}}{V_{final}}\right)^\gamma, \qquad \gamma = \frac{C_p}{C_v}$$

Deltoff ch. 4, Lectures 6-7

- Adiabats are steeper than isotherms on (P,V) plane becasue $\gamma > 1$

Q: Why is $\gamma > 1$?

3.4 Adding all contributions to W and Q

	W	Q
I	$nRT_H \ln\left(V_1/V_2\right)$	$-nRT_H \ln\left(V_1/V_2\right)$
II	$nC_v(T_C - T_H)$	
III	$nRT_C \ln\left(V_3/V_4\right)$	$-nRT_C \ln\left(V_3/V_4\right)$
IV	$nC_v(T_H - T_C)$	

3.5 Calculating the Carnot efficiency

$$W_{TOT} = -\left(nRT_H \ln\left(V_1/V_2\right) + nRT_C \ln\left(V_3/V_4\right)\right) \quad \text{total work done by the engine}$$
$$Q_{IN} = -nRT_H \ln\left(V_1/V_2\right) \quad \text{heat absorbed at } T_H$$
$$\eta = \frac{W_{TOT}}{Q_{IN}} = 1 + \frac{T_c}{T_H}\frac{\ln(V_3/V_4)}{\ln(V_1/V_2)}$$

Using property of adiabat $TV^{\gamma-1} = \text{const}$ can show that $V_3/V_4 = (V_1/V_2)^{-1}$

$$\eta_{carnot} = 1 - T_C/T_H$$

3.6 Considering heat transfers

$v = \frac{V_2}{V_1} = \frac{V_3}{V_4}$

	heat absorbed	heat released
Carnot engine	$Q_{IN} = nRT_H \ln V$	$Q_{OUT,C} = nRT_C \ln V$
less efficient engine that burns the same quantity of fuel	$Q_{IN} = nRT_H \ln V$	$Q_{OUT} > Q_{OUT,C}$

required by conservation of energy if $W_{TOT} < W_{TOT,C}$

3.7 Considering quantity $\delta Q/T$

- Quantity: $\oint \frac{\delta Q}{T} \longrightarrow$ integral around the cycle
- Carnot: $\oint \frac{\delta Q}{T} = \frac{nRT_H \ln V}{T_H} - \frac{nRT_C \ln V}{T_C} = 0$
- Less efficient: $\oint \frac{\delta Q}{T} = \frac{nRT_H \ln V}{T_H} - \frac{Q_{OUT}}{T_C} < 0.$ because $Q_{OUT} > Q_{OUT,C}$

$\Rightarrow$ We will soon see that this is related to entropy generation by the less efficient cycle

3.020 Thermodynamics of Materials
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