3.020 Lecture 4

Prof. Rafael Jaramillo
1 Heat engines, abstracted

- Cyclic machine
- Returns to same state after each cycle

- Thermal reservoirs
- Maintained at hot ($T_H$) and cold ($T_c$) throughout

- Total work and heat
- Work in and out
- Heat in and out
- Efficiency
  \[ \eta = \frac{W_{NET}}{Q_{IN}} \]

- Typical representation:
  "Heat engine with efficiency $\eta$ operating between $T_H$ and $T_c"$

- Each cycle:
  $Q_{IN}$ absorbed from $T_H$
  $Q_{OUT}$ "rejected" to $T_c$
  $W_{OUT}$ performed

Question: What is $Q_{IN} - W_{Out} - Q_{OUT}$?
2 Calculating a cyclic process

- Work and heat are process variables
- Thermodynamics doesn’t describe real-world process

What to do??

⇒ Describe hypothetical process for which system remains in equilibrium at all times.
- can use state variables and equations of state (if available)
- in practice, such a cycle would take infinite time.

\[ \text{Power} = \frac{\text{Work}}{\text{Cyclic time period}} \rightarrow 0 \]

3 Carnot cycle with an ideal gas

(I) Isothermal expansion at \( T_H \)
(II) Adiabatic expansion to \( T_c \)
(III) Isothermal compression at \( T_c \)
(IV) Adiabatic compression to \( T_H \)

Note: \( W_{out} \) is the area enclosed by the cycle
(true of any cycle, not just Carnot)
3.1 Isotherms

\[ PV = nRT \]
\[ PdV + VdP = 0 \]
\[ dV = -\frac{V}{P}dP = -\frac{nRT}{P^2}dP \]
\[ PdV = -\frac{nRT}{P}dP \]

\[ \Rightarrow \int \delta W = +\int dP \frac{nRT}{P} = +nRT \ln \left( \frac{P_{\text{final}}}{P_{\text{initial}}} \right) = +nRT \ln \left( \frac{V_{\text{initial}}}{V_{\text{final}}} \right) \]

Sanity Check: expansion does work on surroundings, so \( \int \delta W < 0 \)

\[ \frac{V_i}{V_f} < 1, \quad \ln \left( \frac{V_i}{V_f} \right) < 0 \quad \checkmark \]

3.2 Isotherms, continued

⇒ For ideal gas, internal energy \( U \) is a function of \( T \) only, \( dU = nC_vdT \)

\[ dU = 0 \text{ for isothermal process} \]
\[ \delta Q = -\delta W \]
\[ Q = -W \]

3.3 Adiabats

\[ \delta Q = 0, \delta W = -PdW \quad - \text{how to calculate?} \]

- For ideal gas:

\[ dU = \delta W + \delta Q = -PdV = nC_vdT \]
\[ W = nC_v(T_{\text{final}} - T_{\text{initial}}) \]

- Adiabatic curves are described by:

\[ TV^\gamma = \text{const}, \quad \frac{P_{\text{final}}}{P_{\text{initial}}} = \left( \frac{V_{\text{initial}}}{V_{\text{final}}} \right)^\gamma, \quad \gamma = \frac{C_p}{C_v} \]

Deltoff ch. 4, Lectures 6-7
• Adiabats are steeper than isotherms on (P,V) plane because γ > 1

Q: Why is γ > 1?

3.4 Adding all contributions to W and Q

<table>
<thead>
<tr>
<th></th>
<th>W</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( nRT_H \ln \left( \frac{V_1}{V_2} \right) )</td>
<td>(-nRT_H \ln \left( \frac{V_1}{V_2} \right) )</td>
</tr>
<tr>
<td>II</td>
<td>( nC_v(T_C - T_H) )</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>( nRT_C \ln \left( \frac{V_3}{V_4} \right) )</td>
<td>(-nRT_C \ln \left( \frac{V_3}{V_4} \right) )</td>
</tr>
<tr>
<td>IV</td>
<td>( nC_v(T_H - T_C) )</td>
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3.5 Calculating the Carnot efficiency

\[ W_{TOT} = -\left( nRT_H \ln \left( \frac{V_1}{V_2} \right) + nRT_C \ln \left( \frac{V_3}{V_4} \right) \right) \]  

total work done by the engine

\[ Q_{IN} = -nRT_H \ln \left( \frac{V_1}{V_2} \right) \]  

heat absorbed at \( T_H \)

\[ \eta = \frac{W_{TOT}}{Q_{IN}} = 1 + \frac{T_c \ln(V_3/V_4)}{T_H \ln(V_1/V_2)} \]

Using property of adiabat \( TV^{\gamma-1} = \text{const} \) can show that \( V_3/V_4 = (V_1/V_2)^{-1} \)

\[ \eta_{\text{carnot}} = 1 - \frac{T_C}{T_H} \]

3.6 Considering heat transfers

<table>
<thead>
<tr>
<th></th>
<th>heat absorbed</th>
<th>heat released</th>
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</thead>
<tbody>
<tr>
<td>Carnot engine</td>
<td>( Q_{IN} = nRT_H \ln V )</td>
<td>( Q_{OUT,C} = nRT_C \ln V )</td>
</tr>
<tr>
<td>less efficient engine that burns the same quantity of fuel</td>
<td>( Q_{IN} = nRT_H \ln V )</td>
<td>( Q_{OUT} &gt; Q_{OUT,C} )</td>
</tr>
</tbody>
</table>

required by conservation of energy if \( W_{TOT} < W_{TOT,C} \)
3.7 Considering quantity $\delta Q/T$

- Quantity: $\oint \frac{\delta Q}{T} \rightarrow$ integral around the cycle

- Carnot: $\int \frac{\delta Q}{T} = \frac{nRT_H \ln V}{T_H} - \frac{nRT_C \ln V}{T_C} = 0$

- Less efficient: $\oint \frac{\delta Q}{T} = \frac{nRT_H \ln V}{T_H} - \frac{Q_{OUT}}{T_C} < 0$. \(\text{because } Q_{OUT} > Q_{OUT,C}\)

$\Rightarrow$ We will soon see that this is related to entropy generation by the less efficient cycle
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