3.020 Lecture 7

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1 Ideal gas processes

1.1 Reversible adiabatic expansion

Q. What to choose for independent variables ?

- $\delta Q = 0$, reversible process $\longrightarrow dS = 0 \longrightarrow$ use S
- expansion $\longrightarrow$ pressure/volume change $\longrightarrow$ use P
  Find equation of state $T(S, P)$

- Use general strategy to find differential form

$$\begin{aligned} dT &= MdS + NdP \\ &= M(\frac{C_P}{T}dT - V\alpha dP) + NdP \\ &= M\frac{C_P}{T}dT + (N - MV\alpha)dP \end{aligned}$$

By inspection, $M\frac{C_P}{T} = 1 \quad \longrightarrow M = \frac{T}{C_P}$

$N - MV\alpha = 0 \longrightarrow N = \frac{TV\alpha}{C_P}$

- Use properties of ideal gases

$$V = \frac{RT}{P}, \quad \alpha = \frac{1}{T}, \quad C_P = 5/2R \quad \leftarrow \text{for monoatomic gas}$$

$$dT\Big|_S = \frac{TV\alpha}{C_P}dP = \frac{TRT}{C_P TP}dP = \frac{R}{C_P}\frac{T}{P}dP = \frac{2}{5}\frac{T}{P}dP$$

- Separate variables and integrate

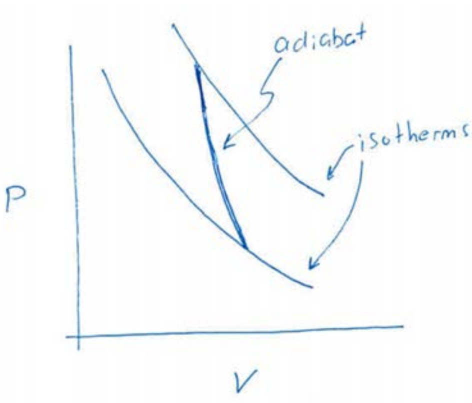
$$\frac{dT}{T} = \frac{R}{C_P}\frac{dP}{P} \quad \longrightarrow \quad \frac{T_f}{T_i} = (\frac{P_f}{P_i})^{R/C_P}$$

- Alternative forms of adiabatic, reversible expansion.

$$PV^r = const. \quad , \gamma = \frac{C_P}{C_V}$$

$$TV^{\gamma-1} = const.$$

$$\frac{P_f}{P_i} = (\frac{V_i}{V_f})^\gamma$$



e.g. compression stroke of internal combustion engine

- Alternative derivation of adiabatic, reversible expansion of ideal gas

$$\delta Q = 0 \quad \longrightarrow dU = \delta W = -PdV$$

  - from last time we know that $dU\big|_T = 0 \longrightarrow \quad dU = C_V dT$ for I.G.

$$C_V dT = -PdV = -\frac{RT}{V}dV$$

  - separate and integrate

$$\frac{T_f}{T_i} = (\frac{V_i}{V_f})^{R/C_v} \quad \longrightarrow \quad TV^{R/C_V} = const.$$

R/C_V
$= \frac{C_P - C_V}{C_V}$
$= \frac{C_P}{C_V} - 1$
$= \gamma - 1$

1.2 Isothermal expansion

Q. What to use for independent variables ?

- Isothermal, $dT = 0 \quad \longrightarrow$ use T
- Expansion $\longrightarrow$ use $P(0 - V)$

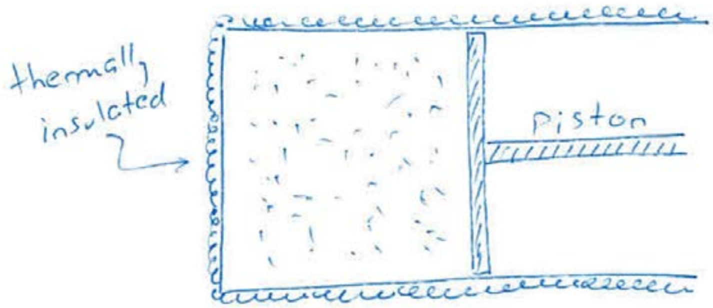
Find equations of state $G(T, P)$

- Use differential form

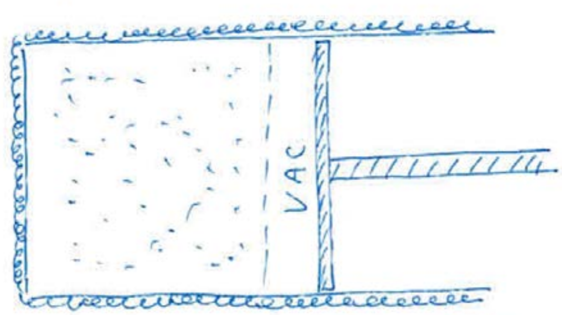
$$dG = -S \underbrace{dT}_{0} + V dP = \frac{RT}{P} dP$$

$$G_f - G_i = \Delta G = RT \ln\left(\frac{P_f}{P_i}\right)$$

1.3 Adiabatic free expansion



withdraw piston instantaneously



gas will spontaneously (freely) expand into the free volume

irreversibly

- Work and heat during free expansion

$$\textbf{Work} = -\int dV P = 0 \quad \longleftarrow \textbf{gas expanding into vacuum}$$

$$\textbf{Heat} = 0 \quad \longleftarrow \textbf{adibatic process}$$

$$\Delta U = W + Q = 0$$

$$\textbf{Implies} \quad dT = 0 \quad \textbf{for ideal gas}$$

- Adiabatic free expansion is spontaneous, so $\Delta S > 0$

  Q. How to calculate ΔS ?

  A. Find differential form $dS = ...$ and integrate

  Q. What to use for independent variables ?

  - Isothermal, $dT = 0 \quad \longrightarrow \quad$ **use** T
  - Expansion $\longrightarrow \quad$ **use** V (**or** P)