3.020 Lecture 27

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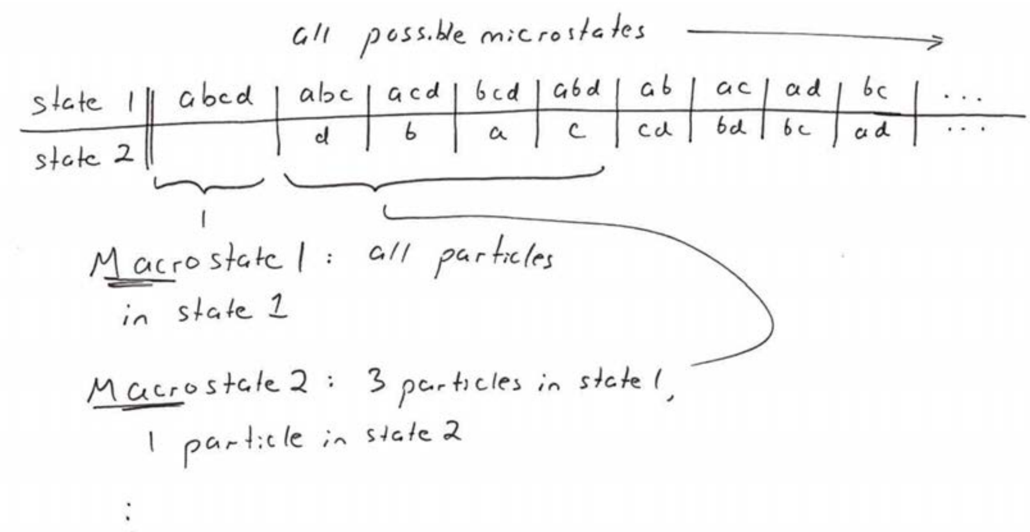
1 Microstates and macrostates

- Microstate (μstate): Description of the state of every molecule in a system
  e.g. O(10^{23}) pairs of position and velocity (r, v)
- Macrostate: Description of system on macroscopic length scale, averaging over microscopic (e.g. molecular) processes.
  e.g. P, T, N

- Simple example, after DeHoff
  4 particles: a, b, c, d
  2 possible staes for each particles: 1, 2



- General case: n particles distributed over r states
  $\Omega = \#$ of microstates in the macrostate defined as :
  - n_1 particles in state 1
  - n_2 particles in state 2
  - n_i describes the macrostate

occupation numbers

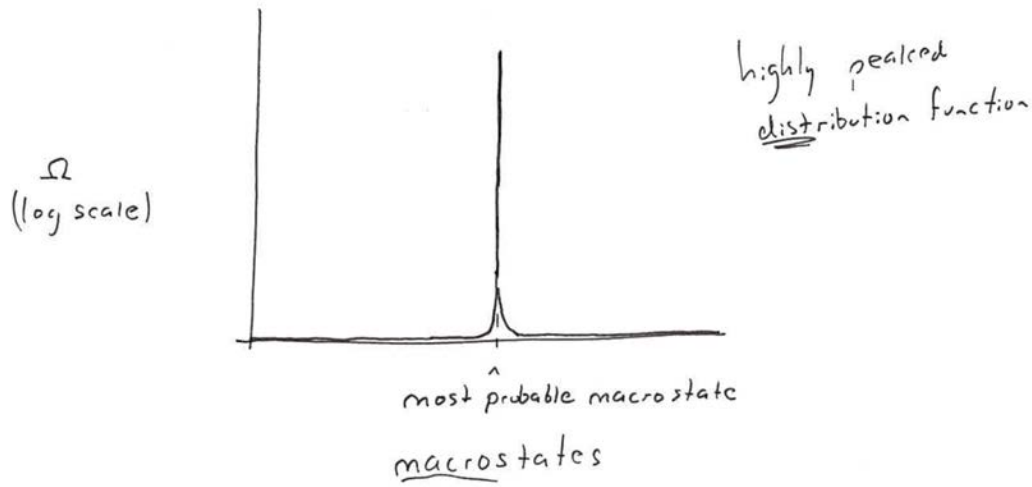
$$\Omega = \frac{n!}{n_1!\, n_2!\, n_3!\, \ldots\, n_r!}$$

Challenge: Work this out!

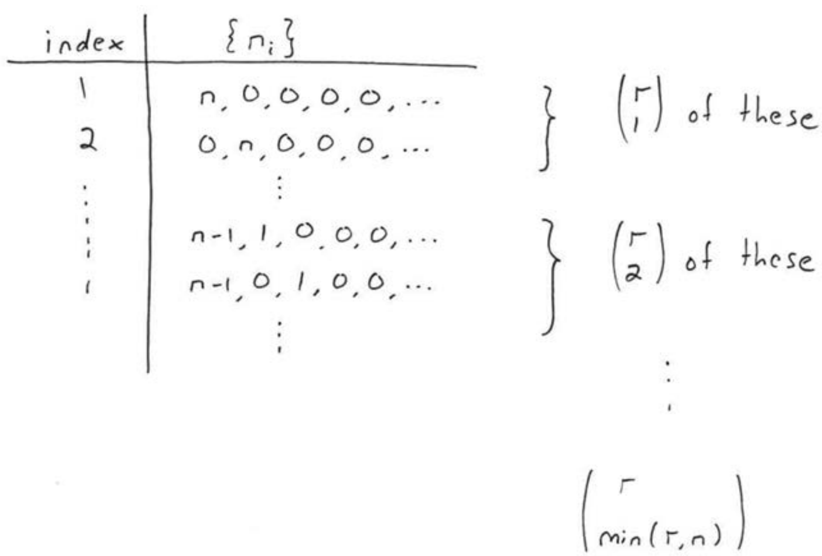
- For large systems :

$$n >> 1, \qquad r >> n$$

Ω is very sharply peaked around some macrostate



- Define (i.e. count) macrostates for n particles in r states or "boxes"



- We now know how to define macrostaes n_i and count them, and we now how to count microstates for each macrostate n_i

$$\Downarrow$$

Can plot distribution logΩ

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2 Ergodic principle: All microstates that are compatible with constraints are equally likely

- Ensembles of μstates that all satisfy given constraints
- Time average = ensemble average
- Frequentist approach to probability and statistics

- Likelihood of finding a given macrostate is proportional to its # of microstates

$$\rho_j = \frac{\Omega_j}{\sum_k \Omega_k}$$

ρ_j: prob. of finding macrostate j
Ω_j: # of microstates in j
$\sum_k \Omega_k$: total # of microstates possible within constraints Q. For what types of cases might the ergodic principle break down ?