3.020, Spring 2021
Thermodynamics of Materials
Problem Set 9

Massachusetts Institute of Technology
Department of Materials Science and Engineering

Due **Thursday** May 6, 2021 at 10am EDT

We encourage you to work in groups. If you do so, please note the names of your groupmates on the first page of your solutions.

Remember to clearly present your solutions, including intermediate steps. Failure to show your work may result in reduced credit. Sloppy presentation may result in reduced credit.

Thermo 9.1: Counting microstates [8 pts]

After DeHoff Problem 6.4 Calculate the number of microstates corresponding to each of the following combinations:

(a) [2 pts] A system with 17 particles and five energy levels

(b) [2 pts] A system with five particles and 17 energy levels

(c) [2 pts] A cluster of 75 particles, each of which may reside in any of 35 energy levels

(d) [2 pts] 1500 particles that may reside in 200 energy levels

Thermo 9.2: Counting microstates [5 pts]

Show that $\ln\binom{r}{n} \approx n \ln r$ for $r >> n$. Clearly indicate each time that you make an approximation. *Hint: Start with Stirling's theorem.*

Thermo 9.3: Ergodic ... what's that? [6 pts]

Give one example of a non-ergodic system.

Thermo 9.4: Thermalization [11 pts]

Suppose you have an isolated system of 20 gas molecules that are distributed over 10 different energy states. The states are labeled by index $k = 1 \ldots 10$, and the energy per state is $E_k = \epsilon_0 k^2$, where ϵ_0 is a prefecator with units of energy. Initially, 19 of the molecules are in the lowest-energy state, and one particle is highly excited. In other words, the occupation numbers N_k are $\{19, 0, 0, 0, 0, 0, 0, 0, 0, 1\}$.

In time, the highly-excited molecule will lose energy to the other molecules and the system and the system will approach thermodynamic equilibrium. We say that the highly-excited molecule "thermalizes". Propose a set of occupation numbers N_k that represent the system at any time during this thermalization process, and show that the Boltzmann entropy of this intermediate state is higher than the entropy in the initial state.

Note: You will study a similar situation in Lab 3.