MITOCW | 1. It's a quantum world: The theory of quantum mechanics

JEFFREY GROSSMAN: That is the point here. This is not a heavy math derivation kind of thing. This is not-- we are not going to go into great mathematical detail in quantum mechanics. Now, we will show some equations, we'll work with equations, because I want you to know what they are. You got to know what's under the hood. But we're not going to go into sort of gory detail in solving complex quantum problems on paper. That's where the tools help lift you away from that.

And it also is kind of the flavor of this class, which is that this is about practice. This is about learning the basic concepts, and then applying them to real problems and materials. And my enjoyment is in energy, so the problems we'll do in this part are going to be energy-related problems.

How many of you have had quantum mechanics? I wouldn't have expected that many. How many of you have never seen quantum mechanics in any form? Raise your hands, because it's good. It's good. We're going to start-- we're going to do quantum mechanics-- we're going to do a couple lectures here-- oh, this is what we're going to do.

That's our outline for the class, for the second half. And you can see that what we're going to do is we're going to kind of gently learn the key aspects of quantum mechanics that I'd like you to know for the modeling that we're going to do of materials. And then we're going to actually apply it. We're going to stop right here in the derivations, in the theory, and then we're going to start applying that to real energy problems. So then we're going to do two lectures on how to apply what we just learned to two different energy topics. One, solar thermal fuels. Very cool and exciting material. And another, hydrogen storage. Also very cool and exciting.

Then we're going to turn to how you go from modeling atoms to modeling solids with quantum mechanics, where different things happen. How many of you have heard of a band structure? Yeah. Now, how many of you can tell me, from the band structure, why silicon solar cells are too expensive? Now, you will be able to tell me that right here. We're going to know. And that's a really exciting thing, because it's all in the band structure. That answer is all in the band structure. So many properties of materials come out of these basic quantum mechanical property maps that you get.

So that's what we're going to do here. And then we'll go more general and do applications in nanotechnology and more broadly.

How many of you have heard of quantum confinement? That's down here. Do you know what it is? Somebody tell me what it is. Yeah.

AUDIENCE: So--

JEFFREY GROSSMAN: Oh, yeah.

AUDIENCE: What is it?

JEFFREY GROSSMAN: Yeah, yeah, let's hear it.

AUDIENCE: So what is it? So you have the [? key, ?] [? and that ?] [? holds ?] electrons, stuff like that, and they both [INAUDIBLE] take a small section, it's so small that it emits different colors-- I mean, I don't know.

JEFFREY GROSSMAN: Yeah. That's exactly right. Now, but why? You have electrons and holes, which are just positive and negative charges, right? You talked about that. And then you make it small, and it emits a different color light. What emits a different color light?

AUDIENCE: Quantum dot.

JEFFREY GROSSMAN: Quantum dot, for example. Now, why? Can anybody tell me why?

AUDIENCE: The [? sides ?] all emit from a certain wavelength?

JEFFREY GROSSMAN: Why? Changes the fundamental gap of the material, but why? It's something called quantum confinement. Actually, it's nothing more than what those two words say. Quantum confinement is the squeezing of quantum things. And in this case, our quantum things are positive and negative charges. Right?

And it turns out, when you make an expectation, the positive and negative charge like to be kind of close to each other. They don't like to go away, necessarily. Sometimes they like to stay close. But they don't like to get too close. So if you make the material smaller than the closeness they like to get, then they're squeezed. They're confined. And when you confine quantum things, you get interesting effects.

That's why I can take something that, say, ordinarily looks like this, and take a small piece of it, and all of a sudden, it turns blue. Like, how cool is that? You changed the color just by changing the size. That's quantum confinement.

That's something that has everything to do with quantum mechanics. Right? Electrons and holes, positive and negative charges.

You're not going to get that-- you're not getting that over here. Mm-mm. That's part one. Now we're going to go to quantum mechanics. Because you can't simulate electrons and holes in part one, but now you can. Well, you will be able to. And we're going to actually-- you're going to be able to tell me what color a quantum dot could be just by solving these equations on a computer. Because now we're going to capture quantum mechanics and apply it to things like nanotechnology. OK, so that's just kind of a flavor of where we're going to go.

Any questions? Mm-hmm. It's a quantum world. That's where we're going to start.

And the reason I put this is because you can do Shakespeare very quickly. There are people who have been doing it for a long time. "Reducing expectations for over 20 years." I just like that quote. There is something called the Reduced Shakespeare Company. And that's what I'm going to try to do today. We're going to go through the basics of quantum mechanics in one lecture that's not really meant to teach you too much about quantum mechanics, or go too much into depth. Again, I want to go through some of the key aspects, but really to get a sense for it, not to go into great mathematical depth. That's what you take a quantum course for. OK?

But I need you to understand some of the key aspects of quantum before we embark on modeling quantum mechanics. And so I want to start with why, and why did we even need this theory? And so we'll talk a little bit about that. And then we'll talk about some of the key aspects, which is this wave-particle duality. And then this is it right here. That's the equation we're going to be solving for all of part two. The Schrodinger equation.

Has anybody seen the Schrodinger equation? Yeah, a little bit. Have any of you solved it for [? A? ?] I saw a so-so. A little kind of like you sort of solved it? What happened there?

AUDIENCE: I solved it, but probably incorrectly.

JEFFREY GROSSMAN: OK, I'll take that. That's good. And what else-- what other problems have you solved with the Schrodinger equation? Yeah.

AUDIENCE: Hydrogen atom?

JEFFREY GROSSMAN: Hydrogen atom, OK. That's where we'll end today. But still listen, because it's going to be really interesting. Any other-- who else has solved the Schrodinger equation? Yeah.

AUDIENCE: Particle in a box.

JEFFREY GROSSMAN: Particle in the box.

AUDIENCE: Periodic potential?

JEFFREY GROSSMAN: Periodic potential. Any others? Yeah.

AUDIENCE: Quantum tunneling.

JEFFREY GROSSMAN: Quantum tunneling, cool. Anyone else solve it?

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: That's what we're going to do at the very end. That's the hydrogen atom. It's a harmonic potential. Somebody said something else? That's not quite a harmonic potential, but.

AUDIENCE: Cat in the box.

JEFFREY GROSSMAN: Any other-- cat in the box! I've got a cat in the box today for you! I put a cat in a box.

The thing is, what's interesting about all those examples you all just said?

AUDIENCE: They're all [INAUDIBLE].

JEFFREY GROSSMAN: Yeah. They're all kind of what?

AUDIENCE: Simple.

AUDIENCE: Exactly solvable.

JEFFREY GROSSMAN: Exactly solvable and simple, and therefore? To someone in course three?

AUDIENCE: Boring.

JEFFREY GROSSMAN: Eh. Right? That's where we have changed so much in what we can do because of computation. So now, because of the approximations that you're going to learn in the next couple of lectures, and the speed of computers, we can actually solve the Schrodinger equation for realistic materials. That's why we can have this class. We would not be having this class-- no. We wouldn't be having part two of this class back in the 1970s, 1980s.

Oh, that reminds me. You should go to Berni Alder. Are all of you aware of Berni Alder's talk? He is really one of the key foundations of computational science in this country and in the world. And he contributed to some of the very first molecular dynamic simulations ever done. And really helped-- and that was in the 1950s. So he's going to be a speaker. Who can tell me when?

AUDIENCE: Tomorrow.

JEFFREY GROSSMAN: Tomorrow at 4:00. Where? Somewhere on this campus, Berni Alder will be speaking at 4:00. Please find him and go if you can. He just really made some really important key contributions to this field and helped to launch it. It gave credibility. He saw-- in molecular dynamics simulations in the 1950s, he could see really interesting phenomena-- phase transitions happening in materials. Really helped to launch the field of computation in materials.

OK, so let's go. So here, you've seen this picture in the first part. And you've also seen this. And, see, this is what? In part one, this is what?

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: Yeah. Force field, Lennard-Jones. And where do you get that? Because you need it, right? Can you do part one without that? Now, so where do you get it?

AUDIENCE: Quantum [INAUDIBLE].

JEFFREY GROSSMAN: Well, you could. Or you could get it from where?

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: Experiment. But see, if I'm making new materials that have never been made, or if I'm making materials where interactions are happening that I can't really measure, because they're really hard to measure, then where could you get something like this? You can't do the experiment. That's where we can go all the way down to here. And that's another place where this whole realm of quantum modeling is going to be important. So that's where we are, the bottom of the barrel. But not any less important.

This is just-- this has nothing to do with quantum mechanics at all, but I thought it was pretty cool. Right? This is just a guy who has this-- he's got a camera pointing that way, and then he's got a little projector and a very specialized material here, and he's projecting the image onto his jacket, and so it looks like he's transparent. But the point being that I thought it was a cool idea, I guess.

Sort of interesting. So you can see, the car's sort of there, but I don't think the car's really there. Anyway, he needs to focus it. But weird things happen in quantum mechanics. Really, really weird things. And it is a very quantum world. If we understand those basic particles like electrons, then we can almost understand everything. Almost everything. Seriously. OK?

So if you want to know the properties, like mechanical properties, and you have a good potential of this piece of iron, for this piece of iron, you can do classical mechanics. But if you want to understand the mechanical properties, and you don't have a good potential, well, you may need quantum mechanics. And certainly, if you want to understand the electrical properties, or the optical properties, can you do that with classical potentials? Tell me why. Why I can't get electrical properties with classical potentials? Yeah.

AUDIENCE: Because you're modeling the entire atoms. You're just basically considering the nucleus and not really talking about the electrons.

JEFFREY GROSSMAN: Exactly. And what do I need for electrical properties? You got to know about the electrons. Band structures! Band structures will tell us about the electrical properties. You will learn to see-- it's like *The Matrix.* You will learn to read band structures as whole cities to explore that tell you all about things like this, and this. It's really powerful.

And it tells you-- and so all we need to do is go down one more scale, because this is where you've been, and this is where we're going. We're going here. Do electrons actually orbit?

AUDIENCE: No.

JEFFREY GROSSMAN: What do they do?

AUDIENCE: Go around [INAUDIBLE]?

JEFFREY GROSSMAN: They go around?

AUDIENCE: No, they fly around.

JEFFREY GROSSMAN: They fly around? Sounds better. What?

AUDIENCE: They have their orbitals that they can-- that there's a possibility that they'll [? be at. ?]

JEFFREY GROSSMAN: Yeah. Oh, yeah. That's where we're going. Why do we call them orbitals? Why do we call them orbitals? We'll get to that.

Here's another motivation. And this is a great example of what I was just talking about in that potential. If I have the right potential, and I do my classical force field calculations, I can feel pretty golden about simulating the behavior of materials, like how it cracks. Did you guys do crack propagation? Ol' Professor Buehler likes crack propagation. There are not many people on the planet who more about crack propagation than Professor Buehler.

But look at this. I've cracked this piece of-- what is this? Graphene, let's say. Silicon?

AUDIENCE: Silicon.

JEFFREY GROSSMAN: Oh, it's silicon. This is silicon just looked at from the side. Yep, there they are, two. I've cracked this piece of silicon. I guess I could have read "silicon" there and known. And I can't measure it experimentally. And so I crack it in the computer with something called an EDIP potential that was fit to certain experimental parameters, and then I crack it in a computer with a tight-binding model, and look at the difference. Is that the same crack? Is that the same property? Is it happening in the same way? Absolutely not. It's completely different. Qualitatively, quantitatively, the results you'll get and the findings you publish will be totally different, depending on which potential you choose.

So that's really-- this is the reason why we need these more accurate-- a way, at least, to get more accurate potentials. And then you can get these other properties, as I said. So you can get things like where the electrons are. You can model things like Raman spectra, which have to do with electrons and light.

You can model things like reactions. Now, sometimes potentials-- like especially reacs. Did you run reacs potentials? Reacs-- the name "reacs" makes me think reaction. Does it make you think that? So they're actually modeled for reactions. And that means that they can break bonds. However, there are plenty of reactions--

AUDIENCE: [SNEEZES]

JEFFREY GROSSMAN: --salute-- where reacs can actually not get the right physics, the right structure of the potential. Why? Well, take a look at this one. This is butadiene plus ethylene going to cyclohexane-- hexene. It's one of the simplest ways you can actually make a ring.

Watch those electrons. Those are the electrons of the part of the system that's doing the reaction. That's where the electrons are. And where those electrons are is what determines everything about the reaction-- the barrier, whether it happens. And let's see if I can get it to play.

And now, watch them. Watch them. It's like balloon animals. See, they pop up on the top, and then they flow across. And then they do a big shape-shift as the reaction actually happens, and they curve back around, and they bubble up on top. And that's the final position probability of those electrons.

It's a very simple reaction, but the behavior is extremely complex. So when you fit a potential, this is what you're ignoring. All of that complexity. And you're trying to fit it to some functional form, and reacs has a whole lot of degrees of freedom, so it actually can fit some of this complexity very well. But that just gives you a sense of the kind of thing that's happening, and why you can miss important aspects with potentials.

So in the Standard Model, atoms are made by these massive point-like nuclei. That's what you've been doing. And now they're going to be surrounded by these electrons. And you're going to have core electrons, and then you're going to have the valence electrons, which do all the work. They're the ones that bind together and do all the interesting stuff.

Somebody tell me, who knows about the chemistry, like s, p, d filling? Good. Somebody tell me, how many electrons are there in carbon?

AUDIENCE: Six.

AUDIENCE: Six.

JEFFREY GROSSMAN: Six. And how many are core and how many are valence? What's what there?

AUDIENCE: Two core, two [? valence-- ?] four core, two valence.

JEFFREY GROSSMAN: OK, so it goes 1s, 2s. That's four. 2p, how many are in the 2p? Four, two? How many? Two? Like that, that's good.

Now, what about silicon? Which number is that? Oh. We really like materials-- material scientists really like silicon. It's good material. 14. How many electrons are in the valence?

Yeah, it's the same-- because in carbon-- you see, you had the 2s and the 2p, four electrons in the valence. And in silicon, you've got the 3s and the 3p, four electrons in the valence. Right?

But you see, here's carbon. And here's 2p. And now here's silicon. This is 1s, 2s, 2p. Is this making sense to people? How many of you have not seen this? It's OK. We'll talk a little bit about this. So these are filled, filled, and so on, up to here. And then you got these guys. And this is the valence, and this is the valence.

Those are the electrons that do all of the interacting. Those are the ones that do the chemical bonding. They're the ones that we're going to care about in this class. We are not going to be working with these core electrons. Those are core electrons. So we're going to freeze those out, because they don't really play much of a role in the kinds of properties we're going to actually care about in this class, OK?

And I won't talk much about how we do that. We do that with something called pseudo potentials, which model the core part of the electron. So this part, we're going to get rid of.

Now, somebody tell me why, in looking at this, why we are made out of carbon and not silicon? The valence electrons look pretty similar, don't they? You got a couple of s and a couple of p in each case. OK, higher. What does that mean?

AUDIENCE: More metallic [INAUDIBLE].

JEFFREY GROSSMAN: More metallic? Maybe. [? Except so, ?] because I'm metallic.

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: What was that?

AUDIENCE: More reactive?

JEFFREY GROSSMAN: More active? Mm, not sure about that. You could argue that if you think about the alkalis, if you're thinking in that way, that it wants to lose these electrons more. Carbon can be a very reactive atom.

Actually, the bond in diamond is considerably higher energy than the bond in silicon. So why are we made out of carbon and not silicon? We'll come back to that. Think about it. It's cheaper?

It's cost-efficient. Actually, nature is always really good about that. Not quite the answer.

It's real. These electrons that we're going to be simulating, they exist! They can be seen, and look at that complex structure! That's the complexity of a bond measured in experiment. That's a real experimental bond. And it shows where that electron density is in the bond, and look at how complicated that is. And that's what I'm talking about.

And we can calculate that-- well, this is a different material, but you can see a similar sort of complexity. We can calculate that complexity with quantum mechanics. And you can see again by looking at this-- look at this. I mean, there's like a ball underneath a donut and a teardrop, and a donut and a ball. Did your classical potential get all of that? No way. It's just averaging over this, right?

And so once we go to trying to calculate properties of materials that are this complex, electrons, we really got to get it right. And that's what the Schrodinger equation will do.

So now, why is this important to solve in the computer? Well, it's sort of the same reasons for-- these are the same reasons for classical, the first part of the class. It provides us with a microscopic understanding. It has predictive power. And in this case, we can really say it's first-principles. And you hear this word "first-principles--" how many of you have had "first-principles?" What does it mean? Take a guess.

AUDIENCE: From the basic-- just from basic axiom. You're not really assuming very much.

JEFFREY GROSSMAN: Exactly. And in this case, what are we going to assume? What is the only input into the simulation?

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: Well, what's the only input into the simulation of that equation?

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: The atom what?

AUDIENCE: Like the atom that you're--

JEFFREY GROSSMAN: Yeah, which is what? Just the atom. The atom number. And maybe its position. That's important. Like where the atom is. And the charge. The charge on the atom. Is it ionized, is it not. And that's it.

Not really, ha, because we have to make approximations. But ideally, that's all we would need to put in to solve first-principles. That is, to solve without any other fits or inputs. Just the atom, which atom is it, and where is it is what you put in.

And the challenges we have is that this is a much more computationally-intensive kind of simulation. So how long-- who was saying that it was too long? And then we'll have classical-- yeah, how long was it taken?

AUDIENCE: Six hours.

JEFFREY GROSSMAN: Six hours, and how many atoms?

AUDIENCE: One--

JEFFREY GROSSMAN: One atom?

AUDIENCE: No, 150,000.

JEFFREY GROSSMAN: Oh, 150,000. OK, good. So if we tried to simulate 150,000 atoms, that's not even a sentence I can start. Actually, in quantum mechanics, you can't even get close to that. So if you try to do 150 atoms in quantum mechanics, it could easily take you six hours, and it could take you even much more.

Our goal in the homeworks will be to make sure the simulations take as long as possible. I'm kidding. But we'll make sure that they don't take too long. But they're slow and painful, and you will feel that pain. Not because we're going to make problems where you have to simulate for a long time, but because you'll just see. You'll be like, wait a second, I only got two atoms in here. Why does this take an hour? Well, it's because even with all the approximations and the computer speed, it just takes longer to do these kinds of calculations that are more complex. OK?

How many people know where the fastest computer in the world is? I asked this question already. You know the answer. Where is it?

AUDIENCE: Google.

JEFFREY GROSSMAN: Google. How many people know how many times I've asked them for an account? It's many. They never gave it to me. I would love to run on Google's computer.

OK, classical mechanics goes back a long way. This is what you've been solving. You've been doing a lot of F equals ma. And there are problems with it that led to quantum mechanics.

I'm just going to give you a couple of examples because they're fun, and then I'm going to show you a really goofy movie. Has anybody seen the really goofy quantum movie that I'm going to show?

AUDIENCE: Is it Dr. Quantum?

JEFFREY GROSSMAN: It's not Dr. Quantum, but that sounds cool. Maybe it's Dr. Quan-- no, I don't think it's Dr. Quantum. Has anybody seen really goofy quantum movies? I love goofy movies. If you had me for 3012, you know I like strange movies. It's a little thing. And they're just a couple, so I promise you there won't be too many. One I'll show you today, and one I'll show you on Thursday.

Some of the problems that led to the need to develop a new theory were the classical atom, the quantization of properties, the wave aspect of matter. We'll talk about those. And then black-body radiation, which I won't talk about, but that's a really cool one because of the word they use to describe the failure. How many people know what I'm talking about? It's called the ultraviolet--

AUDIENCE: Catastrophe?

JEFFREY GROSSMAN: Catastrophe! I love it. You can't really call theories today a catastrophe. You got to have a lot of-- but that was a catastrophe. That was really a catastrophe. Because the energy that was going to be emitted from a black body just kept going up and up and up and up. And it doesn't do that in experiment. That's a catastrophe. Catastrophe, when you use words like that, you know you need a new theory.

A lot of really, really, really smart people were thinking about these problems. They were aware of them. They knew they had these problems back at the time. And they were really working intensely on saying, well, why are we having all these problems describing the world? We've been good to go for 400 years. And now this? 350 years. What's going on?

In the classical atom, the problem there is that, you see, if you think about a hydrogen atom as a plus charge surrounded by a negative charge-- that's the nucleus, and there's the proton, and there's the electron. And if it's orbiting, then it has to be accelerating. And we know that an accelerating charge is going to give off energy in the form of radiation. Right?

Well, if that were true-- and we know it is true. In some situations, they knew it very well to be true. That's what a synchrotron is.

But then this should just spiral, right? And it should just collapse into the center here. Why doesn't it do that?

Do you know how long a hydrogen atom should be stable, according to that classical world, that classical picture of accelerating charge? It's accelerating, by the way, just to stay on course around this circle. So it's giving off radiation, losing energy, meaning its orbit-- it should come closer and closer to the center. How long would that be stable?

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: What fraction? Give me a 10 to the minus something.

AUDIENCE: 12.

JEFFREY GROSSMAN:

Yeah, right around there. 10 to the minus 10, 10 to the minus 12. That's how long hydrogen atoms would be in existence if classical physics described them, which it doesn't.

And then there was this. And this was a really cool thing that also could not be described by the classical world. And again, this is my reduced Shakespeare version of quantum, so we're covering these very general, very interesting topics just to give you a flavor and a feeling. And then, if you want to learn more, there's just so much you can read about these. They're really cool experiments that were done.

This is the experiment that won Einstein the Nobel Prize. It's very important work, but it's probably one of his least profound pieces of work. But it was very important, again, to start to understand this duality between waves and particles, which I'll talk about in a few minutes.

And what he found is that if you take a piece of metal, and you shine a light on it, the energy from the light can kick electrons out. But what was thought is that if you increase the intensity of the light, that you'd increase how much energy these particles get kicked out by. You just increase the intensity of the light, and then you could get these things to come out at higher and higher energies.

But if you plot the energy of an electron coming out of the metal versus the intensity of the light, it's a flat line. It does not depend on the intensity of light. You get more electrons out, but you don't get them to be any more energetic. And that had to lead to a picture of light not necessarily being a wave.

Because instead, what you find is that there is this very strong direct dependence of the energy of the electrons that come out when you shine light on a piece of metal, and the frequency of the light. And so what Einstein said is, well, maybe light can be thought of as particles that have an energy that depends on their frequency. And he wrote this thing down. So they have an energy that depends on this Planck's constant, this thing here, this h, or h bar.

Do you like a bar, do you not? Do you like pi? Do you like 2 pi Do you want to write that, not? This is all-- it's a free world, and you guys can write h or h bar, however you want. That's my feeling about it. I'll stop there. h bar. h. 2 pi But you'll need to get it right for solving problems, right?

But this is the energy, and it's quantized. Well, no, sorry, it's not quantized yet. I didn't say that. But it's a particle. It's based on the frequency. The energy goes as the frequency of the light, and that's what was measured. Kind of building on the understanding of what's going on.

And this is a great one. When you look out-- you see, if you excite an atom, if you excite the electron in an atom, and you let it go back down-- so if you excite an electron, let's say of a hydrogen atom. Here's your hydrogen atom, and here's that electron that's orbiting. It's not orbiting, but we're going to say it's orbiting. And you excite it up in energy, and then it goes back down, it emits light.

And so you can look out into the world-- which is whole lot of hydrogen-- and you can say, well, what kind of light do I see? And you would think, according to classical physics, the classical world said, well, it could be excited to any amount from different reasons, can excite an electron. We won't talk about that now.

Instead, you look out, and you see what wavelengths of light are coming at you from hydrogen and space, and you don't see a continuous spectrum at all. You see only very, very certain wavelengths. So why? How could that possibly be?

That was-- now we say, well, it's just quantization. Particle in a box, which I'll get to in a second. But back then, they didn't have particle in a box. They didn't have Schrodinger. They didn't even really know whether light was a particle, a wave, or what. They were really grappling with that.

Now-- and it actually is even more complex, and we'll see this complexity in the next couple lectures. We'll be able to actually calculate this if we want, than just those discrete energies. There were these big jumps, and then there were these little jumps. What's up with that? They didn't just see discrete things, they saw them sort of bundled up.

And then, if you were lucky enough to have looked at one of these series, and you were like, a-ha, I got the green one, then you got it to be named after you, if it was one of the first five or six. And that's why they have names. And then they stopped naming them, because they were like, well, OK, we kind of get it already. Like, I don't know if there's a name for the sixth one. Maybe there is. Does anybody know? Yeah.

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: See, now, I could take that as ageism, but I won't. Because, now-- I'm kidding. My group makes fun of my age very often.

Now, OK. This is the time to put down your PDAs-- if you're in the back and looking at them-- and to pay attention, because this is a really fun movie and I wouldn't want you to miss it just because you're texting somebody.

[VIDEO PLAYBACK]

- And here we are.

JEFFREY GROSSMAN: Is that quantum guy? OK.

- Granddaddy of all quantum weirdness-- the infamous double slit experiment.

JEFFREY GROSSMAN: Got to know about the double slit.

- To understand this experiment, we first need to see how particles, or little balls of matter, act. If we randomly shoot a small object-- say, a marble-- at the screen, we see a pattern on the back wall where they went through the slit and hit. Now, if we add a second slit, we would expect to see a second band duplicated to the right.

Now let's look at waves. The waves hit the slit and radiate out, striking the back wall with the most intensity directly in line with the slit. The line of brightness on the back screen shows that intensity. This is similar to the line the marbles make. But when we add the second slit, something different happens. If the top of one wave meets the bottom of another wave, they cancel each other out.

So now there is an interference pattern on the back wall. Places where the two tops meet are the highest intensity-- the bright lines-- and where they cancel, there is nothing. So when we throw things-- that is, matter-- through two slits, we get this-- two bands of [? hits. ?] And with waves, we get an interference pattern of many bands.

Good, so far. Now let's go quantum.

JEFFREY GROSSMAN: Now that's what I'm talking about.

- An electron is a tiny, tiny bit of matter, like a tiny marble. Let's fire a stream through one slit. It behaves just like the marble-- a single band. So if we shoot these tiny bits through two slits, we should get, like the marbles, two bands.

What? An interference pattern! We fired electrons, tiny bits of matter through, and we get a pattern like waves, not like little marbles. How? How could pieces of matter create an interference pattern like a wave? It doesn't make sense.

But physicists are clever. They thought, maybe those little balls are bouncing off each other and creating that pattern. So they decide to shoot electrons through one at a time.

JEFFREY GROSSMAN: This is cool.

- There is no way they can interfere with each other. But after an hour of this, the same interference pattern is seen to emerge. The conclusion is inescapable. The single electron leaves as a particle, becomes a wave of potentials, goes through both slits, and interferes with itself to hit the wall like a particle. But mathematically, it's even stranger. It goes through both slits, and it goes through neither, and it goes through just one, and it goes through just the other. All of these possibilities are in superposition with each other.

But physicists were completely baffled by this, so they decided to peek and see which slit it actually goes through. And they put a measuring device by one slit to see which one it went through and let it fly.

But the quantum world is far more mysterious than they could have imagined. When they observed, the electron went back to behaving like a little marble! It produced a pattern of two bands, not an interference pattern of many. The very act of measuring, or observing, which slit it went through meant it only went through one, not both. The electron decided to act differently, as though it was aware it was being watched.

And it was here that physicists stepped forever into the strange neverworld of quantum events. What is matter-- marbles or waves? And waves of what? And what does an observer have to do with any of this? The observer collapsed the wave function simply by observing.

[END PLAYBACK]

JEFFREY GROSSMAN: Nice. That's pretty cool. I mean, it's a little bit-- but does that blow you away? Seriously? I mean, I-- so here's the thing. I strongly recommend you take this movie out the next time you go to a bar, and when you're at the table, maybe you're on your second round-- wait, I keep doing this. You guys shouldn't be drinking. Sorry. I keep making this mistake. OK, you're drinking tomato juice, and you bust out this movie, you're going to blow people away. And it's a really great-- it's a conversation starter, it's an ice breaker.

And that's why Niels Bohr said "anyone who is not shocked by quantum theory has not understood it." And here's what's amazing about it. This is real stuff that matters a lot for material science and every other discipline in engineering. This is not just some theoretical, very super physics-y, probably not going to matter to me kind of thing. Well, this matters a lot, because this is how electrons behave.

And that's what, as I said, dictates so many of the properties and materials that we care about, especially in energy, which we'll spend some time in this part of the class talking about.

So I got to talk about Schrodinger's cat. How many of you know about Schrodinger's cat? Who can tell me what you do with this poor guy? Go ahead. Somebody-- who said they knew about Schrodinger's cat? What's the experiment? Yeah.

AUDIENCE: So you put a cat in the box, and--

JEFFREY GROSSMAN: Put a cat in a box.

AUDIENCE: And you have some sort of radioactive element, or--

JEFFREY GROSSMAN: Radioactive.

AUDIENCE: --Schrodinger liked to particle, or whatever. And you have a Geiger counter or something to measure it that will break a vial of poison if it detects the radiation and won't if it doesn't.

JEFFREY GROSSMAN: Right.

AUDIENCE: And so if it radiates, then it breaks the poison and the cat dies. If it doesn't radiate, then the cat survives. But until you observe it, it hasn't done either, so the cat's both alive and dead at the same time.

JEFFREY GROSSMAN:

And that was-- you see, that's exactly right, very nice. And the rest of that is you say, well-- you put it so that it has a half life something like an hour, roughly, between when it emits a particle. And so then you measure it-- and so after an hour, there's a 50-50 chance the cat has been killed or is alive.

But the key point is what you said there, which is that, until you measure it-- you see, the little radioactive thing is a very quantum mechanical particle. So until you measure it, as with the electron, it's everywhere. It's both emitted and released the poison and not. The state is quantum mechanical.

And so what Schrodinger did is he said, well, that's absurd. Because then if you take that up to the macroscopic levels, you can get these sort of absurdities. And that's why he came up with this, actually to make a point that we don't understand this yet. That was really the point here.

And this paradox has been-- and so, because the point being the cat is both dead and alive according to quantum mechanics. It is both. It is a superposition of dead and alive until you open the box, in which case, you collapse the wave function because you observe it.

That would be really what quantum mechanics tells you. And this paradox has been one of the central talking points of quantum mechanics, or examples used to illustrate how strange quantum mechanics is for many decades. And it's also led to a lot of discussion on how we should actually interpret quantum mechanics.

This is called the Copenhagen interpretation, for those of you who are interested in reading more, but there are many, many others. And I have a sheet at the end-- I'm not going to go through-- where I list like 10 or 15 different interpretations of quantum mechanics. How cool is that? We really don't know what this all means. And that's pretty neat.

Actually, Schrodinger, I think, he made this statement-- which I thought was funny-- about his paradox. "I don't like it, and I'm sorry I ever had anything to do with it." Because I think it got taken out of context a lot as well, and the whole killing cats is not a very PC-- but he very carefully said, pardon my-- in his paper, sorry for being so crude about talking about dead cats.

And the final one to look into, but I won't talk about more, is called the EPR paradox. And this stands for Einstein, Podolsky, and Rosen. And it's a very interesting paradox, again, that comes out of quantum mechanics, which is that you can entangle the states of two particles together-- let's say the spin-- so that its total is zero. But one is one and one is minus one.

And now as you separate them, because they were entangled at one point, you can separate them to very long distances, and according to quantum mechanics, they both have to really be both. One cannot be one and one minus one-- the collection is zero. But one cannot be one and one minus one until you measure one. And then that one is something, and the other one is the other something.

And that actually leads to some very interesting paradoxes involving information, because you've gotten information about something that's more than the speed of light away. Is that possible? [INAUDIBLE] wonderful discussions about this kind of stuff that I won't go into.

OK. So now we're going to get to-- we're going to move-- those are motivations. This is what everybody was at the time grappling with, is how can we explain these things? And what it really came down to is that particles can be both waves and-- everything can be both waves and particles. But especially when you get down to very small masses, the wave nature can dominate. And that is the case for electrons.

So for waves, they have particle-like properties, and for particles, they can have wavelike properties. And that's really the key observation. That's the wave-particle duality.

And as you saw in the video, what that means is that-- you see, a wave is a wave, which means it can have both-- it can have interference. As you saw, the electron, they shoot it one at a time. Not interfering with another electron. It's literally interfering with itself.

And you can see that you can actually get interference patterns, and that diffraction pattern that you see on the plate in the two-slit experiment is a measurable wavelike property of the electron. You know what I love so much about that is-- I don't know if you caught it, but it's shot out as a particle. It's got to go through the slits as a wave. There's no other way to explain it. If you don't observe it, it goes through as a wave.

But then in order to-- the original experiments were done with a photographic plate that can only be-- the track can only be made if it's a particle. The position of the electron has to be a particle to make the dot on the plate. So it goes from particle to wave to particle.

And that's really the nature of quantum mechanics. You have to be able to describe particles as waves. And so we start seeing effects like this, where you can get constructive interference from a particle. That's really cool. And that's what we need to be able to describe to describe the quantum world in a computer.

That's real. That's a water dropping to make waves. This is actually an electron. This is where an electron sits on a surface. I think that's a quantum corral.

And it's been measured experimentally-- this is the experimental setup. It's been measured experimentally for bigger things than electrons. You think electrons are really tiny. Oh, Grossman only cares about them for his electronics and optics, but I care about bigger things, like fullerenes. Well, if you take fullerenes and you shoot them through two slits, you also get a diffraction pattern. This is real stuff. It happens for real bigger things.

OK, but now, could I go through two slits? Is that possible? Am I quantum mechanical?

AUDIENCE: Are you observing yourself?

JEFFREY GROSSMAN: I am-- I--

I don't have a good response to that question. But I am not observing myself and no one else is. Am I quantum mechanical? Yeah. Why don't I create diffraction patterns wherever I go?

AUDIENCE: Too big.

JEFFREY GROSSMAN: Too big? Don't read ahead! Why am I too big? So where does this stuff end? Are you always a wave, no matter how big you are? And that's where we're missing something. We're missing something really important. The answer is, absolutely. I am a wave, and I'm proud of it. My arm right now is a quantum mechanical-- this is a wave. This is not a particle.

If I throw this chalk-- that was fun-- that's not a particle. That was a wave I just threw. It has a wave associated with it. It is very wavelike. But if I throw chalk through two slits, will I get a diffraction pattern? No. And now we're going to see why.

When is a particle like a wave? Well, it always is-- but to varying degrees. And that's where de Broglie came in and generalized what Einstein had been working on, and others, and said, look, everything has a wave. Everything is both a wave and a particle-- everything.

And he said, and we can associate-- we can calculate a wavelength to these things. Here's the electron. The wavelength of the electron is 10 to the minus 10th meters. Now, why is that a distance that is making electrons really quantum mechanical? Or why can't I describe them classically at that distance? What's that distance?

AUDIENCE: Angstrom?

JEFFREY GROSSMAN: It's an angstrom. What happens at the angstrom level? Like, what's an angstrom? What's 4 angstroms? What's 2.447 angstroms?

AUDIENCE: Carbon.

JEFFREY GROSSMAN: Carbon. Yeah, [INAUDIBLE] carbon. So that's the level of bonds. This is it, man. This is where-- that wavelength is the same size as the kind of distances that matter to an electron, like the distance from one atom to another. It's the same. So that means-- because the wavelength is so similar to the world of the electron, then that quantum effect is going to be really important.

And the wavelength is 100 times smaller for a fullerene, which means that it's maybe less important for things we do with fullerenes. What do we do with fullerenes, by the way? Has anybody done anything with fullerenes? Probably not, but what could we do with fullerenes? No. We're still working on it. Make what?

AUDIENCE: Make soccer balls.

JEFFREY GROSSMAN: Soccer balls. You can make pretty high temperature superconductors-- what would have been considered very high temperature before high TC came on. You can also make solar cells. You can make electronics. You can make all kinds of interesting chemistries. And most of the time, the quantum mechanics won't matter, but that experiment showed that you can still measure it.

Now I have a baseball, and here's the wavelength of a baseball. Can I measure that? But it has a wavelength. And it's just-- and so do I. So do you. It's just that this is 20 orders of magnitude smaller than the diameter of the nucleus of an atom. That's how tiny the wavelength is associated with these more larger objects.

So you see, these are very quantum mechanical, and they do have-- they would be able to give you the same effect, if you could pick up-- if you can measure distances that small.

So what we have in classical versus quantum is that we have to describe in a computer the mechanics of waves, as opposed to the mechanics of particles. So what we had before-- OK, so that way, we're going to be able to describe the wave and the particle behavior.

What we had before was this. This is the mechanics of a particle, and you all did a lot of simulating of this. Did you guys do-- so did you do molecular dynamics? Yeah. So you know about that. And you did that with velocity [INAUDIBLE], leapfrog. No? How did you integrate that? Did you pick an integration scheme? How did you solve that equation? You clicked Simulate.

But there are probably-- yeah, go ahead.

AUDIENCE: He showed us one scheme where we used, I think, the two subsequent positions sort of got an average velocity out of that, and--

JEFFREY GROSSMAN: Great.

AUDIENCE: Got it from there.

JEFFREY GROSSMAN: Yeah. And so they're-- right. So here's the thing. There are many ways to solve this equation. And some are more efficient to put on a computer than others. But none of them are really very complicated. We're going to have the same thing with quantum mechanics, it just gets a little more complicated, because the equation is more complicated.

But we're going to have the same thing. We'll have an equation for quantum mechanics, and there'll be many ways to solve it. And we're going to learn a little bit about them, and then just kind of move on and apply it. But we'll learn a little bit about the different ways to solve it.

So this wave is now-- you see, instead of being a particle, it's now an expectation. It's a vibration. So what we need to know about a wave-- oh, yeah, psi. Psi. When you see psi, that's like quantum mechanics, right? Or a fraternity? I don't know.

But anyway, here it's quantum mechanics. And psi is the function. That's it. That's what we want to know. That's the wave function. That's what we're solving for for the next so many weeks-- five weeks? Six weeks. We're going to be solving for this. This is the wave function. This is what tells us where those electrons are going to be and how they're going to behave.

And so the simplest way to write down a wave is what's called the plane wave, which is just a cosine-- it's a cosine plus sine, where you have an amplitude, and then you have this vector k at the spatial position, and then this frequency times the time. So this is the simplest way to write a wave, and you can find little nice animations on Wikipedia of what a wave like that would look like. That's a plane wave.

And you can see that it's called a plane wave because you can look at it as these moving planes of some sort of constant magnitude. It's a very simple way of describing this. Maybe there are more complex ways, but this is going to be one that we like in this class.

Now, what I was telling you about before is what de Broglie did, which he said, look, everything can be described as a wave. Everything can have a wavelength. And the wavelength associated with anything is going to look like this. Did I put it here? Yeah. It's going to go as h divided by the momentum-- the mass times the velocity.

So there is a wavelength that can be associated with anything. That's what de Broglie said. So he generalized what Einstein had been working on with light to say that this would be something that anything can have. It can have a description that can be described by a wave function. And that's what Einstein already did this. He did that part. He said the energy of that wave would be proportional to its frequency. That was part of the photoelectric effect.

OK, so how do we describe it? So in the last 10, 15 minutes, we'll say, this is it. This is where we've been building to, because we're going to end with what we need to do in this class, which is solve an equation. We're going to end just with a little bit of feeling of our F equals ma. And our F equals ma is the Schrodinger equation.

And I won't talk about how it was derived. It's actually-- it's a wonderful story. He was very frustrated, because this didn't actually agree with some of the work that was being done on relativistic effects solving this equation. And so he refused to publish, actually, his equation for quite a time. And he took a retreat to his cabin, and apparently came back and decided he would publish.

I love reading these stories. Those were the days. People really published when they really had something important to say. That's cool. And that was-- it was just really neat to watch the communication between Schrodinger and Einstein, and all these just brilliant people of the time.

This is our equation. This is a wave equation. This tells us what we need to know. This tells us how this function evolved in time and space. And the key here is that this side here has the terms-- this has the momentum squared term, which-- you can look at the previous page. When you look at de Broglie and Einstein, h bar omega is the energy from the frequency, and then the momentum, then you can go back and see why these things make a lot of sense.

But this is going to be-- this second derivative here, spatial derivative, is going to give you the momentum squared. So that looks kind of like a kinetic energy term. And that's the kinetic energy plus the potential energy times the wave function gives you something that's equal to the time dependence of that wave function. The time derivative of the wave function. That's the Schrodinger equation.

And for this class, we will consider that left-hand part that has the parts that give you the potential energy and the kinetic energy. That's basically how you can think of this, of the particle. But oh, that's the kinetic energy of the particle, but for a wave. And again, you can see why, because you can go back and see what this gives you, what k this gives you. So that's the kinetic energy, but it's meant for a wave.

And if we assume that this energy part doesn't depend on time, then the Schrodinger equation is much easier to solve, and we get something that's time-independent. And that's what we'll do in this class, because we can separate it into a spatial part and a time part, and then we can just ignore the time part, and we can call both sides equal to a constant. This is just some very simple algebra. And if we do that, we can set them both equal to some constant, let's call it E, which is-- there's a reason for that. And then this full psi with the time will depend on the spatial part times this exponential, and we'll just ignore this for this class. We're just going to care about this part.

OK. So let's end with-- there's these couple of examples that some of you have seen. How many of you have not seen particle in a box? Couple people. OK. But if you have seen it, you know this is still going to be fun, because particle in a box is always fun.

So all particle in a box is it's the spatial part of the Schrodinger equation solved for one of the simplest systems you can imagine, which is that you have some quantum mechanical particle that can exist inside of this region. And so in this region, there's no potential energy. It's just a free particle. It's a free wave.

But then it has these boundaries that go to infinity. And because it has a potential that's infinite at the boundaries there's only one solution here, which is that the function has to be zero. And you can convince yourself of that just by looking at this. If this is infinity, psi has to be zero. It's the only solution.

So here's what becomes really interesting about this. So you have-- if you look at this equation, and you say, well, OK, outside-- so you have two equations you can write-- one where there's a V, but it's infinite, in this case, and one where V is zero, in which case that part drops out, and this is the equation you're solving. And this has a general solution, which looks like this. It's a sine plus a cosine. And from this general solution, you can actually write down what E is. That's actually very fairly simple algebra to get to that.

But what becomes really interesting and exciting is when you apply the boundary conditions. So when you look at what has to happen here, you have constraints. And those constraints lead to something really important that was really, really frustrating all these people. What is it?

AUDIENCE: Quantization.

JEFFREY GROSSMAN: Leads to quantization. That's it. That's the ticket. It's when you put boundary conditions on the simplest quantum particle, quantum thing imaginable, a thing in a box, you quantize it. You quantize it because, you see, when I put these boundary conditions on, I'm saying that there are only certain allowable values for this, because of this.

So the boundary conditions cause quantization, and that's what gives rise to the whole thing. That's what gives rise to this whole world of energy levels, and a lot of the things that we talked about in the beginning that were so challenging to understand.

So you have quantization. You also have normalization. We're not going to go through this in great detail, but that also helps you understand what A is.

But you see there, that has to be. k can only be n pi over L where n is some integer. It can only be that because of those boundaries. And if you go back to the previous page, you can see why.

So now, your energy, which was nice and general as k, going as k squared, can only go as n squared h bar squared pi squared. So it can only go at certain values. It cannot be anything else.

And what you get, then, is you get energies. So you get energies that have very discrete values. And that starts to remind me of what? What experiment?

AUDIENCE: Photoelectric?

JEFFREY GROSSMAN: Photoelectric? OK. And what else, where you only saw discrete values?

AUDIENCE: The spectrum?

JEFFREY GROSSMAN: The spectra. The lines. You only saw discrete colors. You only see discrete colors. You guys can take a telescope and look, and you'll only see discrete colors.

Now, it also gives you something else. It gives you psi, which gives you these curves. So you see, for the first solution, n equals 1, you get a psi that looks like that. But for the next allowed value, you get a different shape for psi. And for the next allowed value, you get yet another shape for psi.

Oh, boy, is this important. Seriously. Check this out, OK? Because now-- OK, so that helps explain this. It looks like it might. And now we're going to do it for V equals something that looks more like a harmonic potential, and we're going to get equations that look like this. You see? We get energy levels-- the quantized energy levels that look like this, and we get psis that look like this. And these are psi squareds, which I'll tell you about in a minute.

And that starts to look a whole lot-- do I have it? That starts to look a whole lot like the hydrogen atom. You see, look at these psis. Look at the shapes of those. Those have structure, and they also have zeros. They have places where they go to zero.

Now, here's the critical thing. The shape of the psi squared is the answer. It's where the electron is going to be. It is the probability of finding the electron anywhere in space. Psi squared is the probability of finding a quantum mechanical particle anywhere in space. It is the wave function, and it is also where the particle will be with a given probability.

So the probability of finding this particle in a harmonic well is going to be largest here, and then almost zero there. And then it's going to go up again, further out. And you have these different solutions that only happen for certain energies. That is exactly what's observed. So the psi squared is the probability you find a particle. Psi, we don't know what it is. We don't know what psi is. Does anybody know what psi is?

AUDIENCE: [INAUDIBLE]

JEFFREY GROSSMAN: No, that's psi squared. Psi is what we solve for in the Schrodinger equation, and that's we're going to solve for in this class. Psi squared gives us the probability of finding that particle. What is psi? What does it mean? Any guesses? The square of it is the probability that you can find a particle anywhere. But what is psi itself?

People have struggled with this a lot. And you know what I think? I think people have given up. Because when you read old textbooks, old quantum textbooks-- I'm talking from the '30s and '40s, they talk about that a whole lot. And when you read new quantum textbooks, they don't talk about it anymore.

Because we just don't really know, is the answer, what is the physical meaning of psi. Isn't that so cool? We don't really have a good grip on it. There's a lot of different discussions around what it could be. But this is the thing that means something we can measure directly. It's the distribution of where that particle is.

You see, so this is it. This is an electron around-- this is the electron of hydrogen. And you see, when that electron is in-- let's take a look at this. This is really important. When that electron is in this shell, we already know that's called the 1s. The psi for that first allowed energy level, that first allowed energy level has a shape to it, and it's spherical, and it looks like that. And therefore, that is the probability distribution of the electron. That's where it can be in space.

If I measure where that electron is, where's it going to be? Yeah.

AUDIENCE: I mean, if you measure it, you're going to get back some position for it, and it's going to be at that position.

JEFFREY GROSSMAN: And I love that. If I measure it, it will be where I measure it. Which is true. And that's actually the right answer. But where will it be according to that picture? How can I use this picture to tell me where it's going to be? That is psi squared for the electron of hydrogen in the first level. Yeah.

AUDIENCE: You guess as to where it will be within that boundary.

JEFFREY GROSSMAN: Yeah. I mean, basically, the answer is that it tells me where it is more or less likely to be, but it doesn't tell me where it will be. It can't tell me that. Psi squared only tells me a probability. It only tells me-- it can tell me where it can never be. That's what these are. Those are those zeros. When psi squared is zero, that electron can never be there.

But it also can tell me with more likelihood or less where I might find it. So if I measure it 10 times, well, most of the time, I'll find it in here. Maybe once I find it kind of out here. That's the nature of quantum mechanics. That's the relationship between the psi that we're going to solve for and the real deal.

Now, what you find, then, is see, the 1s is here, and then the 2s is here. So these are those allowable energy levels for the atom, which we'll start with that next time. But-- wait, don't go, we're not done yet.

So now, I've said that the system is quantized, which means it can only have certain energies. That's what the boundary conditions do. How does that solve my classical atom problem? The atom only living for 10 to the minus 12th seconds. How does it solve that?

So basically-- yeah.

AUDIENCE: It can't radiate away just a little bit of energy. It can't fall in just a little bit. If it's going to radiate energy, it has to fall down an entire level.

JEFFREY GROSSMAN: Exactly. Yeah. That's exactly right. These electrons-- it solves the problem of the electron collapsing into the core because it says the electron no longer can collapse into the core, basically. We've quantized it, so it just can't do that.

Because you see, here's the 2s. And then the 2p-- you see, there's s, and there's 2s, and then the 2p has some shape to it. I'm just going to put wiggles. It's not just a sphere anymore. It's got shape. When you go up higher in energy, that psi squared has funny shapes.

But the point is that the probability distribution is maybe around here, and it goes pretty much to zero. It goes very close to zero in between, pretty much zero, where the chance is like the age of the universe it won't happen.

And so what that says is that that electron is just not allowed to be here. It cannot be there physically. It cannot be in that part of space. How cool is that? I mean, how weird is that, right? The electrons simply cannot exist in between the 2s and the 1s. They got to be at those energies. You cannot give it an energy in between. It won't take it, because the allowed levels are quantized. There is no in-between.

[? Buber ?] was wrong. How many of you know-- never mind. Nobody reads Buber. Philosopher. It's all in the in-between. Never mind.

Point being that that's really mind-blowing, I think. The electron cannot-- now here's the question. Well, how does it get from 2s to 1s? How does it get there if it's not allowed to go in between?

OK. These are the interpretations-- we'll pick up with that on Thursday. These are the interpretations. Look at that. It's like a whole world. It's incredible, all the discussion on how to interpret psi and the different things.

And this is a great one. They don't know they're doing quantum mechanics, but that is a double slit kind of statement. And finally, this is what we talked about, and there's some great books. There's also many, many places on the web where you can read more about this. And on Thursday, we're going to start going more and more into how we can solve this for real materials, which is where we're going to be in this part of the class.