

Recitation 14

Outline:

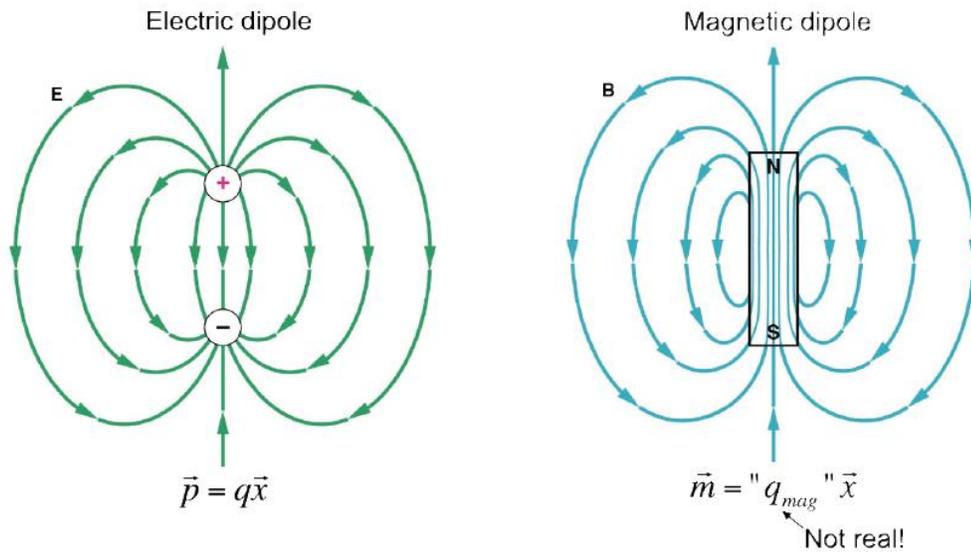
Origins of Magnetism in Materials

- a) Magnetic Dipoles
- b) Paramagnetism
- c) Ferromagnetism, Anti-Ferromagnetism, & Ferrimagnetism
- d) Hysteresis

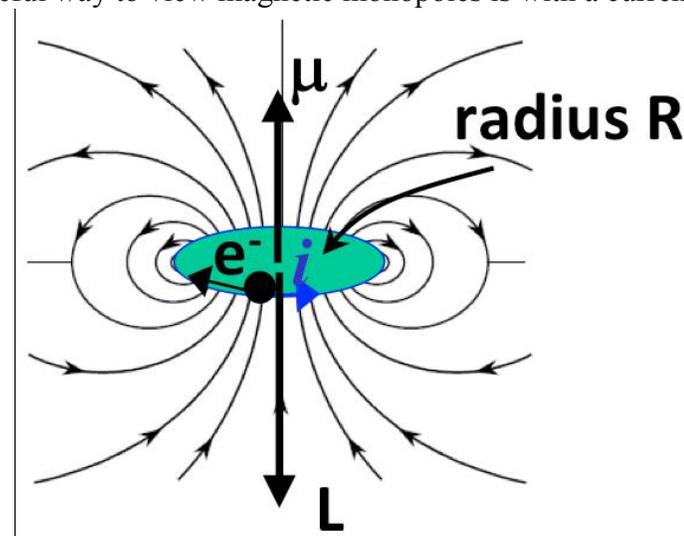
a) Magnetic Dipoles

For comparison, recall for electric fields:	For magnetic fields:
$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$	$\vec{B} = \mu_0 \vec{H} + \vec{M}$
$\vec{P} = \epsilon_0 \chi \vec{E}$	$\vec{M} = \mu_0 \chi_M \vec{H}$
$\vec{P} = N \vec{p} = Nq\vec{x}$	$\vec{M} = N\vec{\mu} = Nq_{mag}\vec{x} ?$

But there are no magnetic monopoles!



A useful way to view magnetic monopoles is with a current loop.



$$\vec{\mu} = i\vec{A} = -\frac{qv}{2\pi R} \pi R^2 \hat{n} = -\frac{qvR}{2} \hat{n} = -\frac{q}{2m} \vec{L} = -\gamma \vec{L}$$

Current loop with area vector  $\vec{A}$  and direction  $\hat{n}$ .

To get the full picture, we must use quantum mechanics, which means using operators.

$$\vec{L} \rightarrow \hat{L}$$

Angular Momentum

$$\hat{L}^2 Y_l^m(\theta, \phi) = \hbar^2 l(l+1) Y_l^m(\theta, \phi)$$

$$\mu_e = |L|\gamma = \gamma \hbar \sqrt{l(l+1)}$$

Spin Angular Momentum

$$\hat{S}^2 \chi = \hbar^2 s(s+1) \chi$$

$$\mu_s = |S|\gamma = \gamma \hbar \sqrt{s(s+1)}$$

Bohr Magneton

$$\mu_B = \gamma \hbar$$

Total Angular Momentum

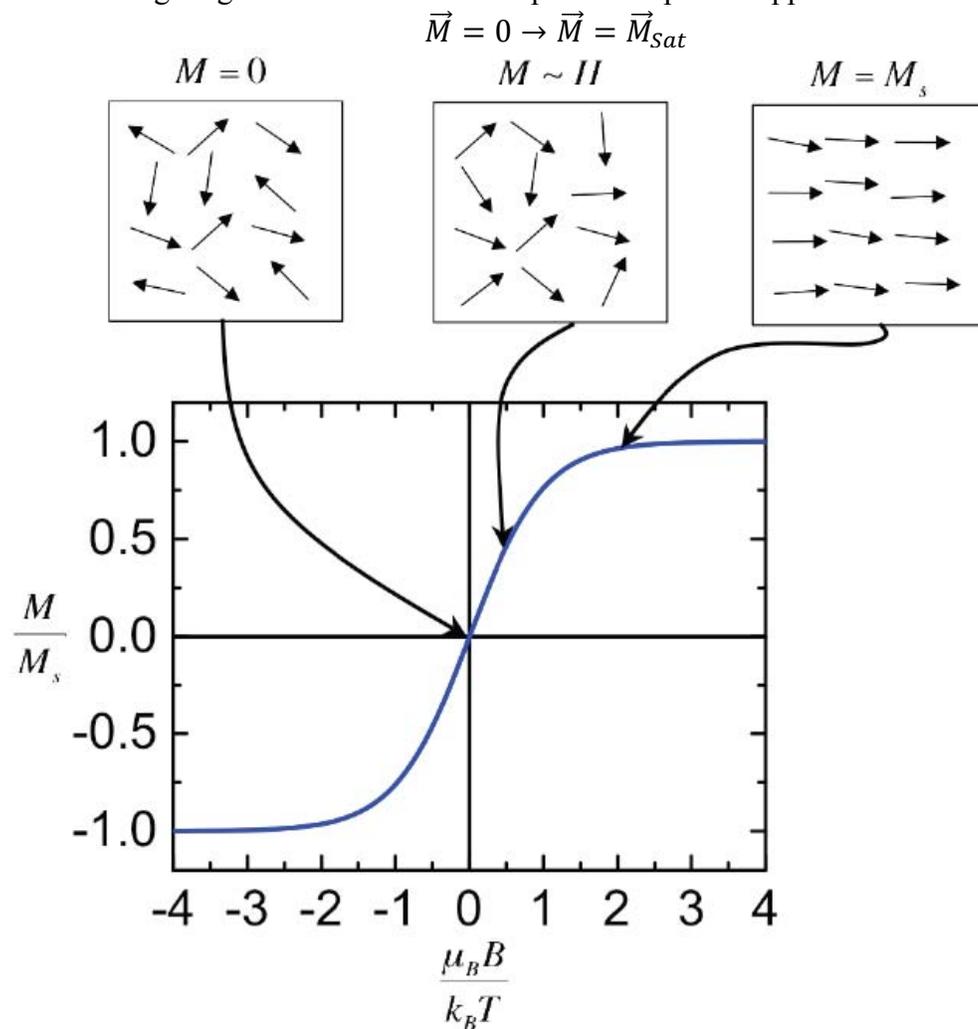
$$\vec{J} = \vec{S} + \vec{L}$$

$$\hat{J}^2 \Psi = \hbar^2 j(j+1) \Psi$$

$$\mu = \mu_B \sqrt{j(j+1)}$$

b) Paramagnetism

Applying an external field  $\vec{H}$  to non-interacting dipoles requires  $\sim 100$  T to align dipoles. This is known as paramagnetism and quantified by  $\mu_r > 1$  (note diamagnetism is the opposite of this where strong magnetic fields cause the dipoles to repel the applied field with  $\mu_r < 1$ ).



c) Ferromagnetism, Anti-Ferromagnetism, & Ferrimagnetism

In certain materials, exchange interactions between electrons cause the electrons' spin to spontaneously align. The manner in which the electrons align is based on the crystal structure of the material and governed by the magnetic exchange Hamiltonian and any externally applied induction field.

$$\hat{H}_{magnetic} = \hat{H}_{exchange} + \hat{H}_{external\_field}$$

For a system of two atoms, this is simply the following:

$$\hat{H}_{magnetic} = -J_{12}\hat{S}_1 \cdot \hat{S}_2 + \frac{\mu_B}{\hbar}\vec{B} \cdot \hat{S}_1 + \frac{\mu_B}{\hbar}\vec{B} \cdot \hat{S}_2$$

For a large crystal system of  $N$  atoms, this becomes the following:

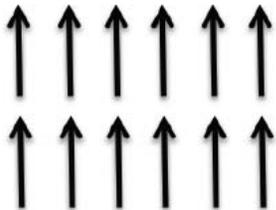
$$\hat{H}_{magnetic} = - \sum_{i,j} J_{ij}\hat{S}_i \cdot \hat{S}_j + \frac{\mu_B}{\hbar} \sum_i \vec{B} \cdot \hat{S}_i = \sum_i \left( - \sum_j J_{ij}\hat{S}_i + \frac{\mu_B}{\hbar}\vec{B} \right) \cdot \hat{S}_i$$

Here the  $J_{ij}$  term is the exchange integral that defines the effective overlap interaction of the electrons.

By defining the exchange term as follows:

$$- \sum_j J_{ij}\hat{S}_i = \frac{\mu_B}{\hbar}\vec{B}_{ex}$$

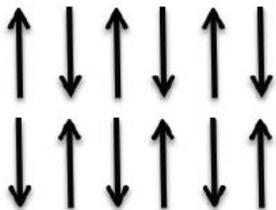
The problem of spontaneous magnetic ordering reduces to a mean field problem where the field  $\vec{B}_{ex}$  is the effective field applied to the system inducing magnetic ordering via the exchange interaction. Depending on the size and sign of this exchange field term and the crystal structure of the material, the spontaneous magnetic dipole alignment will result in 1 of 3 permanent magnetic material types assuming the material is below a critical temperature that corresponds to a thermal energy greater than the exchange energy.



**Ferromagnetic:**

$J_{ex} > 0 \Rightarrow$  exchange energy is minimized when  $\hat{S}_i \uparrow \uparrow \hat{S}_j$

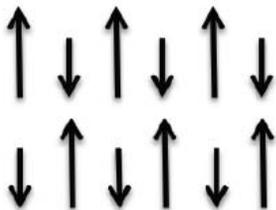
Large spontaneous magnetization at  $T < T_c$



**Anti-ferromagnetic:**

$J_{ex} < 0 \Rightarrow$  exchange energy is minimized when  $\hat{S}_i \uparrow \downarrow \hat{S}_j$

No net magnetization, but ordering at  $T < T_N$



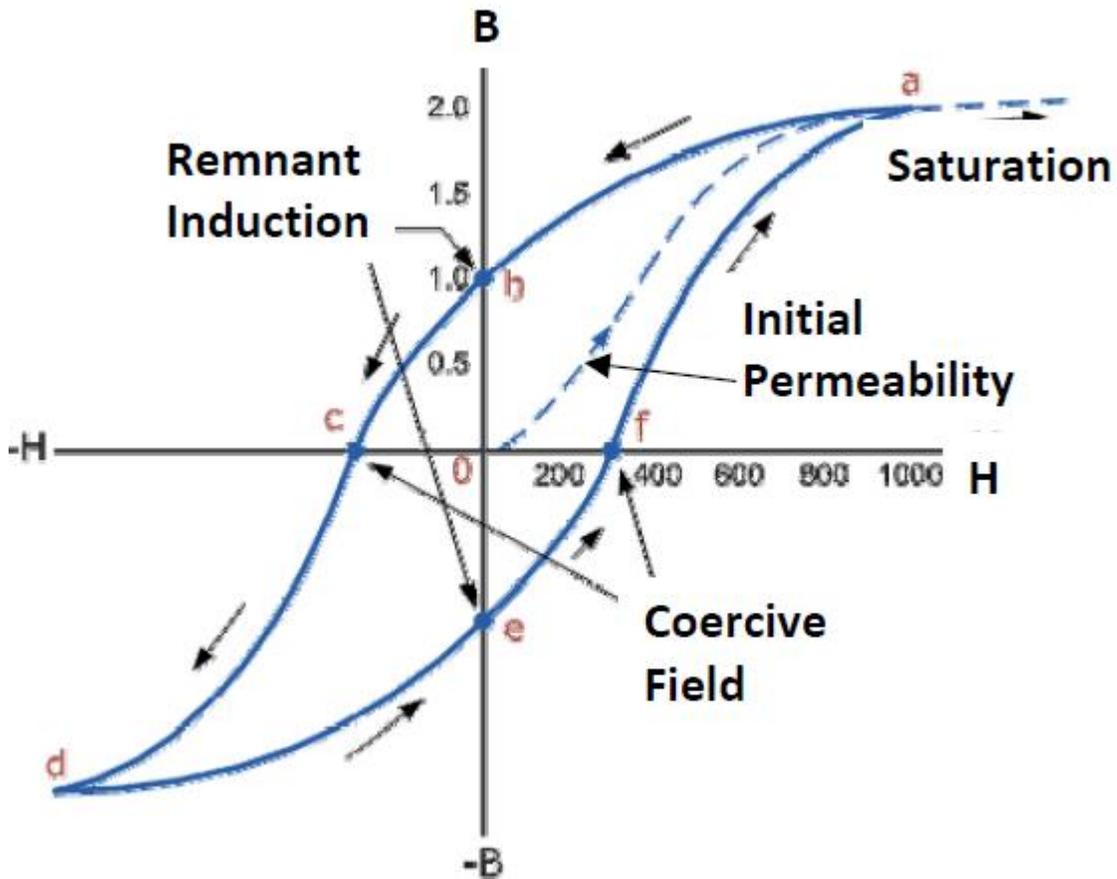
**Ferrimagnetic:**

$J_{ex} < 0 \Rightarrow$  exchange energy is minimized when  $\hat{S}_i \uparrow \downarrow \hat{S}_j$

Reduced net magnetization (as compared to ferromagnetic materials), spontaneous ordering at  $T < T_N$

d) Hysteresis

For materials with exchange interactions, the application of a large external magnetic field can realign the magnetic dipoles. This can be the realignment of local domains aligned in different directions or just realigning the dipoles due to thermal history randomizing the dipole directions. The characterization of this effect is made through a hysteresis curve. A general hysteresis curve is shown below with various points of interest labeled. Materials with low coercivity are known as soft magnets and are useful when demagnetization switching with low fields is required and materials with high coercivity are known as hard magnets are useful for situations where demagnetization is undesired.



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