3.032 Quiz 1
Fall 2006

DO NOT TURN THIS PAGE OVER AND START THE QUIZ UNTIL YOU ARE ASKED TO DO SO.

Guidelines:

1. Show all your work on the sheets included in this stapled document.
2. Use partial credit to your advantage. If you're running short on time, solve algebraically and then solve numerically (plugging in numbers) later.
3. If there is not much space given for you to provide an answer, we want you to be brief.
4. You may not need to use all the information given (e.g., dimensions) to reach your conclusions.
5. Enjoy your dark chocolate before getting started! It lowers blood pressure (*JAMA*, 2003), contains antioxidants (*Nature*, 2003), and supposedly helps neural synapses responsible for memory to fire faster (*Nature Health*, 2003).

NAME (PRINTED):

I agree that this document represents my own independent work on this quiz, using only my own brain, my allowed crib sheet of equations and notes, and my pen / pencil / calculator / protractor / compass / ruler / sliderule.

SIGNATURE:

GOOD LUCK!

1. Harvard University is currently using several large cranes to complete construction along the Charles River. Below is a schematic of one such crane, with dimensions and the position of a counterweight indicated.

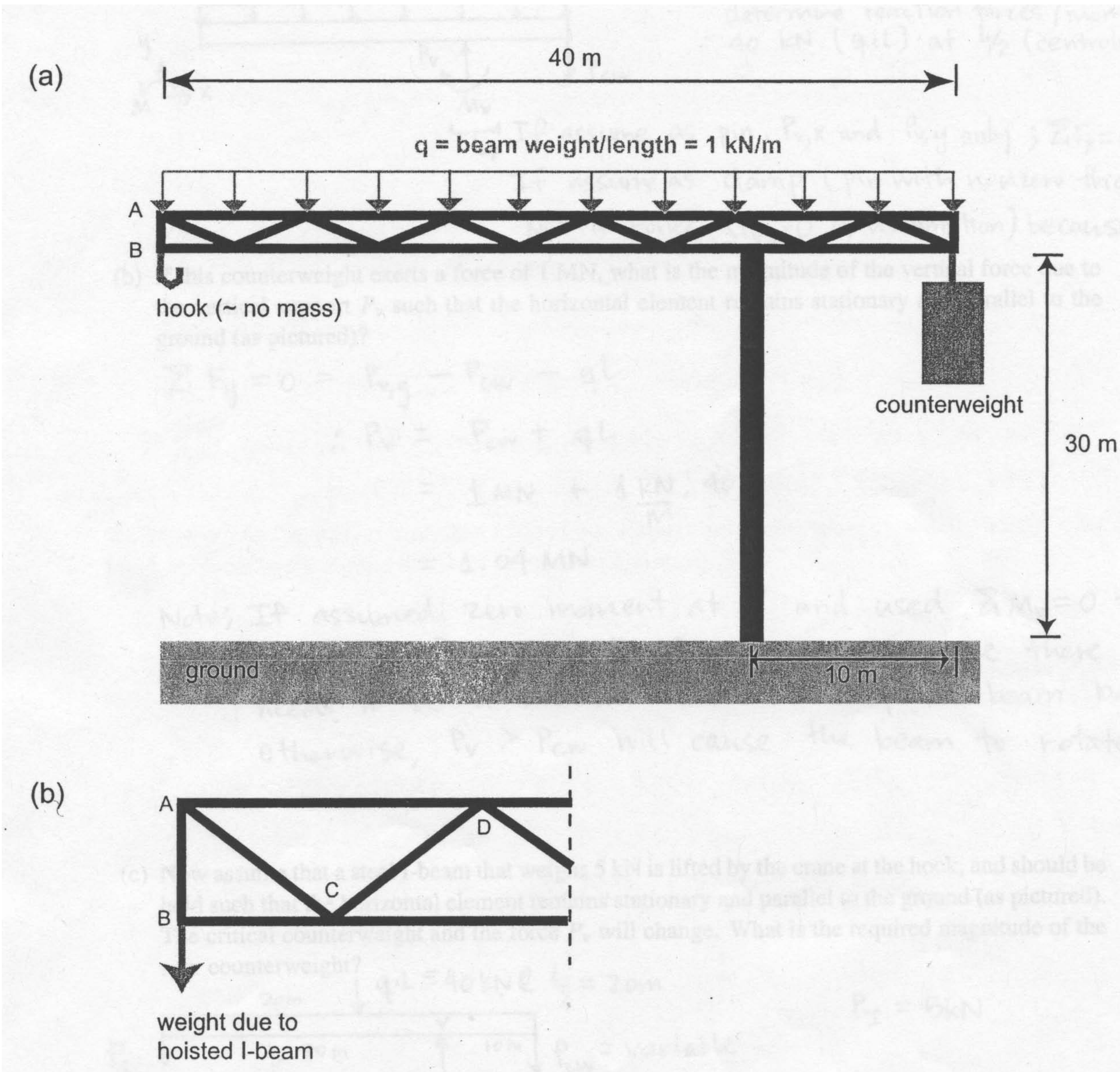

Figure 1: Schematic of crane used to complete construction along the Charles, indicated with the horizontal element of the crane (a trussed beam) parallel to the ground. (a) Entire crane; (b) Hook-end of the horizontal element.

(a) Draw a free body diagram of the horizontal element of the crane (the horizontal span in Fig. 1a), treating it as a beam and indicating all known and unknown applied / reaction forces. You will need to make assumptions about how the crane is connected to the real world, and these should be *briefly* justified.

(b) If this counterweight exerts a force of 1 MN, what is the magnitude of the vertical force due to the vertical support P_v such that the horizontal element remains stationary and parallel to the ground (as pictured)?

(c) Now assume that a steel I-beam that weighs 5 kN is lifted by the crane at the hook, and should be held such that the horizontal element remains stationary and parallel to the ground (as pictured). The critical counterweight and the force P_v will change. What is the required magnitude of the new counterweight?

(d) Consider the hook-end of the crane's horizontal element (Fig. 1b), now loaded by the weight of the steel I-beam. Can member AC be made from a thin steel cable (essentially an elastic rope)? Here, you're considering the elements of the truss explicitly, so their weight as a whole beam structure (i.e., q) can be neglected. Explain your answer fully.

---

2. We know σ_{ij} and ϵ_{ij} can be expressed as quadrics. Explain concisely, but fully and accurately, what this fact immediately implies about tensorial stress and strain.

---

3. Tractor-trailers are used to transport liquid propane in cylindrical tankers made of stainless steel, precisely because the trailer must act as a pressure vessel as the contents change temperature (and thus internal pressure levels) over the course of the trip, and so that accidental combustion could be relatively safely contained.

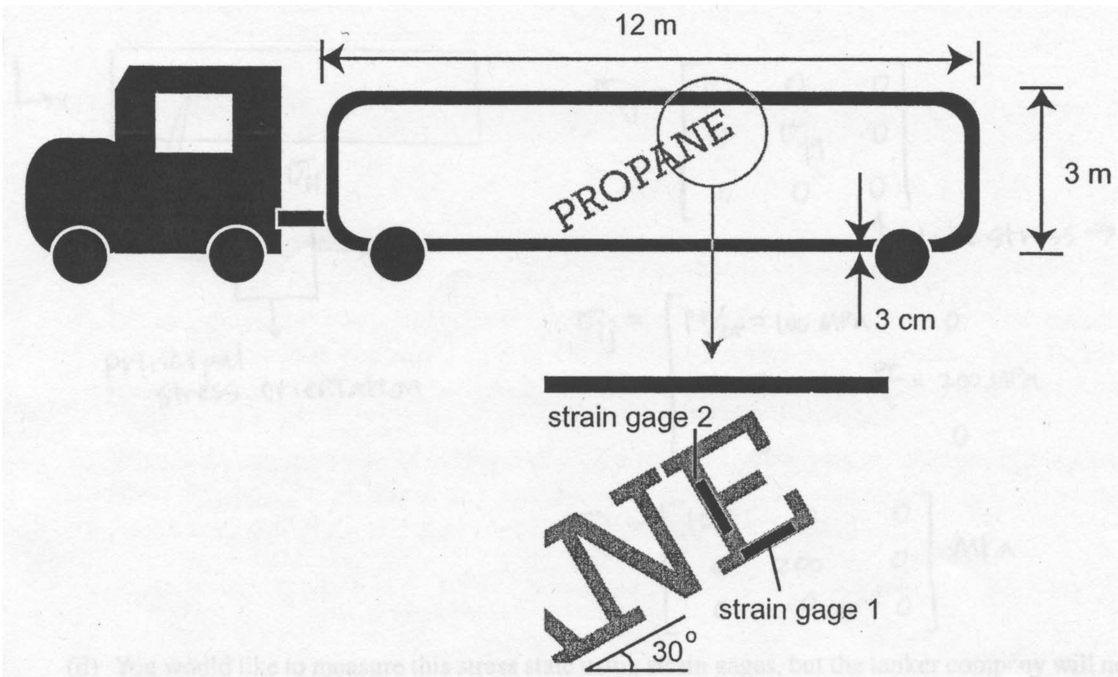

Figure 2: Tractor-trailer transported tanker of propane.

(a) Draw a two-dimensional (side view) free body diagram of the tanker (the part pulled by the cab where the driver sits), indicating supports and corresponding reaction forces as well as applied forces.

(b) Idealizing the tanker as a thin-walled pressure vessel, how much internal pressure P_i can the tanker handle without catastrophic failure? Stainless steel fails at a normal stress (only) of 200 MPa, and at a shear stress (only) of 100 MPa. You can neglect the pressure differential with the atmosphere, such that $P_i = \Delta P_i$.

(c) For this ultimate loading state (i.e., using the value of P_i obtained above), express the tanker wall stress state numerically in tensor/matrix notation: σ_{ij} = [matrix of all zero and nonzero stresses]. Clearly indicate your assumed orthogonal axes with respect to the dimensions of the tanker.

(d) You would like to measure this stress state using strain gages, but the tanker company will not let you place the strain gages anywhere except on the letters of "PROPANE". As shown, these gages are the width of the painted letters, so you're forced to align them with the letter orientation. What is the stress tensor σ_{ij} that you expect to measure at the position of strain gage 1 when the tanker is pressurized to its maximum value P_i? Give the full matrix, and solve this using any of the three approaches discussed in class.

(e) Briefly (but accurately) *describe* how you would use the other two methods you could have chosen to transform this stress tensor.

(f) Let us assume that you could align the strain gage along the longitudinal axis of the tanker, and that this strain gage is a single strip of Cu of initial length $L_o = 1$ cm with a gage factor of 1 (Δresistance = ϵ). The strain in that Cu strip will be equal to the longitudinal strain ϵ of the steel, which is $\epsilon = 0.4\sigma_{longitudinal}/E$.
What is the expected change in length of the Cu strip that you expect to occur under this principal stress $\sigma_{longitudinal} = \sigma_2$ due to this pressure P_i? You can assume engineering strain, and that the elastic moduli E of Cu and steel are 110 GPa and 210 GPa, respectively.

END OF QUIZ 1

USE THIS PAGE FOR EXTRA PROBLEM SOLVING AS REQUIRED.

USE THIS PAGE FOR EXTRA PROBLEM SOLVING AS REQUIRED.