3.044 MATERIALS PROCESSING

LECTURE 13

T	σ, P	c
heat transfer	solid mech., fluid flow	diffusion, phase trans.

Mission for the next part of the course:

Focus on fluid flow, How does fluid flow relate to solid mechanics?
How do we connect all three categories?

Fluid Flow Terminology

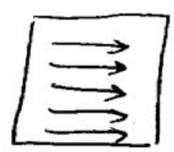
- **free flow** - against low resistance medium (air)
- **confined flow** - against a solid boundary

} VS.

- **incompressible flow** - e.g. water, most melted/liquid matter
- **compressible flow** - e.g. gases, polymer melts (mildly)

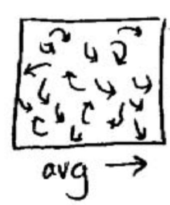
} Does pressure change density?

- **laminar flow** - very uniform motion, low velocity



} VS.

- **turbulent flow** - chaotic motion, high velocity

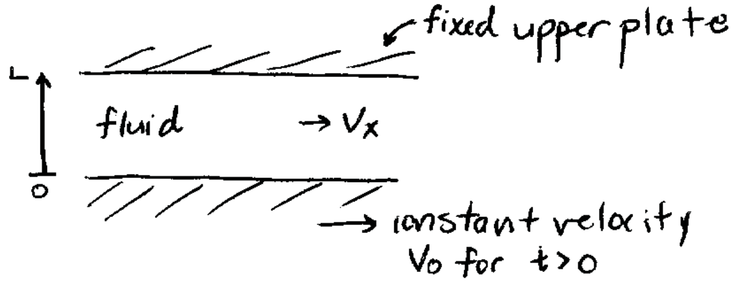


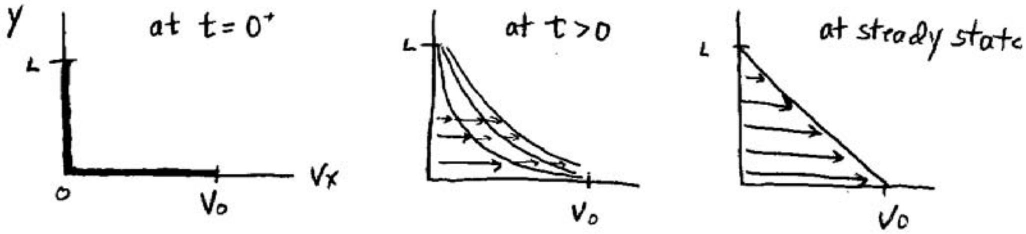
Fluid Flow is the diffusion of momentum

Date: April 2nd, 2012.

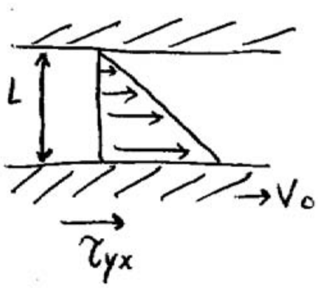
1-D Flow:

Two flat plates:



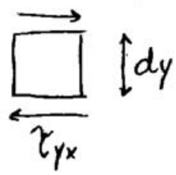


In steady-state → constant v-profile:



Must be applying a constant Force:

$$\overbrace{\underbrace{\frac{F}{A}}_{\frac{\mathrm{N}}{\mathrm{m}^2}=\mathrm{Pa}}}^{\text{driving force}} = \overbrace{\underbrace{\mu\left(\frac{V_0}{L}\right)}_{\frac{\mathrm{N\cdot s}}{\mathrm{m}^2}\left(\frac{1}{\mathrm{s}}\right)=\mathrm{Pa\cdot s}}}^{\text{resulting velocity profile}}$$

Differentially small element: 

Newton's Law of Viscosity: $\boxed{\tau_{yx} = -\mu \frac{\partial V_x}{\partial y}}$

→ only applies to simple fluids, **Newtonian** fluids
→ We'll return to non-Newtonian fluids later

Compare:

$\boxed{1.}$ $\tau_{yx} = -\mu \frac{\partial V_x}{\partial y}$ $\boxed{2.}$ $q = -k \frac{\partial T}{\partial x}$ $\boxed{3.}$ $j = -D \frac{\partial c}{\partial x}$

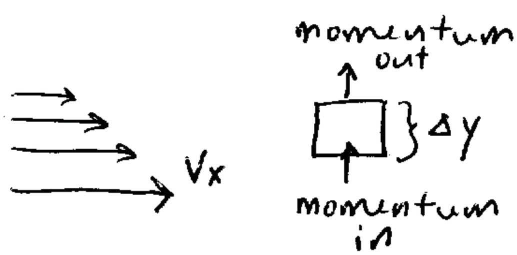
What's Different?

$$\underbrace{\bar{q}}_{\text{vector}} = -k \underbrace{\nabla T}_{\text{grad scalar}} \quad VS. \quad \underbrace{\bar{\bar{\sigma}}}_{\text{2nd rank tensor}} = -\mu \nabla \underbrace{\bar{v}}_{\text{grad vector}}$$

<u>Stress</u> is equivalent to a <u>Flux of Momentum:</u>

Stress	**Momentum Flux**
$\frac{F}{A}$	$m \cdot v$
Units: $\frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$	Units: $\frac{\text{kg}\,\frac{\text{m}}{\text{s}}}{\text{m}^2 \cdot \text{s}} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$

Momentum Balance (1-D):



$$\overbrace{\underbrace{A\;\tau_{yx}\big|_{y}}_{\frac{\mathrm{N}}{\mathrm{m}^2}\cdot\mathrm{m}^2}}^{\text{momentum in}} - \overbrace{\underbrace{A\;\tau_{yx}\big|_{y+\delta y}}_{\frac{\mathrm{N}}{\mathrm{m}^2}\cdot\mathrm{m}^2}}^{\text{momentum out}} + \overbrace{\underbrace{F_x\cdot\;V}_{\frac{\mathrm{N}}{\mathrm{m}^3}\cdot\mathrm{m}^3}}^{\text{momentum generated}} = \overbrace{\underbrace{V\frac{\partial\left(\rho v_x\right)}{\partial t}}_{\mathrm{m}^3\cdot\frac{\mathrm{kg}}{\mathrm{m}^3}\cdot\frac{\mathrm{m}}{\mathrm{s}^2}}}^{\text{momentum accumulated}}$$

Divide through by $V = A\delta y$

$$\frac{\tau_{yx}\big|_{y} - \tau_{yx}\big|_{y+\delta y}}{\delta y} + F_x = \frac{\partial\left(\rho v_x\right)}{\partial t}$$

Plug in Newton's Law of Viscosity

$$\frac{\mu}{\Delta y}\left(\frac{\partial v_x}{\partial y}\bigg|_{y+\Delta y} - \frac{\partial v_x}{\partial y}\bigg|_{y}\right) + F_x = \frac{\partial\left(\rho v_x\right)}{\partial t}$$

$$\frac{\mu}{\Delta y}\,\Delta\frac{\partial v_x}{\partial y} + F_x = \frac{\partial\left(\rho v_x\right)}{\partial t}$$

1-D Fluid Flow Equation

$$\boxed{\mu\frac{\partial^2 v_x}{\partial y^2} + F_x = \frac{\partial\left(\rho v_x\right)}{\partial t}}$$

Assume ρ is constant

$$\mu\frac{\partial^2 v_x}{\partial y^2} + F_x = \rho\frac{\partial v_x}{\partial t}$$

$$\frac{\partial v_x}{\partial t} = \left(\frac{\mu}{\rho}\right)\frac{\partial^2 v_x}{\partial y^2} + \frac{F_x}{\rho}$$

Compare:

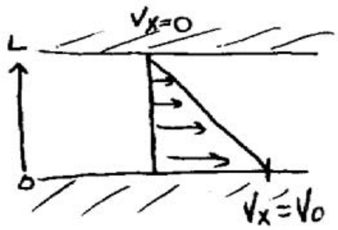
1. $$\frac{\partial v_x}{\partial t} = \underbrace{\left(\frac{\mu}{\rho}\right)}_{\text{diffusivity of momentum}}\frac{\partial^2 v_x}{\partial y^2} + \frac{F_x}{\rho}$$
2. $$\frac{\partial c}{\partial t} = \underbrace{D}_{\text{diffusivity}}\frac{\partial^2 c}{\partial x^2}$$
3. $$\frac{\partial T}{\partial t} = \underbrace{\alpha}_{\text{thermal diffusivity}}\frac{\partial^2 T}{\partial x^2}$$

Diffusivity of Momentum is also known as **Kinematic Viscosity**

$$\boxed{\nu = \frac{\mu}{\rho}}$$

$$\textbf{Units:} \qquad \frac{\mathrm{Pa}\cdot\mathrm{s}}{\frac{\mathrm{kg}}{\mathrm{m}^3}} = \frac{\mathrm{N}\cdot\mathrm{s}\cdot\mathrm{m}^3}{\mathrm{m}^2\cdot\mathrm{kg}} = \frac{\mathrm{m}^2}{\mathrm{s}}$$

Momentum diffuses through a flowing fluid



At steady state:

$$\frac{\partial v_x}{\partial t} = \frac{\mu}{\rho}\frac{\partial^2 v_x}{\partial y^2} = 0$$

$$\frac{\partial^2 v_x}{\partial y^2} = 0$$

Boundary Conditions:

$$@\ y = 0, \quad v = v_0$$

$$@\ y = L, \quad v = 0$$

Solve:

$$v_x = A_y + B$$

$$\boxed{v_x = v_0 - \frac{v_0}{L}\,y}$$

In order to solve simply:

⇒ Use diffusion or conduction solutions and compare $\boldsymbol{v_x}$ to $\boldsymbol{c}$ or $\boldsymbol{T}$