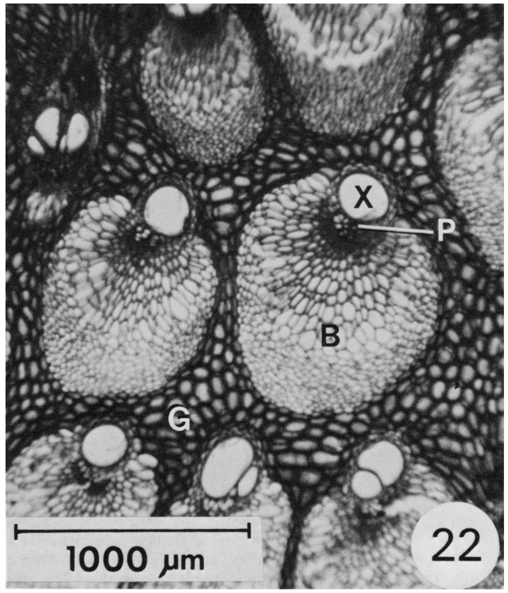
Plant Stems with Radial Density Gradients

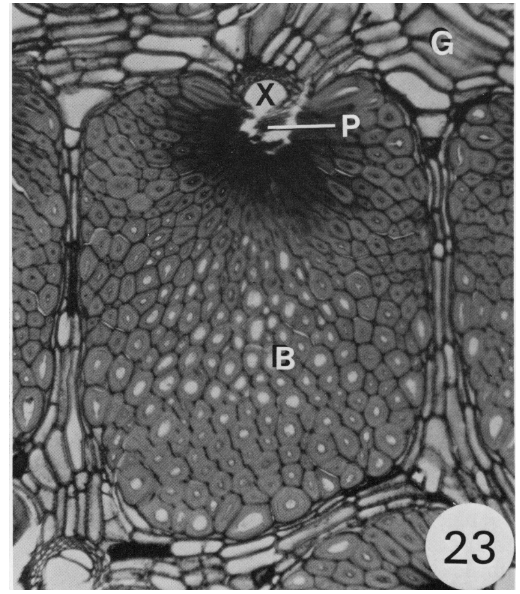
Coconut Palm
http://en.wikipedia.org/wiki/Image:Palmtree_Curacao.jpg

Palm: Density Gradient

Vascular bundles: Honeycomb

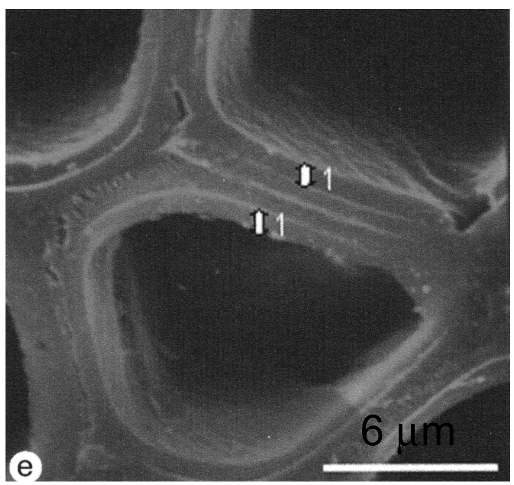
Ground tissue (Parenchyma): Foam

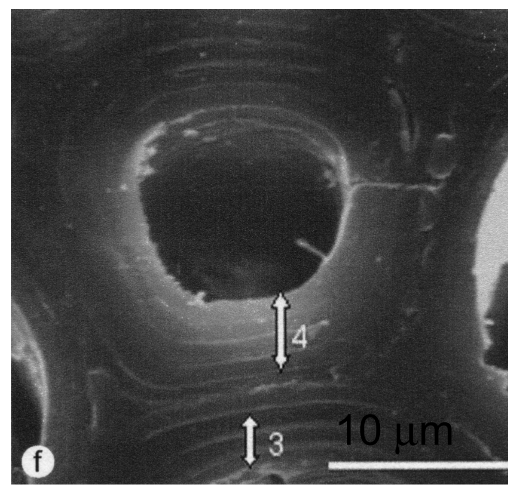




Peripheral Stem Tissue

Rich, 1987



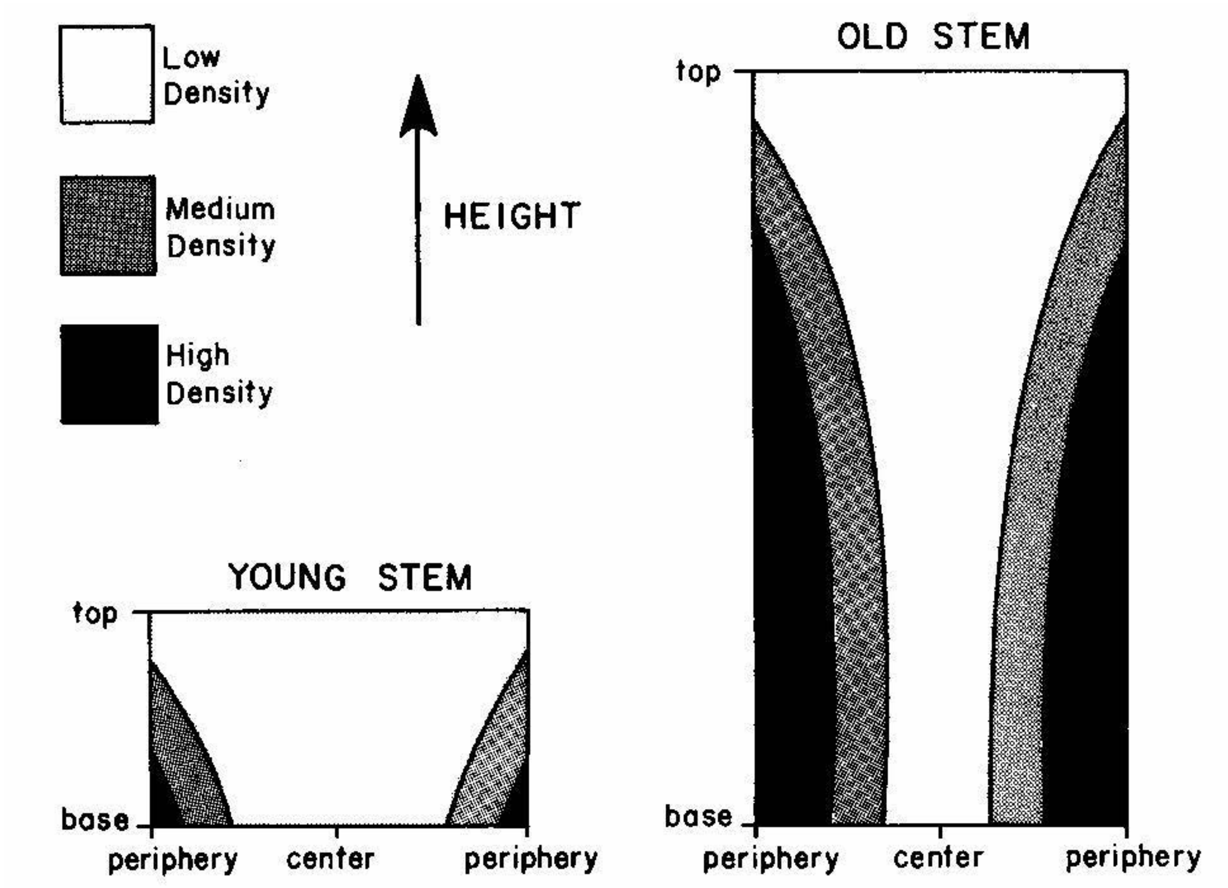


Kuo-Huang et al., 2004

Young

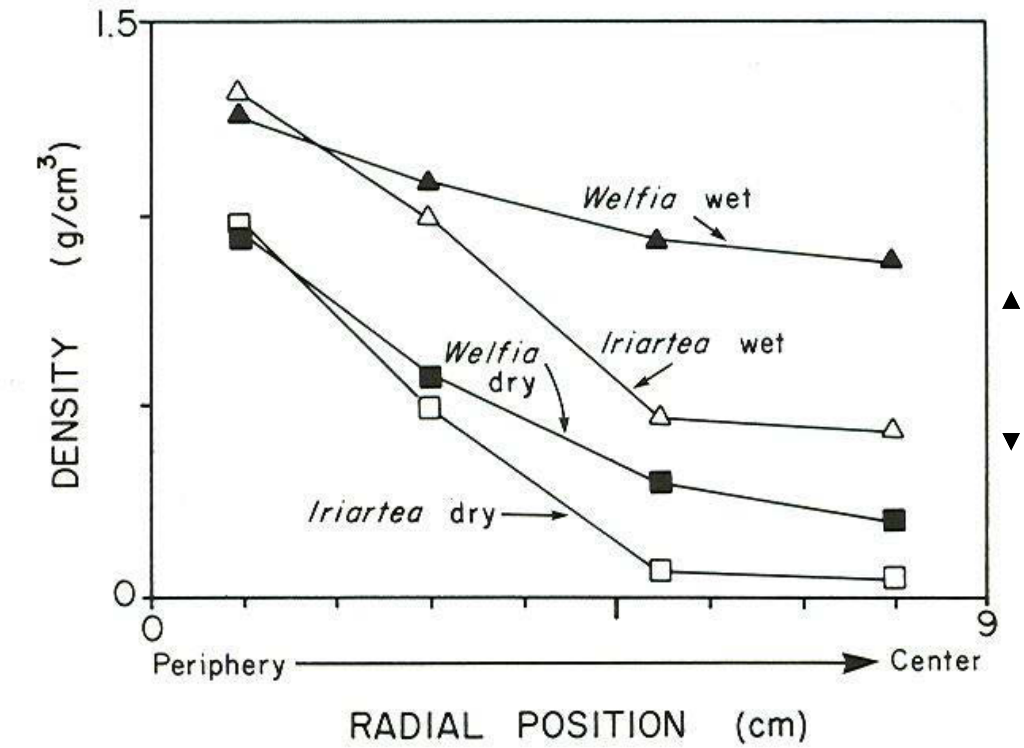
Old

Palm Stem: Density Gradient



Rich, PM (1987) Bot.Gazette 148, 42-50.

Palm Stem: Density at Breast Height



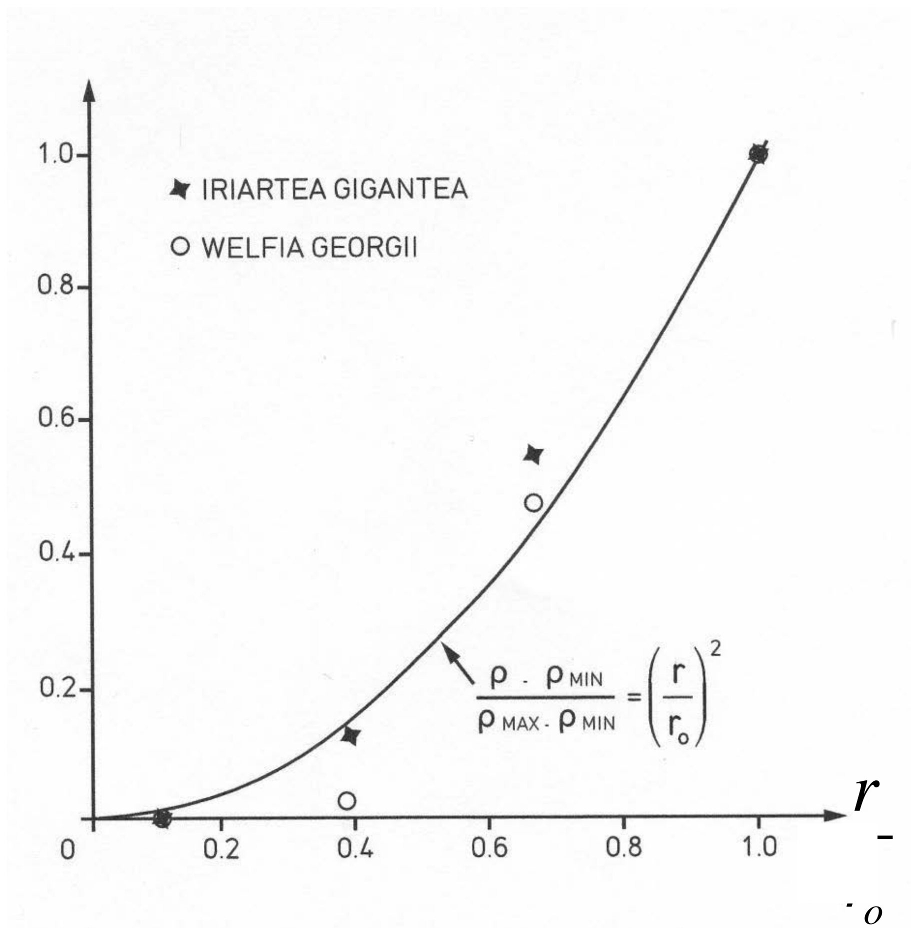
Densities of common woods

A single mature palm has a similar range of density as nearly all species of wood combined

Rich, PM (1987) Bot.Gazette 148, 42-50.

Palm Stem: Density Gradient

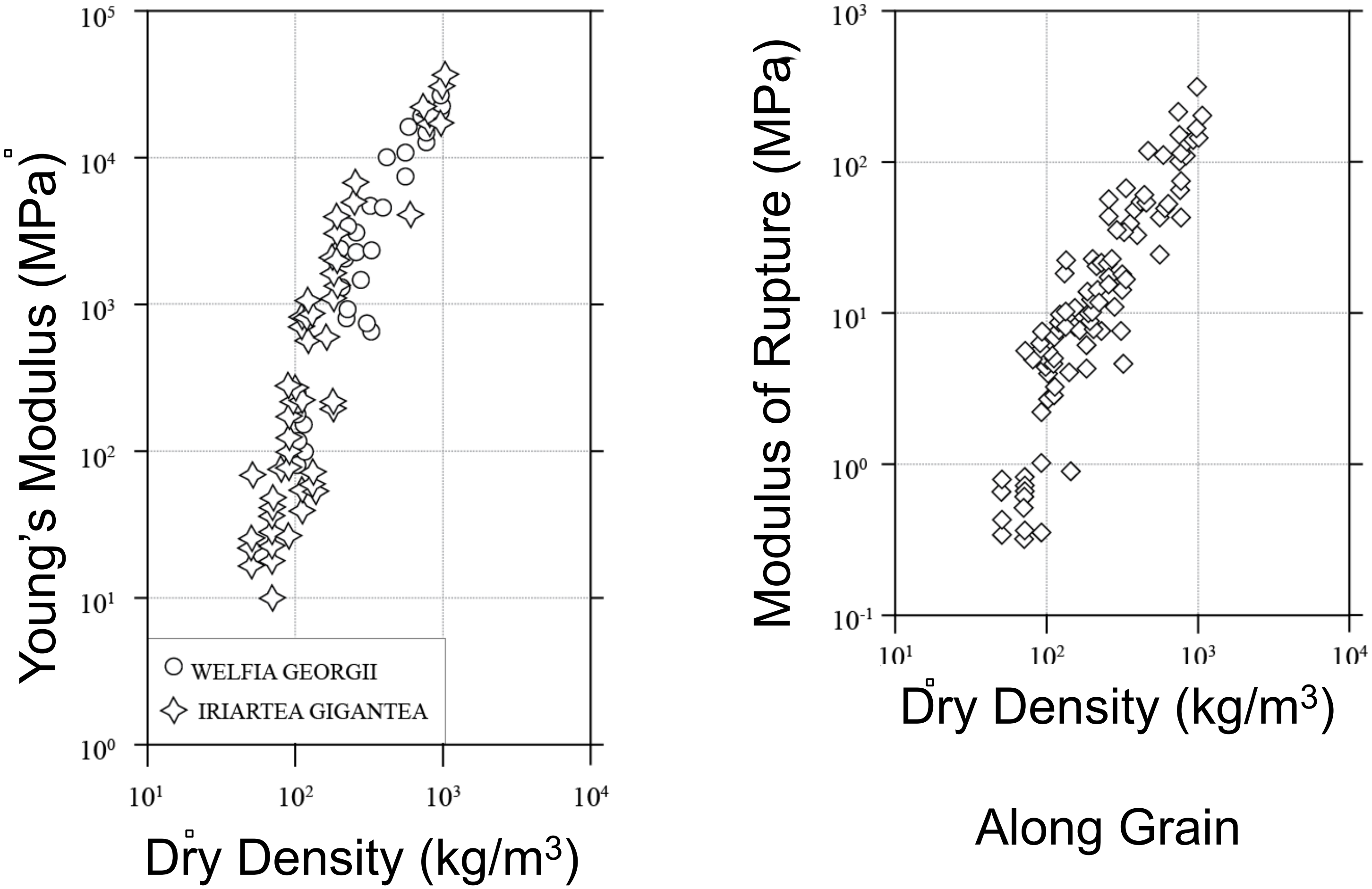
$$\frac{\rho - \rho_{\min}}{\rho_{\max} - \rho_{\min}}$$



Iriartea gigantea:
$\rho_{\min} \approx 0$

$$\frac{\rho^*}{\rho_{\max}} = \left(\frac{r}{r_o}\right)^2$$

r_o is the outer radius



Along Grain

Iriartea gigantea

$$E^* = C\left(\frac{\rho^*}{\rho_{\max}}\right)^{2.5}$$

$$\sigma^* = C\left(\frac{\rho^*}{\rho_{\max}}\right)^{2}$$

Palm Properties

- Prismatic cells in palm deform axially (like wood loaded along the grain)

- If E_s was constant, would expect: $E^* = E_s\left(\rho^*/\rho_s\right)$

- But measure: $E^* = C\left(\rho^*/\rho_{\max}\right)^{2.5}$

- Similarly with strength

Palm Properties

- E_s = 0.1-3.0 GPa in low density palm tissue from *Washingtonia robusta* (Rueggeberg et al., 2008)
- Estimate in dense tissue (E^* = 30 GPa; ρ^*= 1000 kg/m^3) E_s = 45 GPa
- Large variation in E_s due to additional secondary layers in cell walls of denser tissue and increased alignment of cellulose microfibrils in those layers

Palm: Mechanical Efficiency Bending Stiffness

$$\rho = \left(\frac{r}{r_o} \right)^n \rho_{\max}$$

$$(EI)_{gradient} = \frac{C \pi r_o^4}{mn + 4}$$

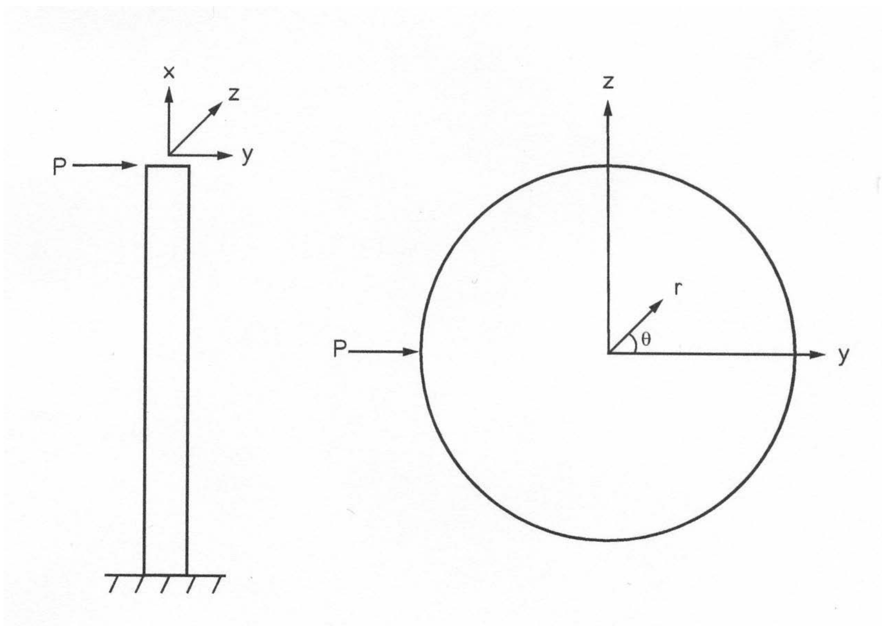
$$E = C \left(\frac{\rho}{\rho_{\max}} \right)^m = C \left(\frac{r}{r_o} \right)^{mn}$$

$$\frac{(EI)_{gradient}}{(EI)_{uniform}} = \frac{4}{mn + 4} \left(\frac{n+2}{2} \right)^m$$

Iriartea gigantea: n = 2, m = 2.5

$(EI)_{gradient}/(EI)_{uniform} = 2.5$

Palm: Mechanical Efficiency Bending *Stress* Distribution



$$\sigma(y) = E\varepsilon = E\kappa y$$

$$\sigma(r,\theta) = C\left(\frac{r}{r_o}\right)^{mn} \kappa r \cos\theta \propto r^{mn+1}$$

I. gigantea: n =2, m = 2.5

$$\sigma \propto r^6$$

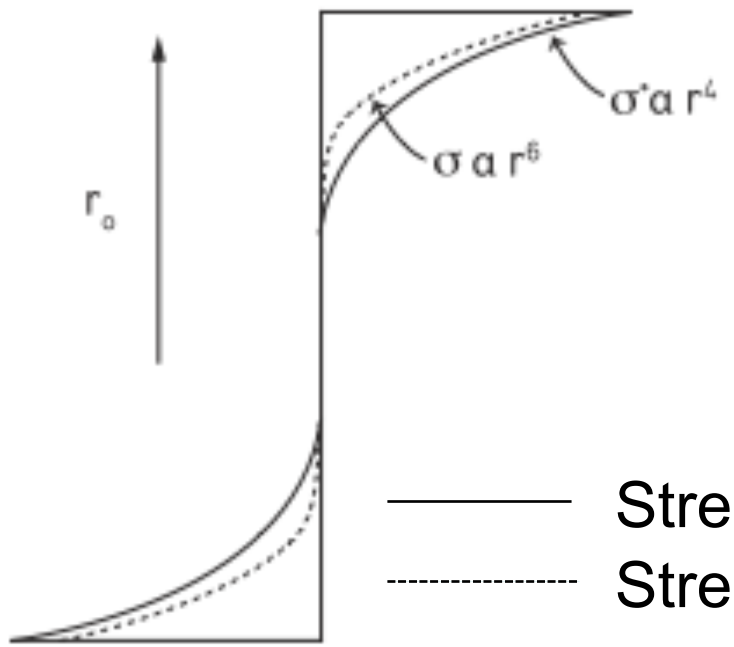
Palm: Mechanical Efficiency Bending *Strength* Distribution

$$\sigma^* \propto \left(\frac{\rho}{\rho_{\max}}\right)^q \propto \left(\frac{r}{r_o}\right)^{nq}$$

Iriartea gigantea: n = 2, q = 2

$$\sigma^* \propto r^4$$

Palm bending stress, strength



Strength distribution
Stress distribution

Figure sources

Sources for all figures in:
Cellular Materials in Nature and Medicine (2010)

Circular sections with radial density gradients: Palm Stems

- Palms can grow up to 20-40m - largest stresses from hurricane winds
- Unlike trees, palms do not have a cambium layer at the periphery, with dividing cells to allow increase in diameter as palm grows in height
- Instead, diameter of palm roughly constant as it grows in height
- Increasing stress resisted by cell walls increasing in thickness
- Add additional layers of secondary cell wall
- Produces radial density gradient
  - Density higher at periphery and at base of stem
  - A single stem can have densities from 100-1000 kg/m^3, nearly spanning the density range of all woods (balsa $\sim$200 kg/m^3 $\rightarrow$ lignum vitae $\sim$ 1300 kg/m^3)
- Specimen of palm taken from different radial positions tested in bending (Paul Rich, 1980s)
- Found $E^*_{\text{axial}} = C' \, \rho^{*\,2.46}$
- Might expect $E^*_{\text{axial}} \propto \rho$ - vascular bundles honeycomb-like

- But additional cell wall layers change E_s: data E_s=0.1-3 GPa
- Also: lower density palm has more ground tissue (parenchyma) with $E \propto \rho$ if at high turgor, but $E \propto \rho^2$ if at low turgor. (bending specimens dry)
- Modulus of rupture $\sigma^* = C'' \rho^{*^{2.05}}$
- Radial density gradient increases flexual rigidity
- Compare (EI) with density gradient to (EI) of section of same mass+radius but uniform density
- For Iriartea gigantea:

$$\left(\frac{\rho^*}{\rho_{\max}}\right) = \left(\frac{r}{r_0}\right)^n \qquad r_0 = \text{outer radius}$$

$$n = 2$$

$$E = C\left(\frac{\rho}{\rho_{\max}}\right)^m = C\left(\frac{r}{r_0}\right)^{mn}$$

$$\int r^2\, 2\pi r dr = J = 2I$$

$$(EI)_{\text{gradient}} = \int_0^{r_0} = C\left(\frac{\rho}{\rho_{\max}}\right)^m \frac{2\pi r\, r^2 dr}{2}$$

$$= \int_0^{r_0} C\left(\frac{r}{r_0}\right)^{mn} \pi r^3 dr$$

$$
\begin{aligned}
(EI)_{gradient} &= \frac{C\pi}{r_0^{mn}} \int_0^{r_0} r^{mn+3} dr \\
&= \frac{C\pi}{r_0^{mn}} \frac{r_0^{mn+4}}{mn+4} \\
&= \frac{C\pi r_0^4}{mn+4}
\end{aligned}
$$

Equivalent mass, r_0, uniform density $\bar{\rho}$:

$$
\frac{\bar{\rho}}{\rho_{\max}} = \frac{1}{\pi r_0^2} \int_0^{r_0} \left(\frac{r}{r_0}\right)^n 2\pi r \, dr = \frac{1}{\cancel{\pi} r_0^2} \frac{2\cancel{\pi}}{r_0^n} \frac{r_0^{n+2}}{n+2} = \frac{2}{n+2}
$$

$$
\begin{aligned}
\Big(EI\Big)_{\substack{\text{uniform}\\ \text{density}}} &= C \left(\frac{\bar{\rho}}{\rho_{\max}}\right)^m \quad \frac{\pi r_0^4}{4} \\
&= C \left(\frac{2}{n+2}\right)^m \quad \frac{\pi r_0^4}{4}
\end{aligned}
$$

$$
\boxed{\frac{\Big(EI\Big)_{\text{gradient}}}{\Big(EI\Big)_{\text{uniform}}} = \frac{C\pi r_0^4}{mn+4} \quad \frac{4}{C\,\pi r_0^4} \left(\frac{n+2}{2}\right)^m = \frac{4}{mn+4} \left(\frac{n+2}{2}\right)^m}
$$

$$
\text{I.gigantea } m = 2.5 \quad n = 2 \qquad \boxed{\frac{\Big(EI\Big)_{\text{gradient}}}{\Big(EI\Big)_{\text{uniform}}} = 2.5}
$$

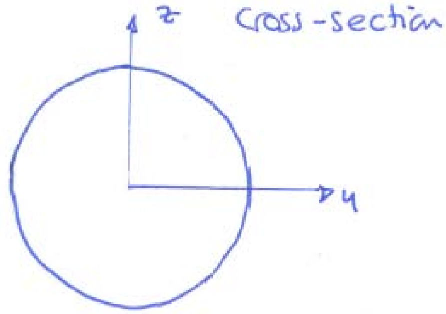
Stress and Strength distribution

$$\frac{\rho}{\rho_{\max}} = \left(\frac{r}{r_0}\right)^n$$

Stress $\sigma(y) = E\,\epsilon = E\,\mathcal{K}\,y$

$$= C\left(\frac{\rho}{\rho_{\max}}\right)^m \mathcal{K}\,y$$

$$= C\left(\frac{r}{r_0}\right)^{mn} \mathcal{K}\,r$$



$\mathcal{K}$=curvature at the cross-section

y=distance from neutral axis

$E = C\left(\frac{\rho}{\rho_{\max}}\right)^m$

$\sigma(r) \propto r^{mn+1}$ I. gigantea m=2.5 n=2 $\sigma(r) \propto r^6$

Strength $\sigma^*(r) = C'\left(\frac{\rho}{\rho_{\max}}\right)^q = C\left(\frac{r}{r_0}\right)^{nq}$

$\sigma^* r \propto r^{nq}$ I. gigantea q=2 n=2 $\sigma^*(r) \propto r^4$

Figure: if max normal stress at $r = r_0$ is $\sigma = \sigma^*$ then bending stress distribution closely follows strength distribution!