MIT 3.071
Amorphous Materials

8: Mechanical Properties

Juejun (JJ) Hu

After-class reading list

- Fundamentals of Inorganic Glasses
  - Ch. 18
- Introduction to Glass Science and Technology
  - Ch. 9



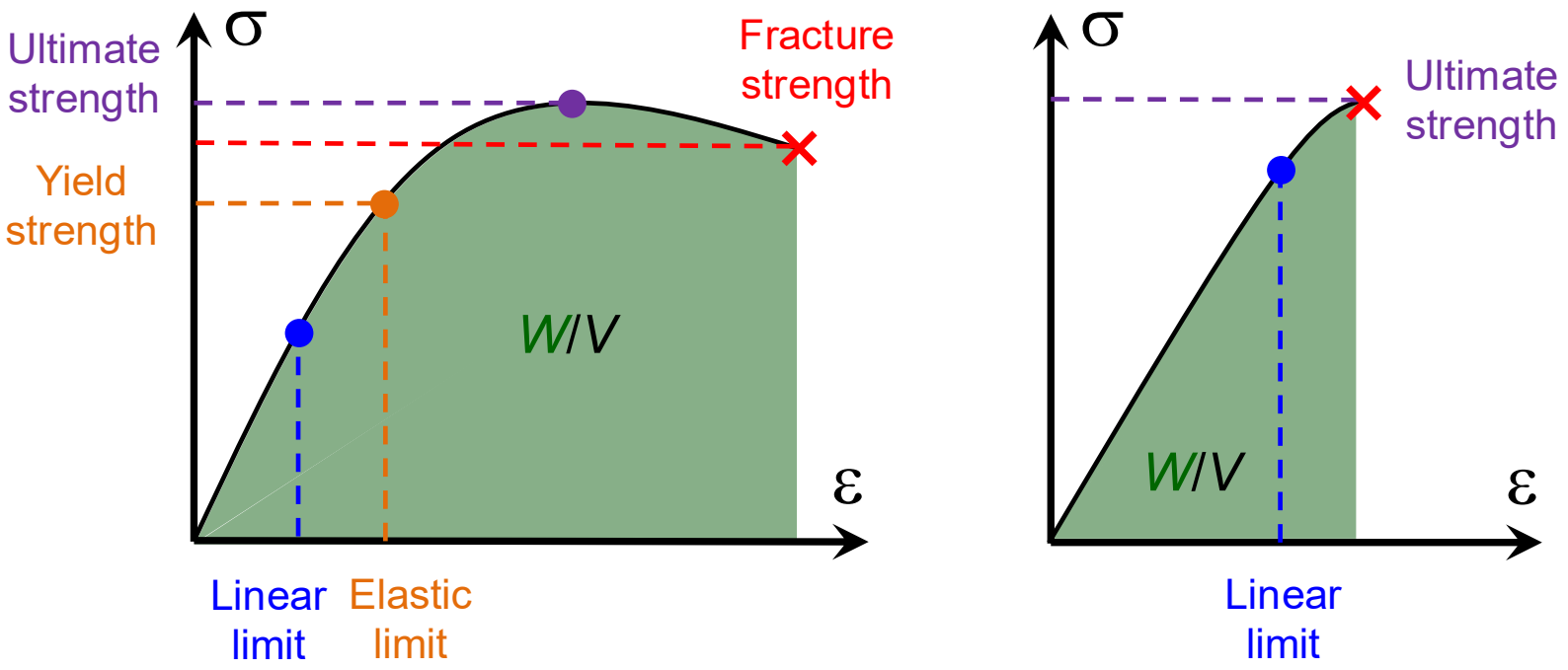
Glass = fragile?

Material	Iron	Structural steel	Glass fiber
Ultimate tensile strength	35 MPa	550 MPa	4890 MPa

Glass

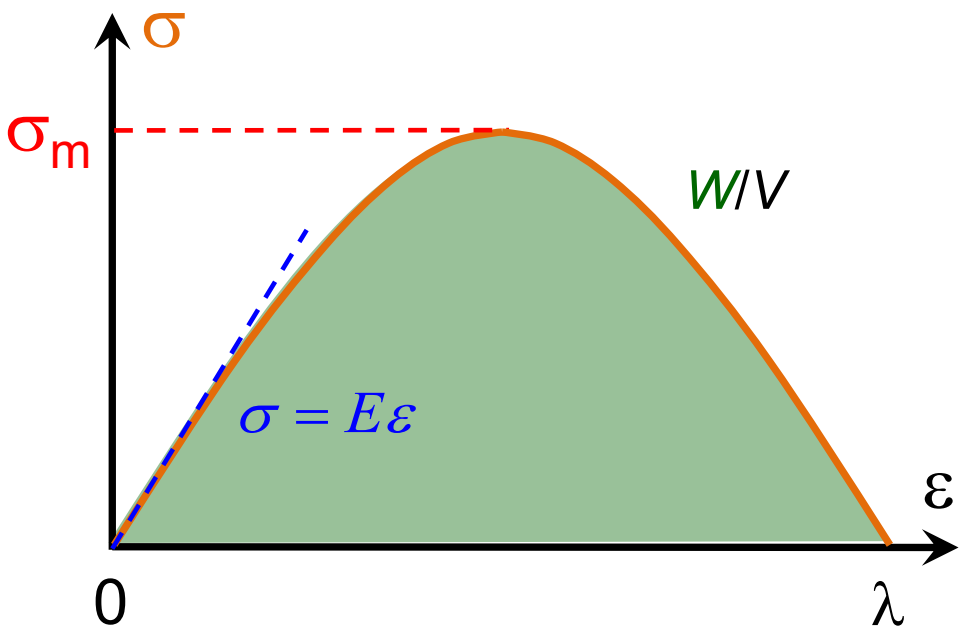
Strength and toughness

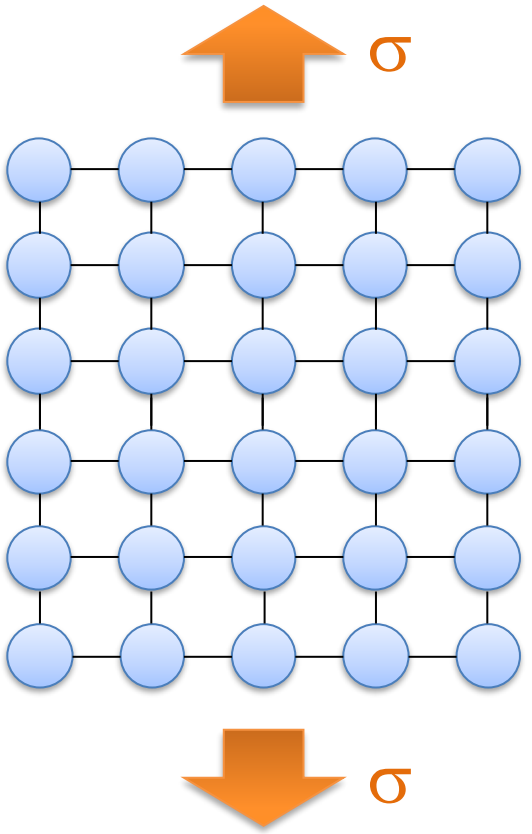
- Strength: applied stress a material can withstand
- Toughness: energy absorbed by (work performed to) a material per unit volume before fracture



Theoretical strength of a brittle material

- Theoretical strength is determined by the cohesive force between atoms
- Work *W* performed to separate the solid equals to the energy of the fresh surfaces created during fracture





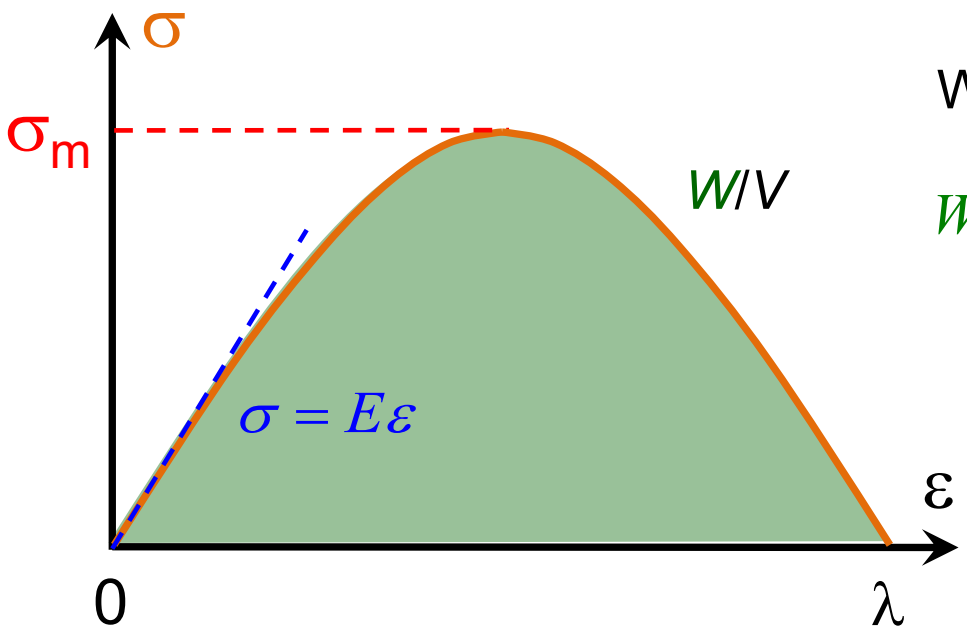
Theoretical strength of a brittle material

- Theoretical strength is determined by the cohesive force between atoms
- Work *W* performed to separate the solid equals to the energy of the fresh surfaces created during fracture

$$\sigma = \sigma_m \sin\frac{\pi\varepsilon}{\lambda}$$

When $\sigma << \sigma_m$, $\sigma = E\varepsilon$

$$\sigma_m = \frac{\lambda E}{\pi}$$



Work *W* performed:

$$W = V \cdot \int_0^{\lambda} \sigma d\varepsilon = \frac{2\lambda^2 EV}{\pi^2} = 2\gamma S$$

$$V = a_0 S \Rightarrow \lambda = \pi\sqrt{\gamma / E a_0}$$

$$\Rightarrow \sigma_m = \sqrt{\frac{\gamma E}{a_0}}$$

Theoretical strength of a brittle material

- Consider silica glass
  - γ = 3.5 J/m^2, E = 70 GPa, a_0 = 0.2 nm

$$\sigma_m = \sqrt{\frac{\gamma E}{a_0}} = 35{,}000 \text{ MPa}$$

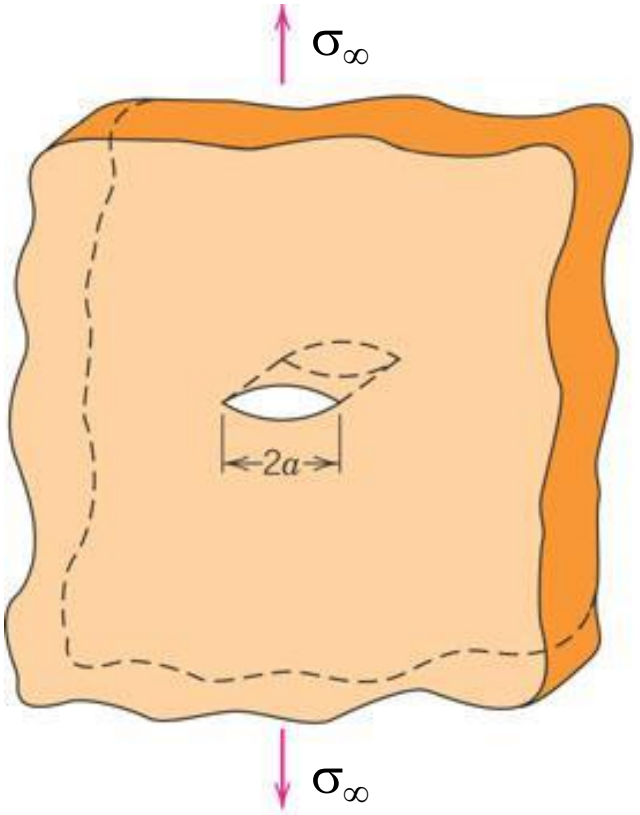
Material	Glass	Silica glass	Silica nanowire
Ultimate tensile strength	~ 30 MPa	110 MPa	26000 MPa†

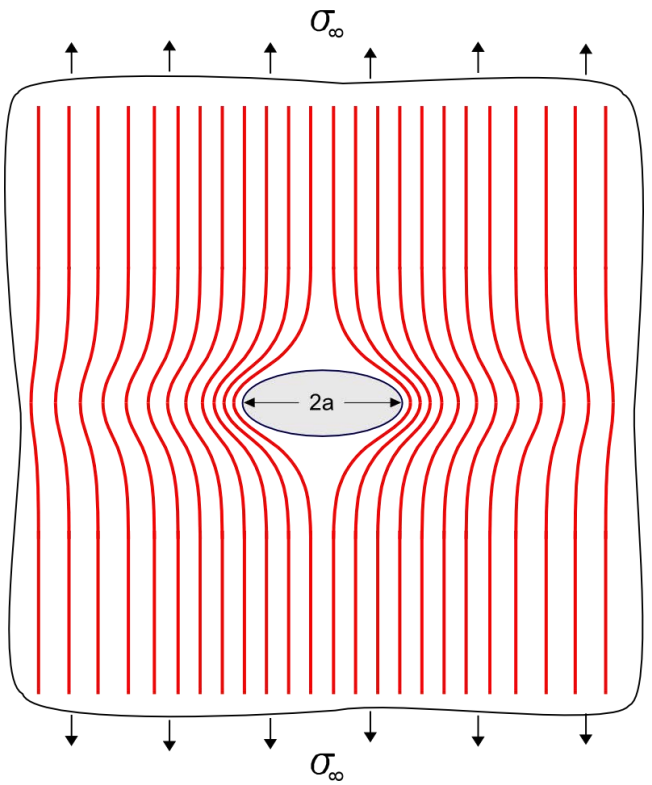
Practical strength of engineering materials is much less than their theoretical strength

† "The Ultimate Strength of Glass Silica Nanowires," *Nano Lett.* **9**, 831 (2009)

Griffith's theory

- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)





Griffith's theory

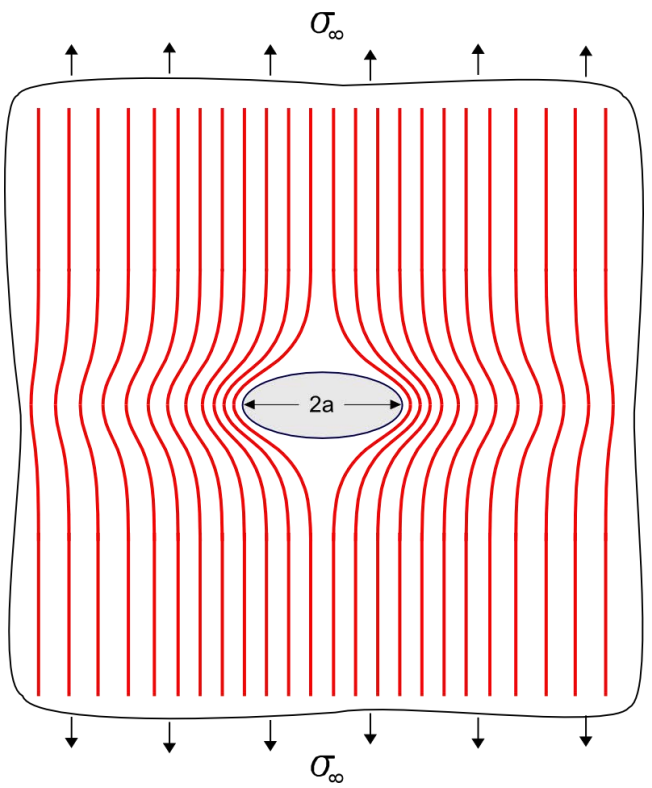
- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)

Stress concentration factor:

$$\frac{\sigma_{\max}}{\sigma_\infty} = 2\sqrt{\frac{a}{\rho}} = 2\sqrt{\frac{a}{a_0}}$$

Fracture strength of a flawed material:

$$\sigma_f = \frac{1}{2}\sqrt{\frac{\gamma E}{a}}$$



Griffith's theory

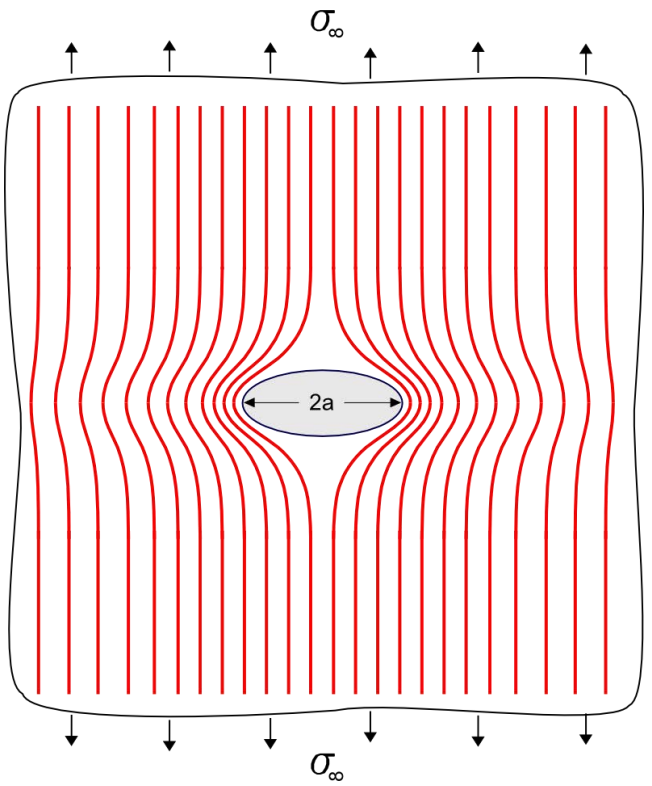
- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)

In flawed silica glass:

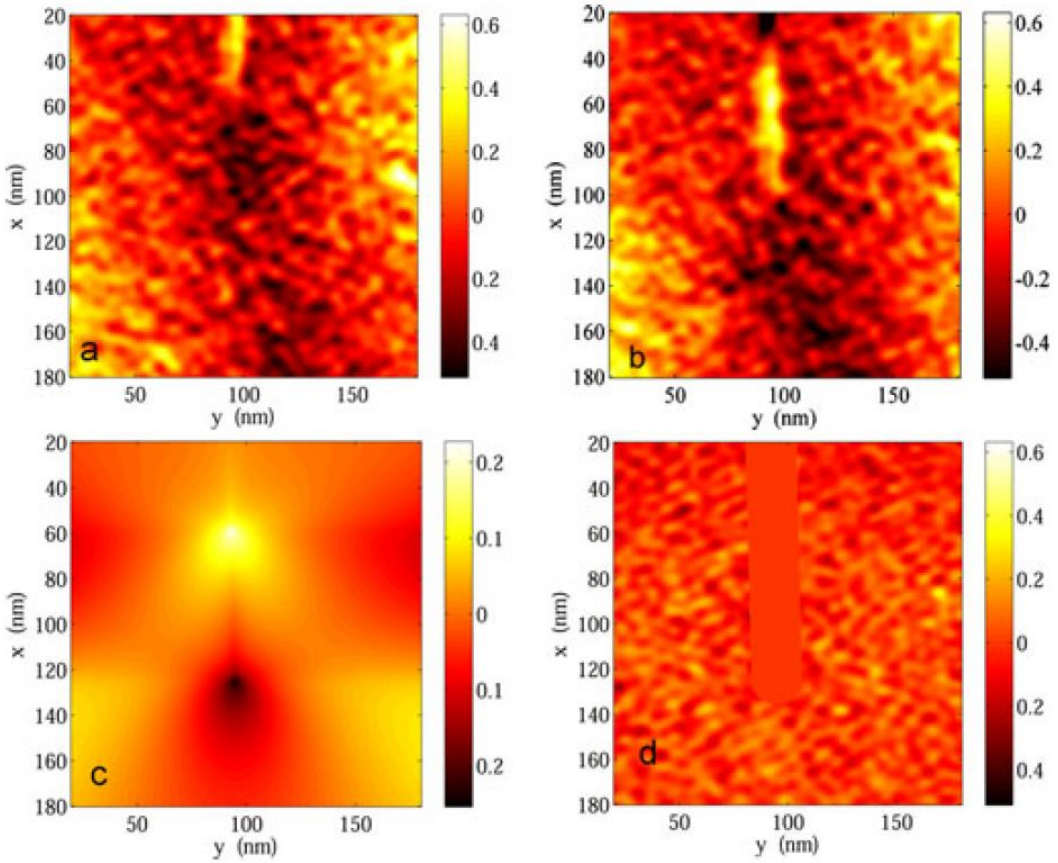
$$\sigma_f = \frac{1}{2}\sqrt{\frac{\gamma E}{a}} = 110 \text{ MPa}$$

$$\Rightarrow \quad a = 5 \text{ } \mu\text{m}$$

A. Griffith, "The Phenomena of Rupture and Flow in Solids," *Philos. Trans. Roy. Soc. London, A* **221**, 163 (1921)

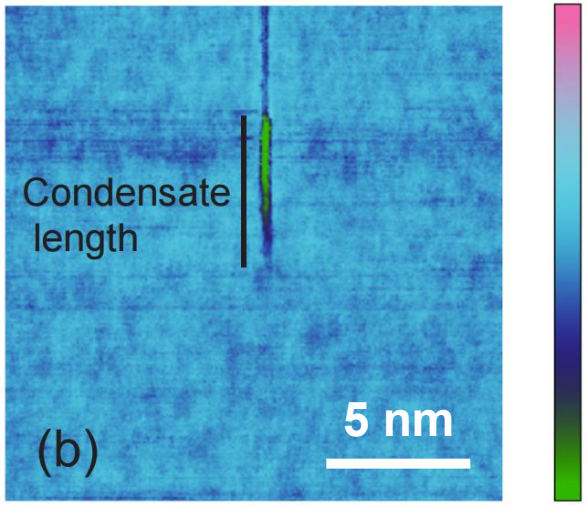


Visualizing Griffith cracks in glass



Displacement field near a crack tip

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AFM phase image

Water condensation at crack tip

Europhys. Lett. **89**, 66003 (2010)

J. Am. Ceram. Soc. **94**, 2613 (2011)

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Stress intensity factor and fracture toughness

- Stress intensity factor (tensile):

$K_I = \sigma_\infty \sqrt{\pi a}$ $K_{Ic} = \sigma_f \sqrt{\pi a}$ critical stress intensity factor

- Strain energy release rate (plane stress):

$G_I = K_I^{\,2} / E$ $G_{Ic} = K_{Ic}^{\,2} / E$ work of fracture

- Fracture condition:

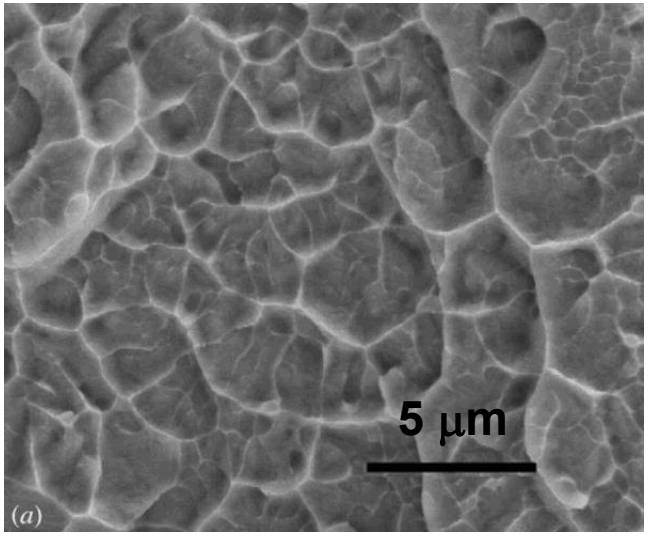
$K_I > K_{Ic}$ $G_I > G_{Ic}$

K_{Ic} is a material constant and is independent of crack length

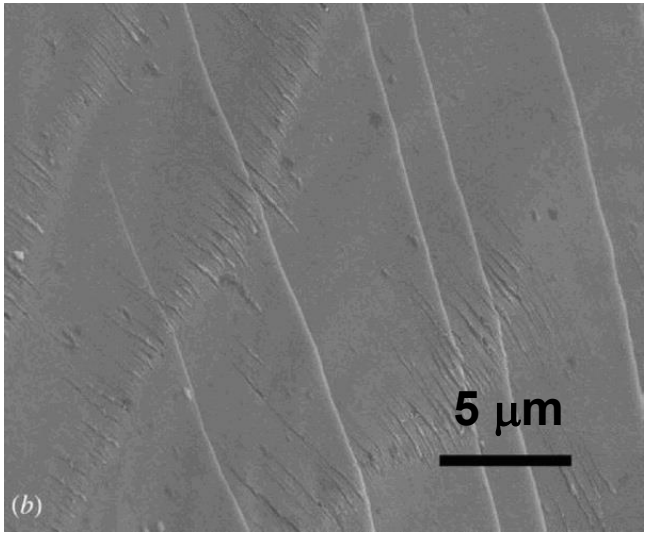
Intrinsic plasticity in amorphous metals

- Lack of global plasticity
- Intrinsic plasticity
- When $G/B < 0.42$: plastic; $G/B > 0.42$: brittle (G: shear modulus; B: bulk modulus)

Phil. Mag. Lett. **85**, 77 (2006)



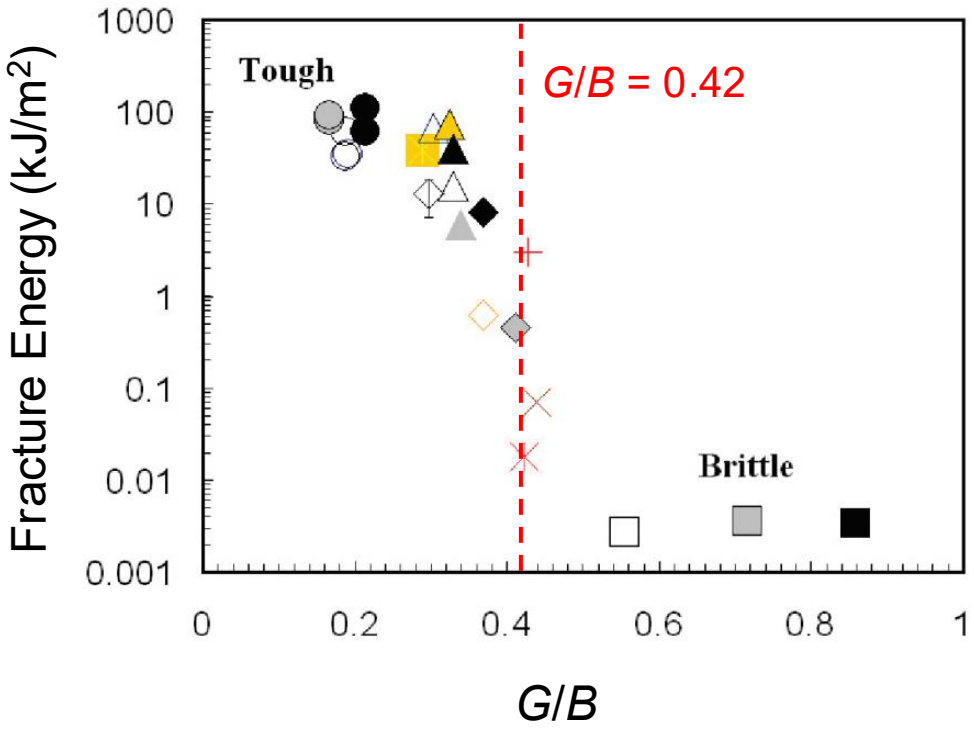
As-cast Vitreloy-1



Annealed at 350 °C for 12 h

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Intrinsic plasticity in amorphous metals



Phil. Mag. Lett. **85**, 77 (2006)

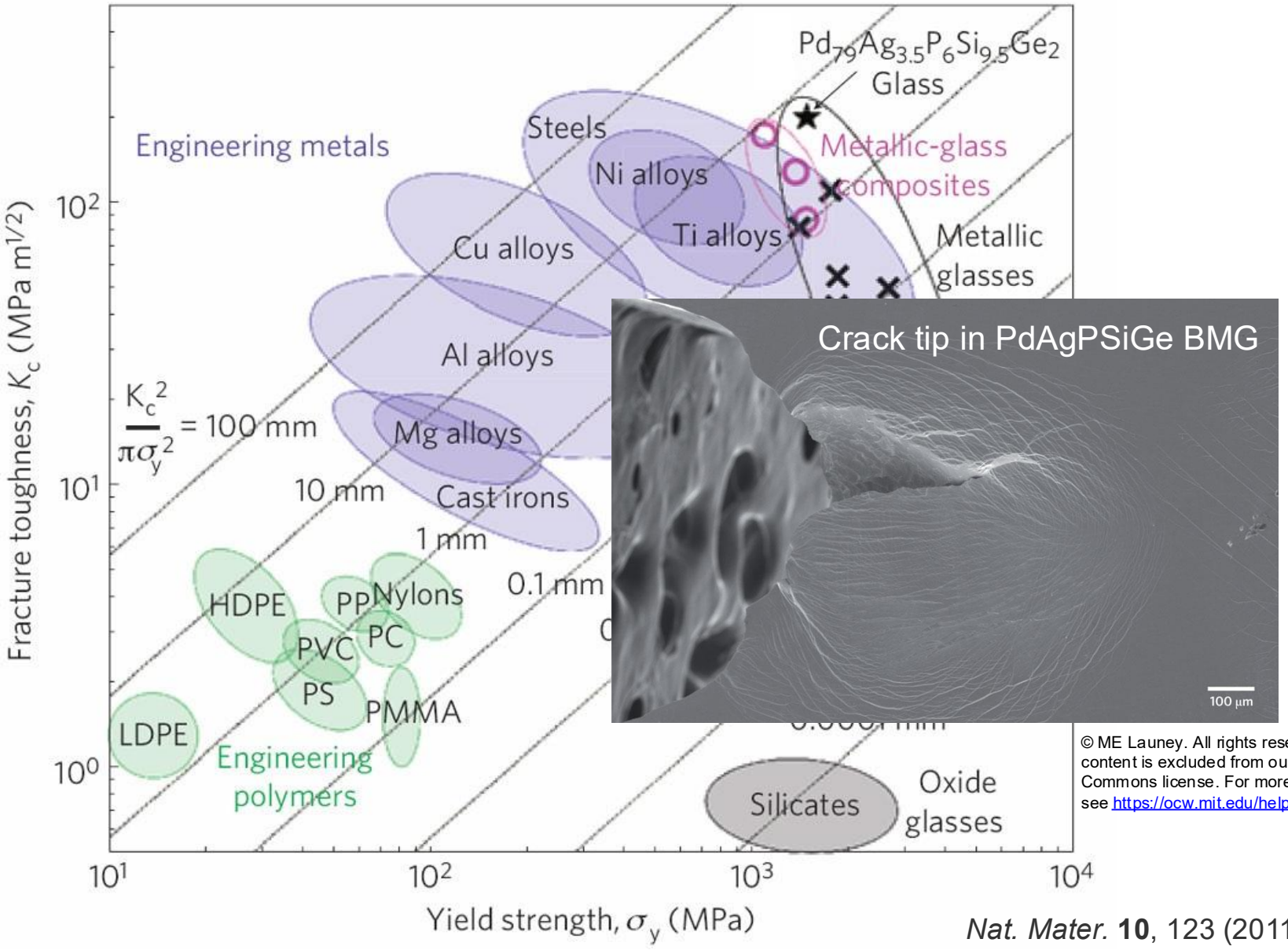
□ Toughened (partially crystallized) glass ■ Window glass ■ Fused silica

× $Mg_{65}Cu_{25}Tb_{10}$ + $Ce_{70}Al_{10}Ni_{10}Cu_{10}$ * $Fe_{50}Mn_{10}Mo_{14}Cr_4C_{16}B_6$

△ $Zr_{57}Ti_5Cu_{20}Ni_8Al_{10}$ ▲ $Zr_{41}Ti_{14}Cu_{12.5}Ni_{10}Be_{22.5}$ ◇ $Zr_{57}Nb_5Cu_{15.4}Ni_{12.6}Al_{10}$

■ $Cu_{60}Zr_{20}Hf_{10}Ti_{10}$ ● $Fe_{80}P_{13}C_7$ ○ $Pd_{77.5}Cu_6Si_{16.5}$ ● $Pt_{57.5}Cu_{14.7}Ni_{5.3}P_{22.5}$

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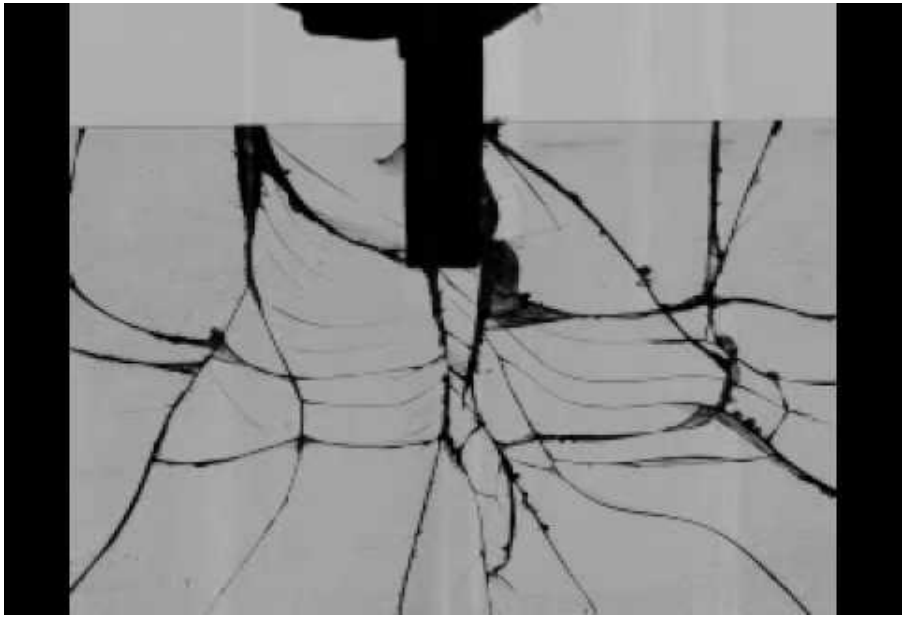
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Nat. Mater. **10**, 123 (2011)

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Brittle fracture of glass

- When a crack exceeds the critical length, the crack becomes unstable and propagates catastrophically through the material



Glass cracking at 231,000 fps

Crack propagation velocity:

1540 m/s

J. Am. Cer. Soc. **22**, 302-307 (1939)

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Brittle fracture of glass

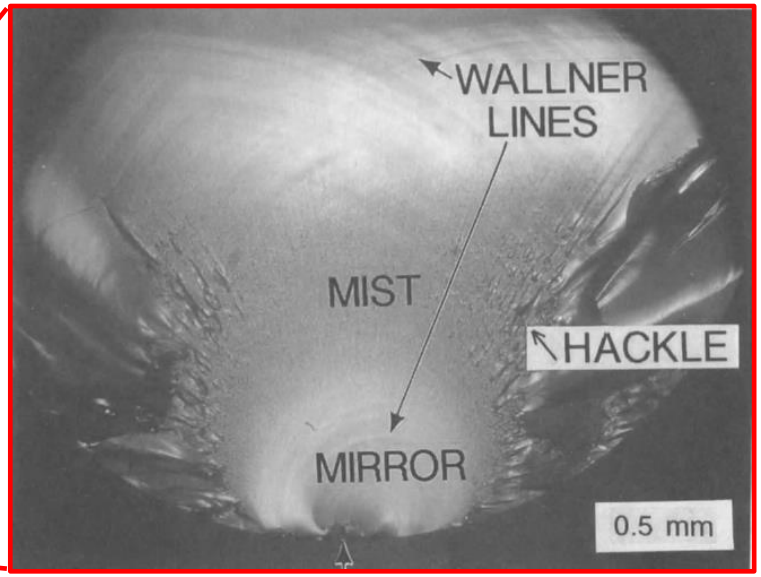
Ceramic Tech Today article link



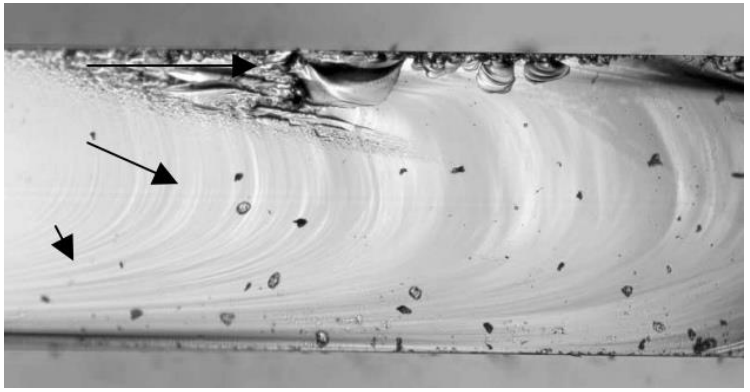
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Fractography







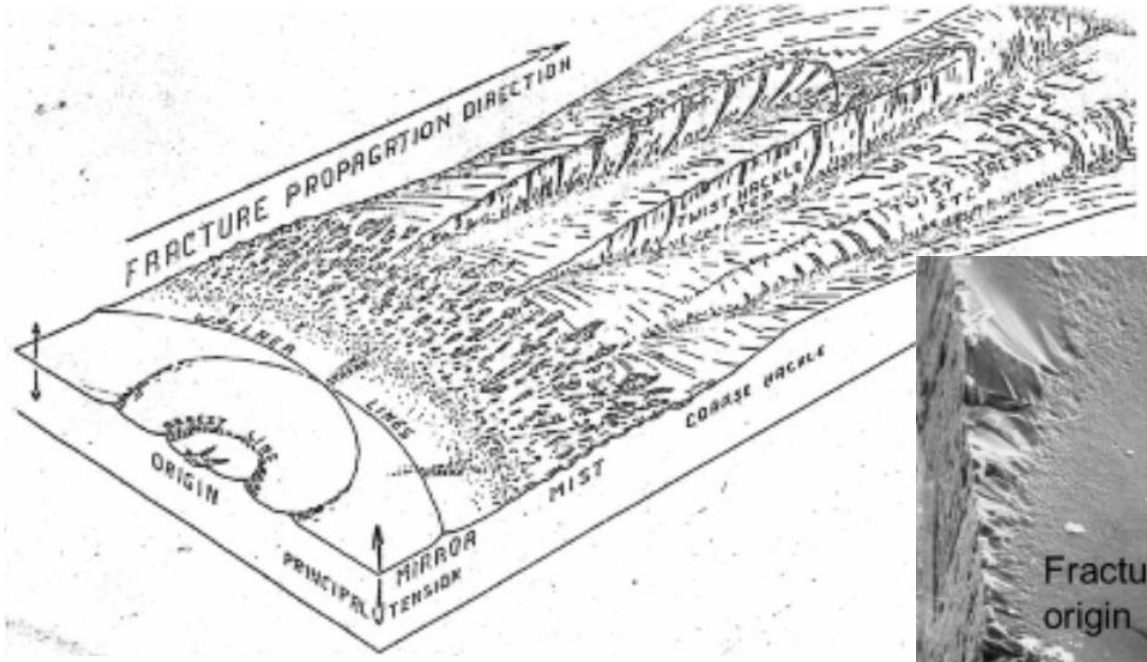


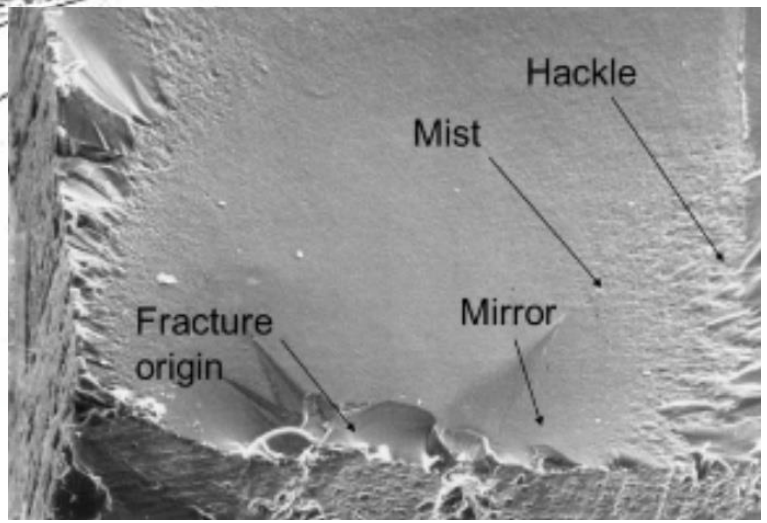
Conchoidal fracture

Image from "*Fracture analysis, a basic tool to solve breakage issues*"

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The fractured surface of glass

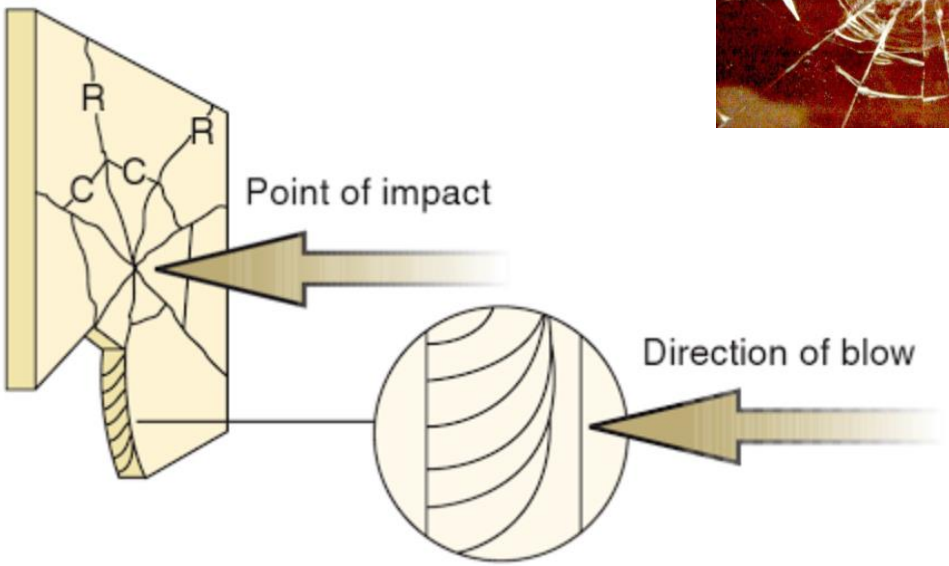




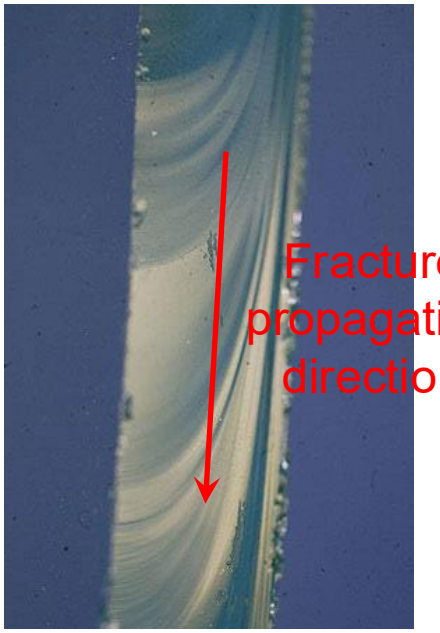
Source: public domain.

Peter L. Bocko, Cornell University / Corning Glass Technologies

The 4R rule







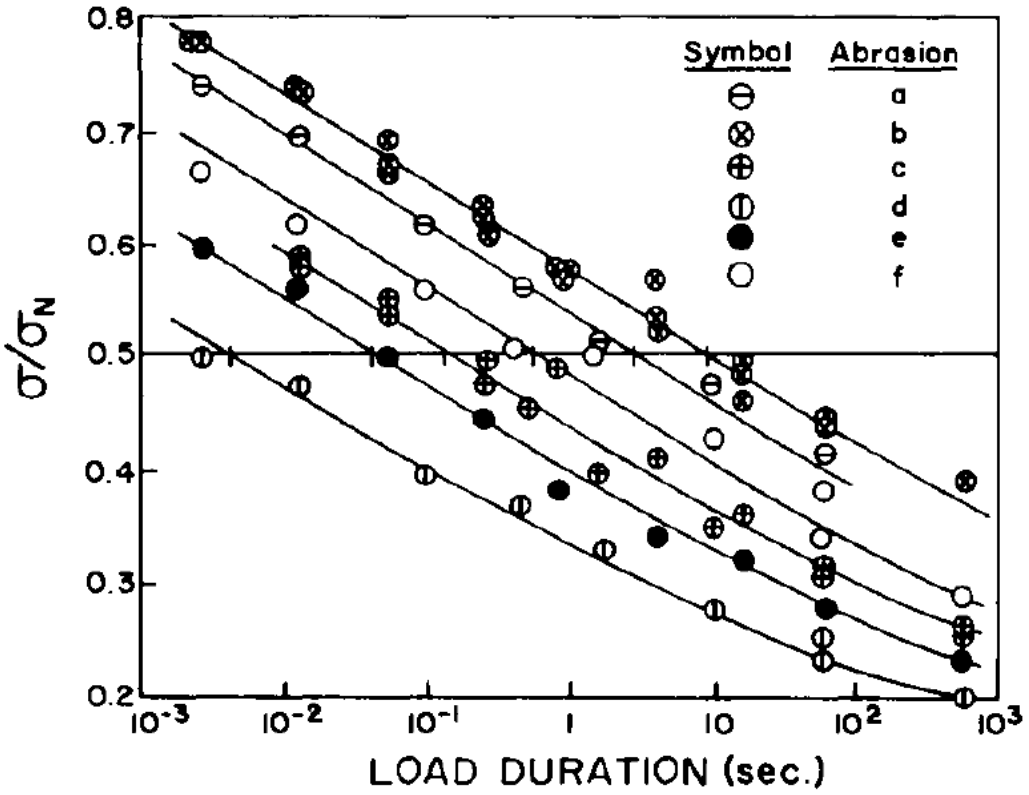
The 4R Rule: Ridge lines on Radial fractures are at Right angles to the Rear

K. Hess, and C. Orthmann, *Criminal Investigation*, Cengage Learning (2016)

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Static fatigue in glass

- Under constant load, the time-to-failure varies inversely with the load applied in a logarithm scale



Sub-critical crack growth: crack length increases over time even when $\sigma_\infty < \sigma_f$

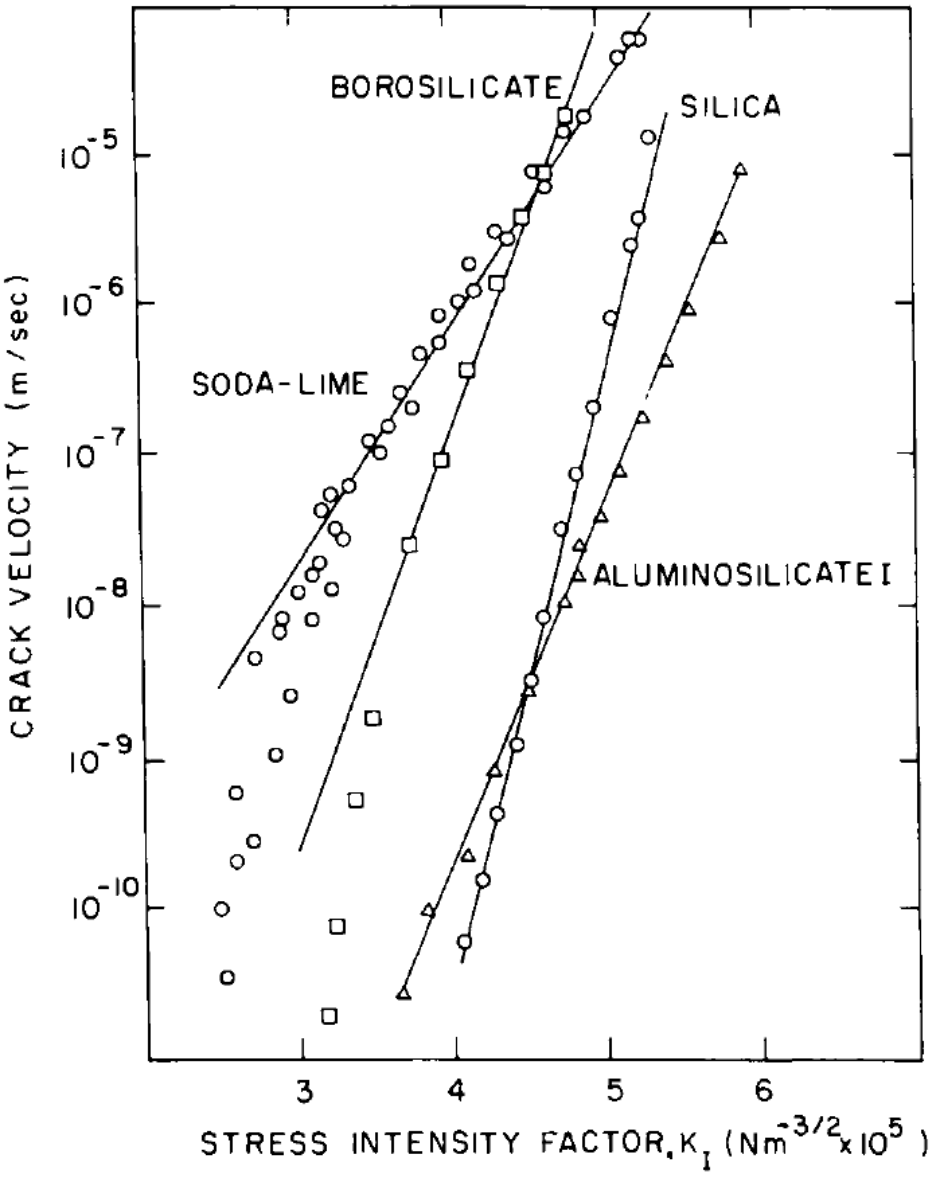
Stress corrosion

- Reaction at crack tip:

  $$\text{-Si-O-Si-} + H_2O \rightarrow$$
  $$\text{-Si-OH} + \text{HO-Si-}$$

- Higher alkaline content generally reduces fatigue resistance
- Higher susceptibility to stress corrosion in basic solutions
- Thermally activated process

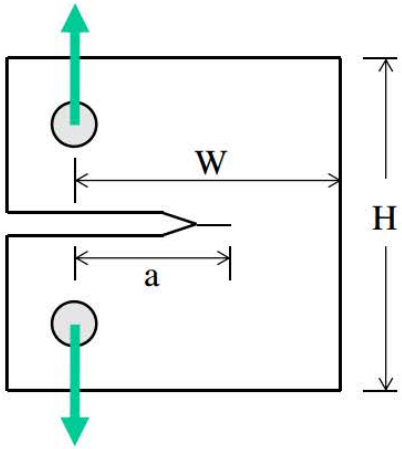
J. Non-Cryst. Solids **316**, 1 (2003)
J. Am. Ceram. Soc. **53**, 544 (1970)



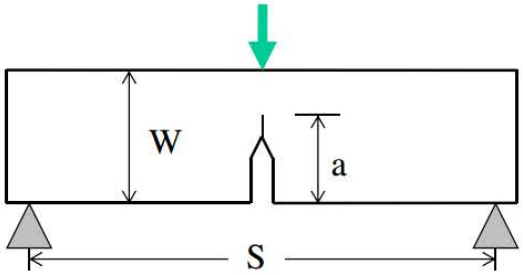
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Fracture toughness measurement

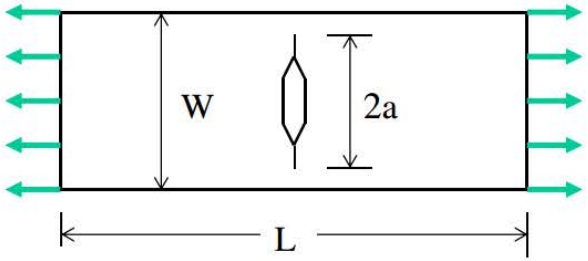
- ASTM Standard E1820-15: Standard Test Method for Measurement of Fracture Toughness
- Standard specimen geometries to obtain load-displacement plot



Compact tension specimen



Single edge-notched bend specimen (for three-point bending)



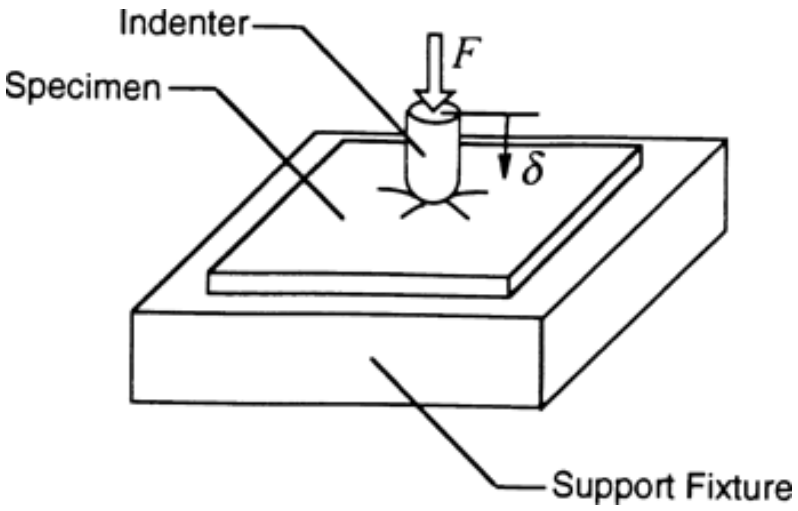
Middle-cracked tension specimen

Eng. Fract. Mech. **85**, 1 (2012)

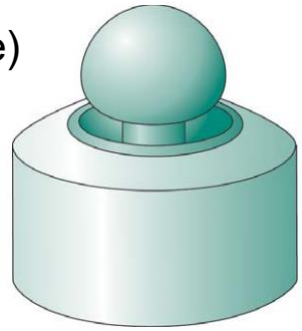
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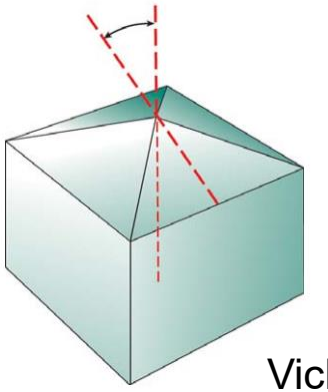
Indentation of glass samples

- Mechanical properties evaluated through indented crack size or crack-opening displacement based on empirical equations
- Poor correlation with conventional test results can be a concern



Hertzian (sphere) indenter tip





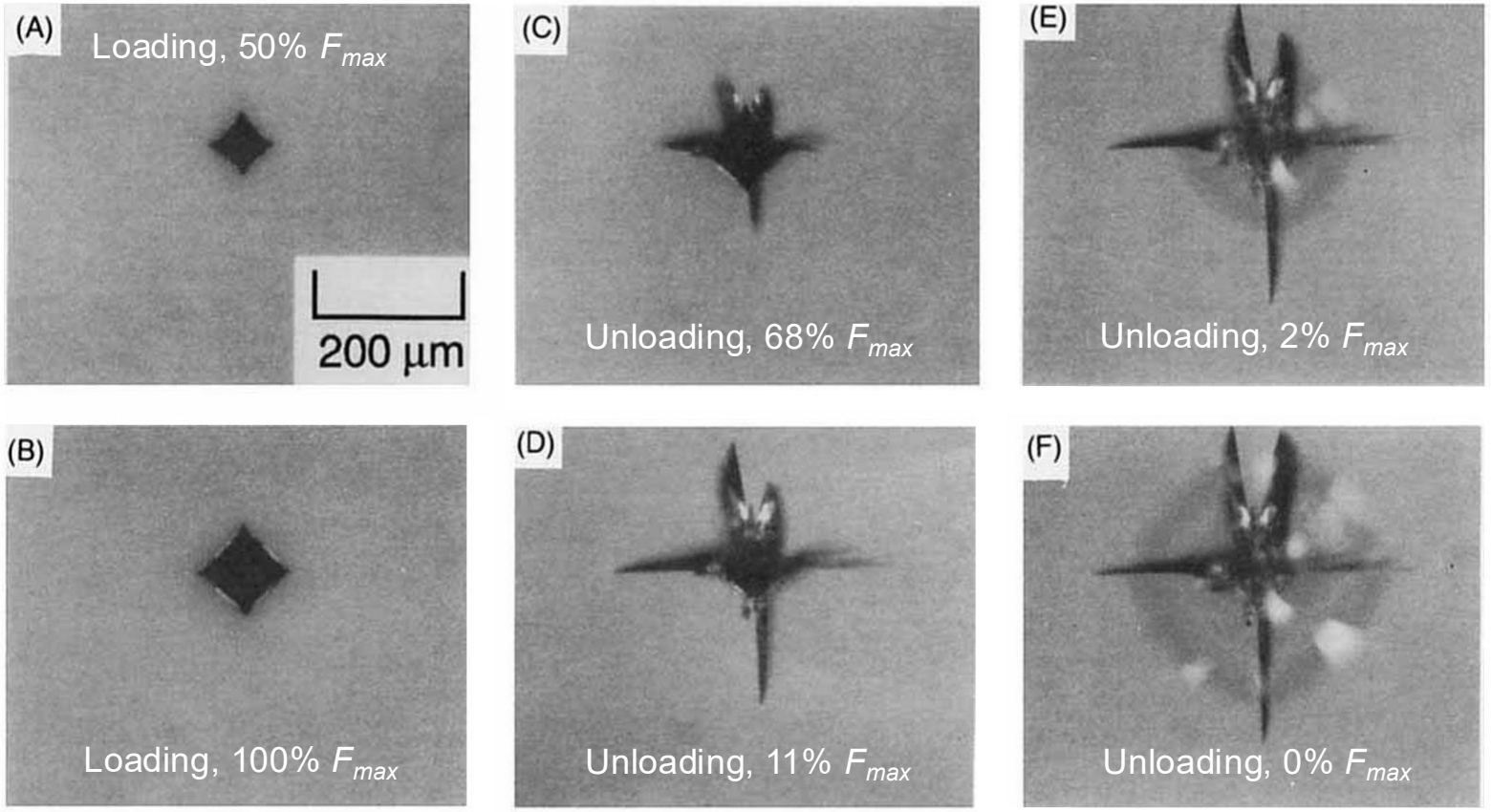
Vickers indenter tip

J. Mech. Behav. Biomed. Mater. **2**, 384 (2009)

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Indentation of glass samples

- Vickers indentation of soda-lime glass



J. Am. Ceram. Soc. **73**, 787 (1990)

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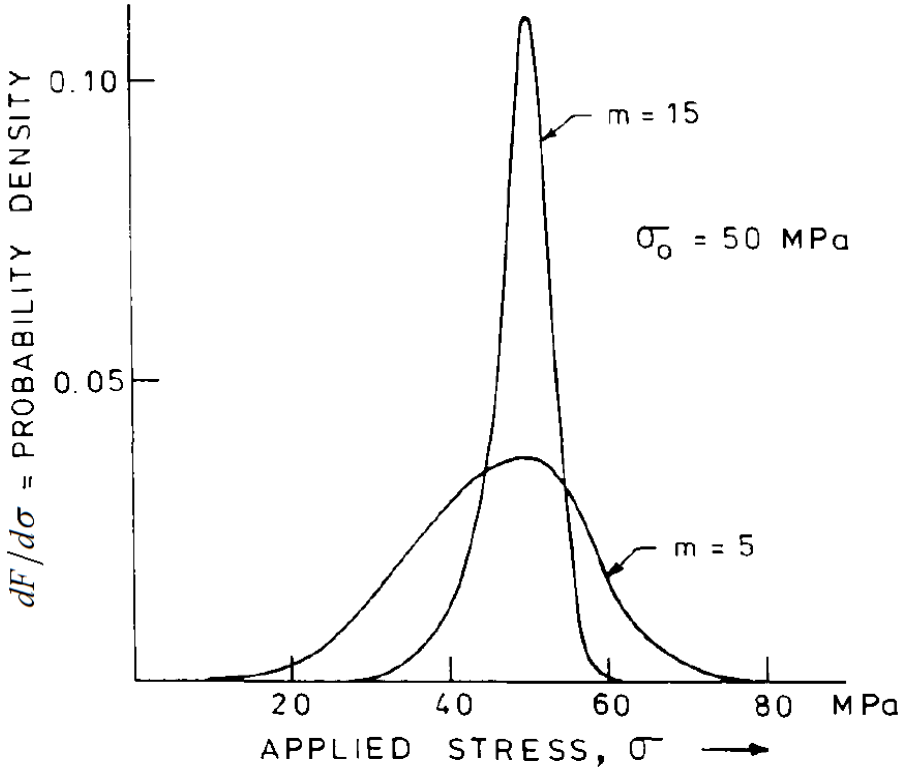
Fracture statistics

- Experimental results of fracture strength can often be described by the Weibull distribution
- The fraction F of samples which fracture at stresses below σ is given by:

$$F = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

m : Weibull modulus

- Probability density $dF/d\sigma$
  - Probability of samples fracture at stress σ

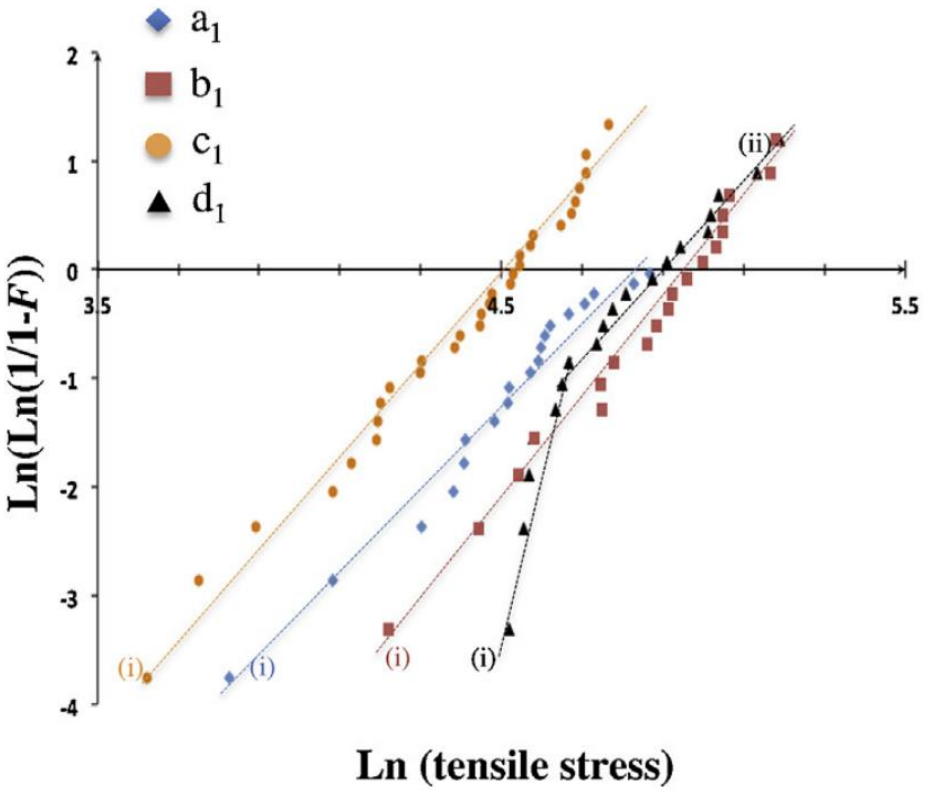


Weibull plot

$$F = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] \quad \Rightarrow \quad \ln\left[-\ln\left(1-F\right)\right] = m\left(\ln\sigma - \ln\sigma_0\right)$$

m : slope of the Weibull plot

σ_0 : intercept with horizontal axis



Mater. Res. Bull. **49**, 250 (2014)

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Summary

- Theoretical and practical strengths of materials
  - Practical strength of brittle materials is usually much lower than the theoretical strength due to the presence of defects
  - Oxide glasses are extremely sensitive to surface defects
  - Intrinsic ductility in select BMGs contributes to high toughness
- Basics of fracture mechanics
  - Griffith crack theory
  - Fracture toughness $K_{Ic} = \sigma_f \sqrt{\pi a} = \frac{1}{2}\sqrt{\pi \gamma E}$
- Fatigue and stress corrosion
- Fracture toughness measurement
- Fracture statistics: Weibull plot

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3.071 Amorphous Materials
Spring 2025

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