## Welcame to 3.091

## Lecture 18

October 21, 2009

X-Ray Diffraction Techniques


$$
\lambda_{\mathrm{sWl}}=\begin{aligned}
& \text { Mo target } \\
& \text { chantinuous } \\
& \text { chacteristic } \\
& \text { radiation }
\end{aligned}
$$



Image by MIT OpenCourseWare.


Averill, B., and P. Eldredge.
.Flat World Knowledge, 2011. ISBN: 9781453331224.

Selection Rules for Reflection in Cubic Crystals

| (hkl) | $h^{2}+k^{2}+l^{2}$ | (SC) | $\mathrm{BCC}$ | (FCC) |
| :---: | :---: | :---: | :---: | :---: |
| 100 | 1 | $\checkmark$ | \% | * |
| 110 | 2 | $\checkmark$ | $\checkmark$ | \% |
| 111 | 3 | $\checkmark$ | x | $\checkmark$ |
| 200 | 4 | $\checkmark$ all |  | +k+ $\mathbf{~} \mathrm{h}+\mathrm{k}+1$ |
| 210 | 5 | $\checkmark$ | $\boldsymbol{X}$ | $\boldsymbol{x}$ |
| 211 | 6 | $\checkmark$ | $\checkmark$ | even: all even |
| 220 | 8 | $\checkmark$ | $\checkmark$ | $\checkmark$ or all odd |
| 300 | 9 | $\checkmark$ | \% | x unmixed |
| 310 | 10 | $\checkmark$ | $\checkmark$ | \% |
| 311 | 11 | $\checkmark$ | X | $\checkmark$ |
| 222 | 12 | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 320 | 13 | $\checkmark$ | X | X |
| 321 | 14 | $\checkmark$ | $\checkmark$ | X |
| 400 | 16 | $\checkmark$ | $\checkmark$ | $\checkmark$ |

## diffractometry




Cu target, $\lambda_{\mathrm{K}_{\alpha}}=1.5418 \AA$
$2 \theta$
44.48
51.83
76.35
92.90
98.40
121.87
144.54
155.51

# Sadoway's Five-step Program for Determining Crystal Structure 

## Step 1 Start with $2 \theta$ values and generate a set of $\sin ^{2} \theta$ values.

Cu target, $\lambda_{\mathrm{K}_{\alpha}}=1.5418 \AA$
$2 \theta \quad \sin ^{2} \theta$

| 44.48 | 0.143 |
| ---: | ---: |
| 51.83 | 0.191 |
| 76.35 | 0.382 |
| 92.90 | 0.525 |
| 98.40 | 0.573 |
| 121.87 | 0.764 |
| 144.54 | 0.907 |
| 155.51 | 0.955 |

## Sadoway's Five-step Program for Determining Crystal Structure

Step 1 Start with $2 \theta$ values and generate a set of $\sin ^{2} \theta$ values.

Step 2 Normalize the $\sin ^{2} \theta$ values by generating $\sin ^{2} \theta_{n} / \sin ^{2} \theta_{1}$.

Cu target, $\lambda_{\mathrm{K}_{\alpha}}=1.5418 \AA$
$2 \theta \quad \sin ^{2} \theta \quad$ normalized

| 44.48 | 0.143 | 1.00 |
| ---: | ---: | ---: |
| 51.83 | 0.191 | 1.34 |
| 76.35 | 0.382 | 2.67 |
| 92.90 | 0.525 | 3.67 |
| 98.40 | 0.573 | 4.01 |
| 121.87 | 0.764 | 5.34 |
| 144.54 | 0.907 | 6.34 |
| 155.51 | 0.955 | 6.68 |

## Sadoway's Five-step Program for Determining Crystal Structure

Step 1 Start with $2 \theta$ values and generate a set of $\sin ^{2} \theta$ values.

Step 2 Normalize the $\sin ^{2} \theta$ values by generating $\sin ^{2} \theta_{\mathrm{n}} / \sin ^{2} \theta_{1}$.

Step 3 Clear fractions from "normalized" column.

Cu target, $\lambda_{\mathrm{K}_{\alpha}}=1.5418 \AA$
$2 \theta \quad \sin ^{2} \theta$ normalized clear
fractions

| 44.48 | 0.143 | 1.00 | 3 |
| ---: | :---: | :---: | :---: |
| 51.83 | 0.191 | 1.34 | 4 |
| 76.35 | 0.382 | 2.67 | 8 |
| 92.90 | 0.525 | 3.67 | 11 |
| 98.40 | 0.573 | 4.01 | 12 |
| 121.87 | 0.764 | 5.34 | 16 |
| 144.54 | 0.907 | 6.34 | 19 |
| 155.51 | 0.955 | 6.68 | 20 |

## Sadoway's Five-step Program for Determining Crystal Structure

Step 1 Start with $2 \theta$ values and generate a set of $\sin ^{2} \theta$ values.

Step 2 Normalize the $\sin ^{2} \theta$ values by generating $\sin ^{2} \theta_{\mathrm{n}} / \sin ^{2} \theta_{1}$.

Step 3 Clear fractions from "normalized" column.

Step 4 Speculate on the $h k l$ values that, if expressed as $h^{2}+k^{2}+l^{2}$, would generate the sequence of the "clear fractions" column.

Cu target, $\lambda_{\mathrm{K}_{\alpha}}=1.5418 \AA$
$2 \theta \quad \sin ^{2} \theta$ normalized clear (hkl)?
fractions

| 44.48 | 0.143 | 1.00 | 3 | 111 |
| ---: | :---: | :---: | :---: | :---: |
| 51.83 | 0.191 | 1.34 | 4 | 200 |
| 76.35 | 0.382 | 2.67 | 8 | 220 |
| 92.90 | 0.525 | 3.67 | 11 | 311 |
| 98.40 | 0.573 | 4.01 | 12 | 222 |
| 121.87 | 0.764 | 5.34 | 16 | 400 |
| 144.54 | 0.907 | 6.34 | 19 | 331 |
| 155.51 | 0.955 | 6.68 | 20 | 420 |

## Sadoway's Five-step Program for Determining Crystal Structure

Step 1 Start with $2 \theta$ values and generate a set of $\sin ^{2} \theta$ values.

Step 2 Normalize the $\sin ^{2} \theta$ values by generating $\sin ^{2} \theta_{n} / \sin ^{2} \theta_{1}$.

Step 3 Clear fractions from "normalized" column.

Step 4 Speculate on the $h k l$ values that, if expressed as $h^{2}+k^{2}+l^{2}$, would generate the sequence of the "clear fractions" column.

Step 5 Compute for each $\theta$ the value of $\sin ^{2} \theta /\left(h^{2}+k^{2}+l^{2}\right)$ on the basis of the assumed $h k l$ values. If each entry in this column is identical, then the entire process is validated.

Cu target, $\lambda_{\mathrm{K}_{\alpha}}=1.5418 \AA$
$2 \theta \quad \sin ^{2} \theta \quad$ normalized clear (hkl)? $\quad \frac{\sin ^{2} \theta}{h^{2}+k^{2}+l^{2}}$ fractions

| 44.48 | 0.143 | 1.00 | 3 | 111 | 0.0477 |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 51.83 | 0.191 | 1.34 | 4 | 200 | 0.0478 |
| 76.35 | 0.382 | 2.67 | 8 | 220 | 0.0478 |
| 92.90 | 0.525 | 3.67 | 11 | 311 | 0.0477 |
| 98.40 | 0.573 | 4.01 | 12 | 222 | 0.0478 |
| 121.87 | 0.764 | 5.34 | 16 | 400 | 0.0477 |
| 144.54 | 0.907 | 6.34 | 19 | 331 | 0.0477 |
| 155.51 | 0.955 | 6.68 | 20 | 420 | 0.0478 |

Selection Rules for Reflection in Cubic Crystals

| (hkl) | $h^{2}+k^{2}+l^{2}$ | SC | BCC | FCC |
| :---: | :---: | :---: | :---: | :---: |
| 100 | 1 | $\checkmark$ | $\boldsymbol{x}$ | $\boldsymbol{\chi}$ |
| 110 | 2 | $\checkmark$ | $\checkmark$ | $x$ |
| 111 | 3 | $\checkmark$ | X | $\checkmark$ |
| 200 | 4 | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 210 | 5 | $\checkmark$ | x | x |
| 211 | 6 | $\checkmark$ | $\checkmark$ | $\boldsymbol{x}$ |
| 220 | 8 | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 300 | 9 | $\checkmark$ | \% | $\mathfrak{x}$ |
| 310 | 10 | $\checkmark$ | $\checkmark$ | $\mathfrak{N}$ |
| 311 | 11 | $\checkmark$ | \% | $\checkmark$ |
| 222 | 12 | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 320 | 13 | $\checkmark$ | \% | $\boldsymbol{x}$ |
| 321 | 14 | $\checkmark$ | $\checkmark$ | $\boldsymbol{\chi}$ |
| 400 | 16 | $\checkmark$ | $\checkmark$ | $\checkmark$ |





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$120^{\circ}$ rotation


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## 4-fold


$90^{\circ}$

## 4-fold <br> 3-fold


$90^{\circ} \quad 120^{\circ}$

## 4-fold <br> 3-fold <br> 2-fold


$\begin{array}{lll}90^{\circ} & 120^{\circ} & 180^{\circ}\end{array}$






## Taxonomy of Solids

ordered

- unit cell
- periodic
- "crystal"


## disordered

- no building block
- no long-range order
- "glass"

1982 Dan Schechtman (Technion) working at National Institute of Standards and Technology (NIST), Gaithersburg, MD

Al - Mn alloy:

- highly ordered
- symmetries impossible in a true crystal (5-fold rotational symmetry)
- lacks translational symmetry: aperiodic





## 5-fold

$72^{0}$
Courtesy of Dan Shechtman. Used with permission.



5-fold
$72^{0}$
"Mission Impossible" - an example of 5:4 time

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3.091SC Introduction to Solid State Chemistry

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