

Massachusetts Institute of Technology

Department of Physics

Physics 8.033

17 October 2006

Quiz 1

Name: (Last, First) _____ (please print).

Recitation number (circle one): 1 2 3

- Record all answers and show all work in this exam booklet. If you need extra space, use the back of the opposing page.
- All scratch paper must be handed in with the exam, but will not be graded.
- No materials besides pencils and erasers are allowed (no calculators, notes, books, pets, etc.)
- Whenever possible, try to solve problems using general analytic expressions. Plug in numbers only as a last step.
- Please make sure to answer all sub-questions.
- Good luck!

Problem	Max	Grade	Grader
1	25		
2	25		
3	25		
4	25		
Total	100		

- (a). (4 pts) Professor F. Ishy claims to have discovered a new force of magnitude $F = A|\mathbf{r}_1 + \mathbf{r}_2|$ between protons, where \mathbf{r}_1 and \mathbf{r}_2 are the position vectors of the protons and A is a constant that he's trying to get named in his honor.
- (A) Is this force translationally invariant? YES / NO (circle one)
- (B) Is this force invariant under rotations around $\mathbf{r} = \mathbf{0}$? YES / NO (circle one)
- (b). (4 pts) Professor C. Rank claims that a charge at (\mathbf{r}_1, t_1) will contribute to the air pressure at (\mathbf{r}_2, t_2) by an amount $B \sin[C(|\mathbf{r}_2 - \mathbf{r}_1|^2 - c^2|t_2 - t_1|^2)]$, where B and C are constants.
- (A) Is this effect Galilean invariant? YES / NO (circle one)
- (B) Is this effect Lorentz invariant? YES / NO (circle one)
- (c). (1 pt) To 1 significant figure, the speed of light in meters/second is _____
- (d). (2 pts) From which two postulates did Einstein derive special relativity?
- (e). (11 pts) Indicate whether each of the following statements are true or false.
- (A) The proper length of a ruler is Lorentz invariant. TRUE / FALSE (circle one)
- (B) The wave equation is Galilean invariant. TRUE / FALSE (circle one)
- (C) The kinetic energy of a particle is Lorentz invariant. TRUE / FALSE (circle one)
- (D) The acceleration of a particle is Galilean invariant. TRUE / FALSE (circle one)
- (E) A ping-pong ball moving near the speed of light still looks spherical.
TRUE / FALSE (circle one)
- (F) X-rays travel faster than microwaves. TRUE / FALSE (circle one)
- (G) If you could send a signal faster than light, then there's a frame where you could send a signal backward in time. TRUE / FALSE (circle one)
- (H) If two twins are reunited, who is oldest may be frame-dependent.
TRUE / FALSE (circle one)
- (I) No experiment inside an isolated sealed lab in space can determine its orientation.
TRUE / FALSE (circle one)
- (J) No experiment inside an isolated sealed lab in space can determine its velocity.
TRUE / FALSE (circle one)
- (K) No experiment inside an isolated sealed lab in space can determine its acceleration.
TRUE / FALSE (circle one)
- (f). (3 pts) List three pieces of observational evidence supporting special relativity.

Question 2: Pole Vault Deluxe

[25 Points]

Your goal in this problem is to completely fill out the table below.

	S (Chris)	S' (Zoe)	S'' (Train)
x_B			
ct_B			
x_C			
ct_C			

To save time, note that all three frames have the same spacetime origin (at event A) and that there is no need to draw spacetime diagrams.

- (a). **(2 pts)** Based on the text below, fill in the entries corresponding to \mathbf{X}_B and \mathbf{X}'_C in the table above (no calculation needed!). Chris is standing next to a barn taking measurements as Zoe runs through the barn in the x -direction holding a pole horizontally in the direction of motion. In Zoe's frame S' , the pole has length L , the event when the rear of the pole is aligned with the entrance to the barn is $\mathbf{X}'_A = (x'_A, ct'_A) = (0, 0)$, and the location of the front of the pole at that same time is $\mathbf{X}'_C = (L, 0)$. In Chris' frame S , Zoe is running at speed β_1 , the rear of the pole is aligned with the entrance to the barn (event A) at $\mathbf{X}_A = (x_A, ct_A) = (0, 0)$, and the front of the pole is aligned with the barn exit (event B) at $\mathbf{X}_B = (x_B, ct_B) = (L/\gamma_1, 0)$, where $\gamma_1 = 1/\sqrt{1 - \beta_1^2}$.
- (b). **(7 pts)** Compute \mathbf{X}'_B and \mathbf{X}_C and fill in the corresponding entries in the table above.
- (c). **(8 pts)** A train with the rest of the 8.033 students passes by, and Chris measures its speed to be β_2 in the x -direction. The train's frame S'' is aligned such that $\mathbf{X}''_A = (0, 0)$. Compute \mathbf{X}''_B and \mathbf{X}''_C in terms of L , γ_1 , β_1 , γ_2 , and β_2 , and fill in the corresponding entries in the table above.
- (d). **(4 pts)** Show that for $\beta_2 = 0$, the students on the train agree with Chris.
- (e). **(4 pts)** Show that for $\beta_2 = \beta_1$ the students on the train agree with Zoe.

Question 3: Variational calculus: Newton's second law modified

[25 Points]

For a particle of rest mass m_0 in a potential $V(x)$, show that out of all trajectories $x(t)$ between two events A and B , the one maximizing the quantity

$$\int_{t_A}^{t_B} \left[\frac{1}{\gamma} + \frac{V(x)}{m_0 c^2} \right] dt$$

satisfies the relation

$$\frac{d}{dt}(m_0 \gamma \dot{x}) = -V'(x)$$

(the relativistic version of Newton's second law). Here $\gamma \equiv 1/\sqrt{1 - \dot{x}^2/c^2}$, $\dot{x} \equiv dx/dt$ and $V' = dV/dx$.

Question 4: Relativistic Red Sox

[25 Points]

In a parallel universe, the Boston team made the playoffs.

- (a). **(6 pts)** Manny Relativirez hits the ball and starts running towards first base at speed β . How fast is he running, given that he sees third base 45° to his left (as opposed to straight to his left before he started running)? Assume that he is still very close to home plate.
- (b). **(7 pts)** A player standing on third base is wearing red socks emitting light of wavelength λ_{red} . What wavelength does Manny see? What color are the socks according to Manny, in the approximation that $\lambda_{\text{green}} = \lambda_{\text{red}}/2^{1/4}$, $\lambda_{\text{blue}} = \lambda_{\text{red}}/\sqrt{2}$, $\lambda_{\text{violet}} = \lambda_{\text{red}}/\sqrt{3}$?
- (c). **(5 pts)** In Manny's frame, the ball is moving with speed $c/\sqrt{2}$ towards first base. How fast is it going in the rest frame of the stadium?
- (d). **(5 pts)** Later in the game, a squabble erupts. An outfielder catches a fly ball at $(x_A, y_A, z_A, ct_A) = (50\text{m}, 40\text{m}, 2\text{m}, cT)$ (event A) and Manny starts to run from second base at $(x_B, y_B, z_B, ct_B) = (30\text{m}, 30\text{m}, 0, 0)$ (event B), where $cT = 30\text{m}$, which corresponds to $T \approx 90$ nanoseconds. The outfielder claims that Manny is out by virtue of running too soon, whereas Manny claims that B preceded A in his frame. You are the umpire and must settle this. Compute the spacetime interval between A and B and indicate whether it is (circle one) TIMELIKE / SPACELIKE / NULL
- (e). **(2 pts)** Is Manny correct? YES / NO / DEPENDS ON HIS VELOCITY (circle one)
In one sentence, why?