

Problem 3.5

Two tanks with equal volume are connected with a Joule-Thompson valve. Tank I contains an ideal gas at 10 atm pressure and 300K while tank II is empty ($P = 0$). The valve is opened slowly so that gas can flow from tank I into tank II. The valve is closed once the pressure in tank I has reached 5 atm.

- What is the temperature of the gas in tank I when the pressure there is 5atm ?
- What is the temperature of the gas in tank II when the pressure in tank I has reached 5 atm ?
- Prove that this process is irreversible by calculating the total entropy change of the system.

DATA:

C_p for the ideal gas is $7/2 R$

C_v for the ideal gas is $5/2 R$

Note that pressure changes in the tank may be considered to be reversible processes. The expansion across the Joule-Thompson valve is irreversible but conserves enthalpy, as discussed in class. The complete enclosure of the tank is insulated so that no heat exchange with the environment occurs.

Solution

In all thermodynamic problems, it is useful to first identify the boundaries of the system and the resulting restrictions regarding the energy flows that can occur.

From the problem statement, we already identify that the combined $Tank_I - Tank_{II}$ system does not exchange energy with the environment, since the system is rigid (no PV work) and insulated Q . Therefore:

$$\Delta U_{Total} = 0$$

Now the tricky part:

A common source of confusion is the fact that the number of moles in both $Tank_I$ and $Tank_{II}$ are not constant. This means that, if we want to write the first law expression, considering the *instantaneous* amount of gas that is in each tank, the first law would be in a differential equation form, which might be somewhat complicated.

It is much easier, however, to consider the system as follows:

- System 1: $n_I \rightarrow$ Number of moles in the first tank, as soon as the pressure reaches 5 atm..
- System 2: $n_{II} \rightarrow$ Number of moles that end up in the second tank, once the valve is closed.

Once we have defined the system in such a way, it is much easier to find the solution to the problem, as we shall see below:

Part a)

We are told that the Joule-Thompson valve opens very slowly. Additionally, there is no heat exchange with the environment. Therefore, we can use the expression for an adiabatic, reversible

expansion:

$$\frac{T_{I,f}}{T_{I,0}} = \left(\frac{P_{I,f}}{P_{I,0}} \right)^{R/c_P}$$

From this, we can calculate the final temperature of the system n_I , since we already know the initial and final pressures. Note that, since $P_{I,f} > P_{I,0}$, $T_{I,f} < T_{I,0}$. The gas cools as it expands.

Part b)

Since energy is conserved in the system, any change in energy in system n_I has to be related to any change in energy in system n_{II} through:

$$\begin{aligned}\Delta U_{TOT} &= \Delta U_I + \Delta U_{II} = 0 \\ \Delta U_I &= -\Delta U_{II}\end{aligned}$$

We know that a change in the internal energy for an ideal gas is only a function of its temperature change, and therefore:

$$\begin{aligned}\Delta U_I &= n_I c_V \cdot (T_{I,f} - T_{I,0}) \\ \Delta U_{II} &= n_{II} c_V \cdot (T_{II,f} - T_{I,0})\end{aligned}$$

We can then write

$$n_I \cdot (T_{I,f} - T_{I,0}) = n_{II} \cdot (T_{II,0} - T_{II,f})$$

This equation has two unknowns: one is n_{II} and the other is $T_{II,f}$. To calculate n_{II} we have to note that the total number of moles is conserved and that is equal to the initial amount of mass in *Tank I*, before the valve was opened.

$$\begin{aligned}n_{Tot} &= n_I + n_{II} = \frac{P_{I,0} \cdot V}{R \cdot T_{I,0}} \\ n_{II} &= \frac{P_{I,0} \cdot V}{R \cdot T_{I,0}} - \frac{P_{I,f} \cdot V}{R \cdot T_{I,f}}\end{aligned}$$

From this and the previous equation, we obtain $T_{II,f}$. Note that $T_{II,f} > T_{II,0}$.

Part c)

To prove that the *whole* process is irreversible, we have to prove that the entropy of the universe is *greater* than zero, i.e. $\Delta S > 0$:

$$\Delta S_{Universe} = \Delta S_I + \Delta S_{II} + \Delta S_{Environment} > 0$$

Since the total system does not exchange heat with the environment, $\Delta S_{Environment} = 0$. Moreover, since the system n_I undergoes an *adiabatic, reversible* expansion, $\Delta S_I = 0$. Thus, we just have to calculate ΔS_{II} :

Using the generalized expression for the change in entropy for an ideal gas,

$$\Delta S_{II} = n_{II} \left[c_P \ln \left(\frac{T_{II,f}}{T_{II,0}} \right) - R \ln \left(\frac{P_{II,f}}{P_{II,0}} \right) \right]$$

And since, $P_{II,f} = 5 \text{ atm.}$, $P_{II,0} = 10 \text{ atm.}$ and $T_{II,f} > T_{II,0}$, $\Delta S_{II} > 0$ and

$$\Delta S_{Universe} > 0$$

The *whole* system undergoes an irreversible process. The source of irreversibility is the Joule-Thompson valve.