

NAME: _____

COURSE 3.20: MATERIALS AT EQUILIBRIUM
90 minute EXAM, Nov 20, 2000

NOTE: Exam is annotated by instructor to comment on differences in course content between 2000 and this year (2001)

PROBLEM 1 (20 POINTS) _____

PROBLEM 2 (20 POINTS) _____

PROBLEM 3 (20 POINTS) _____

PROBLEM 4 (20 POINTS) _____

PROBLEM 5 (20 POINTS) _____

TOTAL (100 POINTS) _____

You can either write your answer on the question sheets or use separate pages. In each case make sure your answer is clearly marked.

Problem 1:

Assume a system is at constant temperature T , volume V and number of particles N . Derive the relationship between the heat capacity C_v and the energy fluctuations $\langle (E - \langle E \rangle)^2 \rangle$. You may remember from the first part of the course that C_v is the temperature derivative of the internal energy.

Problem 2

Consider a system of N independent particles with N very large. Each particle has only two energy levels, 0 and ϵ .

- a) Find the number of microstates, Ω_M , with total energy $E = M\epsilon$.
- b) By using Stirling's approximation, derive the entropy per particle for this system as function of the fraction of particles, $x\epsilon$, that are in the highest energy state. Sketch the entropy and mark important points (maxima, minima, etc.) on the $x\epsilon$ axis (i.e. at what value of $x\epsilon$ do they occur?).
- c) Calculate the temperature as function of $x\epsilon$. Discuss the result.

Note: Stirling approximation is: $\ln(N!) = N \ln(N) - N$

Problem 3

Consider a mixture of A and B molecules.

a) If I choose to write the number of ways one can arrange the A and B molecules on a

model lattice by the formula $\Omega_N = \frac{N!}{N_A!N_B!}$ where N is the total number of molecules

($N_A + N_B$), and express the *configurational* entropy of mixing as, $\Delta S_{mix} = k_B \cdot \ln(\Omega_N)$,

what assumptions have I made for the configurational entropy of mixing? Discuss.

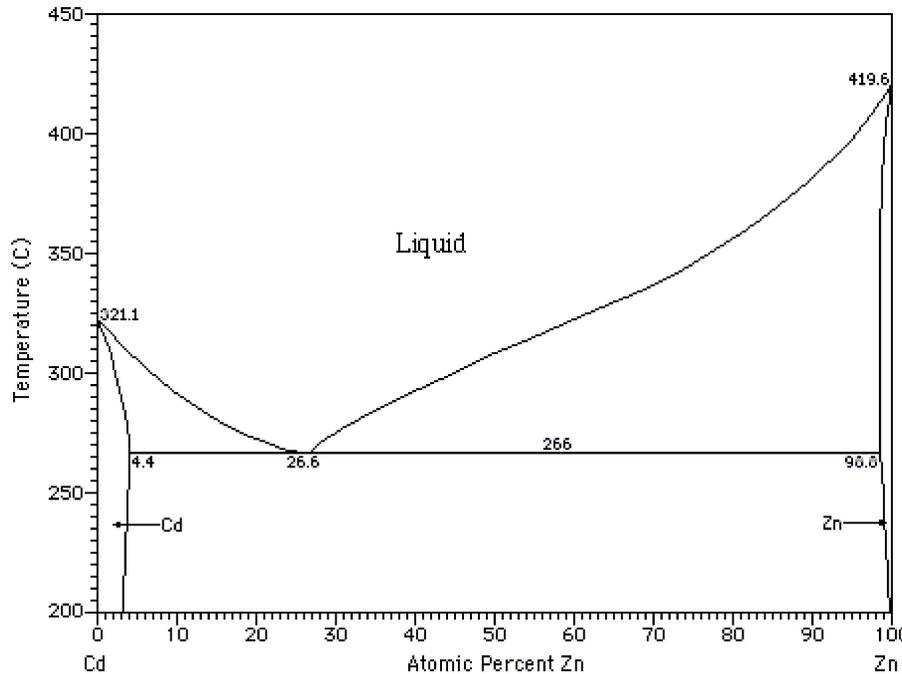
b) If A is a solvent and B is a polymer with n segments, is the above formula for Ω still valid? Discuss. How would you improve it? Discuss. *No equations necessary.*

c) Are there other contributions to the entropy of mixing? Discuss. *No equations necessary.*

NOTE: We may not get to this material in class before exam 2 (in 2001) see further detail in class

Problem 4:

Below is the Cd-Zn phase diagram.



A researcher from University Maxima claims that this phase diagram is incomplete, and that a compound with stoichiometry CdZn with melting point of 350°C exists, but that it usually is not observed because of very difficult nucleation conditions. Draw the phase diagram if the researcher were correct and this compound is present. **Draw the correct phase diagram on top of the diagram shown above so that the relative location of old and new lines is clear.** Explain relative shifts in lines as necessary.

Please make a readable drawing. Extra copies of this question will be available if you want to start over.

**NOTE: We may not get to this material in class before exam 2 (in 2001)
see further detail in class**

Problem 5:

Researchers at MIT are working hard to understand the possible phases of the element Donorium (Do) a recently discovered element which is important because it enhances the properties of Endowium (En) when alloyed with it.

Below, the zero-temperature enthalpy and the Einstein temperature (at 1atm pressure) are given for three crystal structures, speculated to be stable for pure Donorium. Donorium is not magnetic at any temperature.

phase	enthalpy at zero K	Einstein temperature
α	0 J/mole	350K
β	5 kJ/mole	475K
γ	7 kJ/mole	200K

What is the stable state at zero K ? Speculate on the possible phase transitions of Do as function of temperature at constant 1atm pressure. **Justify** your answer (possibly with sketches of free energies) and **explain** why each phase transition occurs. There is no need to perform numerical calculations.