

3.205 Thermodynamics and Kinetics of Materials—Fall 2006

November 2, 2006

Kinetics Lecture 3: Solutions to the Diffusion/Heat Equation

Lecture References

1. Porter and Easterling, *Phase Transformations in Metals and Alloys*, 1981, pp. 71–75.
2. Balluffi, Allen, and Carter, *Kinetic Processes in Materials*, Chapters 4–5.
3. Poirier and Geiger, *Transport Phenomena in Materials Processing*, 1994, see Example 9.8 on p. 310. Also, read Ch. 13, “Diffusion in Solids,” pp. 463–480.

Key Concepts

- Fick’s second law with D independent of c : $\frac{\partial c}{\partial t} = D\nabla^2 c$.
- In one-dimensional diffusion when $D \neq D(c)$, $\frac{\partial c}{\partial t}$ is proportional to the *curvature* of the $c(x)$ profile.
- *Steady-state* conditions by definition have $\frac{\partial c}{\partial t} = 0$, and when $D \neq D(c)$, c is thus a solution to Laplace’s equation $\nabla^2 c = 0$.
- Fick’s second law is a second-order partial differential equation. To obtain solutions, one needs *two boundary conditions* and *one initial condition*.
- When Fick’s second law is linearized (e.g., $D \neq D(c)$), solutions to simpler problems can be superposed to obtain solutions to more complex problems.
- The *error-function solution* describes interdiffusion between two semi-infinite solids for concentration-independent D .
- The *thin-film solution* is valid for a semi-infinite system with a thin deposit applied to the free surface at $t = 0$.
- *Trigonometric series solutions* can be obtained to problems in finite or periodic systems with arbitrary initial conditions.
- It is only possible to obtain algebraic “closed-form” solutions to diffusion problems in *very simple* cases. However, with good software and diffusion data it is possible to solve realistic non-linear problems using numerical methods.
- If a diffusion couple is made from elements or alloys that do not display complete solubility at all intermediate compositions, then the resulting $c(x)$ profile will change discontinuously at a sharp interface in the couple. The compositions on each side of the interface correspond to the tie-line compositions across the two-phase region that must exist between the terminal compositions of the couple. Note that although $c(x)$ changes discontinuously, the chemical potential profile $\mu(x)$ will be continuous at the couple interface.