

3.21 Kinetics of Materials—Spring 2006

April 3, 2006

Lecture 18: Particle Coarsening.

References

1. Balluffi, Allen, and Carter, *Kinetics of Materials*, Sections 15.1 and C.4.

Key Concepts

- The *Gibbs–Thomson equation* relates changes in equilibrium values of chemical potential to interface curvature. In two-phase systems a consequence of the Gibbs–Thomson equation is that solubilities (i.e., tie-line compositions) characteristic of equilibrium in a system with planar interfaces will differ from those in a system consisting of small spherical particles embedded in a matrix.
- In a two-phase system consisting of spherical particles of phase β in a matrix of phase α , concentration gradients will arise in the matrix phase if the β particles are not all *exactly* the same size. The resulting gradients lead to dissolution of the small β -phase particles and growth of larger nearby particles.
- A theory for the volume-diffusion controlled coarsening of a distribution of particles in a matrix can be formulated in the limit of zero volume fraction of the particle phase by using the *mean-field approximation*. Each particle is sufficiently far from its neighbors that it can be assumed to be interacting with a matrix whose far-field concentration is $\langle c \rangle$ (see *KoM* Fig. 15.2). An analysis of the resulting diffusion field, using the Gibbs–Thomson relation as a boundary condition, gives the growth rate of a particle of size R :

$$\frac{dR}{dt} = \frac{2\tilde{D}\gamma\Omega^2 c^{\text{eq}}(\infty)}{kT R} \left(\frac{1}{\langle R \rangle} - \frac{1}{R} \right) \quad (1)$$

This equation predicts that a particle's fate is determined by how its size compares to the mean size $\langle R \rangle$: particles smaller than the mean size dissolve, while those larger than the mean size grow.

- Prediction of the evolution of the *particle size distribution* function $f(R, t)$ is possible from the expressions of flux and accumulation of particle density (see *KoM* Eqs. 15.15 and 15.16). Several important results follow from this analysis, including: (1) the cube of the mean particle size $\langle R \rangle$ varies linearly with coarsening time; (2) a steady-state normalized particle size distribution function $f(R/\langle R \rangle, t)$ is predicted to be approached asymptotically with sufficiently long coarsening time.
- One deficiency of the mean-field theory is the lack of consideration of local particle–particle interactions which become increasingly important as the particle volume fraction takes becomes significant. Detailed analyses show that the rate constant for the $t^{1/3}$ -law is an increasing function of particle volume fraction. Another deficiency is ignoring the possibility of coherency stresses which can lead to many interesting phenomena such as “rafting” and “inverse coarsening” (see *KoM* pp. 372–373).

Related Exercises in *Kinetics of Materials*

Review Exercises 15.1–15.2, pp. 384.