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3.21 Kinetics of Materials—Spring 2006

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Lecture 26: Diffusional Growth.

References

1. Balluffi, Allen, and Carter, *Kinetics of Materials*, Chapter 20.

Key Concepts

- Heat conduction-limited melting of a pure substance serves to illustrate basic principles of formulation of a *moving-boundary problem* in which the system consists of two or more phases separated by a moving interface. Differential equations govern transport within each phase, and the boundary conditions are applied at the borders of the system and at the interphase boundary. Differences in the diffusional fluxes on each side of the interface result in interface motion, described by an equation known as the *Stefan condition.*

- For heat conduction-limited growth the Stefan condition relates the rate at which heat is delivered to the boundary to the latent heat required for a given rate of melting (see *KoM* Eqs. 20.3–4). For one-dimensional heat flow and equal densities in the liquid and solid phases, the temperature profile in each phase has an error-function form (see *KoM* Eq. 20.5) and the boundary condition at the moving interface requires that *the interface advances parabolically in time*. The temperature profiles in the liquid and solid and the parabolic rate constant are given by *KoM* Eqs. 20.8–10, respectively.

- Diffusion-limited growth in one dimension is described in an analogous way to heat conduction-limited growth. In this case the usual approach is to assume equilibrium at the interface and use the equilibrium diagram at the growth temperature to specify the boundary conditions to the diffusion field in each phase. The Stefan condition arises because the concentration on each side of the interface is different, and accumulation of solute at the interface is required for interface motion (see *KoM* Eq. 20.11). Making reasonable simplifying assumptions, the solution to the layer growth problem illustrated in *KoM* Fig. 20.2 parallels that for heat conduction-limited melting, and the interface motion is again parabolic in time (see *KoM* Eqs. 20.18–21)

- Modeling of alloy solidification is considerably more complex because it requires simultaneously solving differential equations for heat flow and mass diffusion, subject to two interface Stefan conditions.

- Growth resulting after nucleation from supersaturated solution often involves three-dimensional evolution of spherical particles. For diffusion controlled-growth the growth kinetics are again parabolic in time (*KoM* Eq. 20.40) with the rate constant η_R given by the solution to the rather messy expression in *KoM* Eq. 20.45. The spatial dependence of c can then be calculated from the function $c(\eta)$ in *KoM* Eqs. 20.43, and $c(r)$ can be calculated at a given time from the scaling relation $\eta \equiv r/(4\widetilde{D}t)^{1/2}$.

Related Exercises in *Kinetics of Materials*

Review Exercises 20.2 and 20.4, pp. 526–527.