

3.21 Kinetics of Materials—Spring 2006

May 1, 2006

Lecture 27: Morphological Stability of Moving Interfaces.

References

1. Balluffi, Allen, and Carter, *Kinetics of Materials*, Chapter 20.
2. *Mathematica* notebook for Lecture 27

Key Concepts

- Perturbation methods can be applied to investigate the morphological stability of particles growing by heat flow and by diffusion. Capillarity acts to make simple interface shapes (e.g., planar and spherical shapes) stable. Both thermal gradients and concentration gradients can act to destabilize the simple shapes, possibly leading to cellular or dendritic growth morphologies.
- Solidification of a single-component liquid is stable if the liquid is everywhere above its melting temperature (see *KoM* Fig. 20.10). If the liquid is undercooled, the solid/liquid interface is unstable because a small protuberance causes the thermal gradient in the liquid to become steeper, amplifying the growth of the protuberance (see *KoM* Fig. 20.11).
- Diffusion-limited growth in an alloy during precipitation is analogous to solidification of an undercooled melt, in that a protuberance on the precipitate/matrix interface locally steepens the concentration gradient in the matrix ahead of the protuberance, tending to amplify its growth.
- In alloy solidification, when the composition of the solid differs from that in the liquid (the usual case), solute rejection into the liquid ahead of the interface may lead to *constitutional supercooling* of the liquid. Melts become constitutionally supercooled when the thermal gradient in the liquid is relatively shallow, and the solute "spike" in the liquid results in liquid compositions that are undercooled by virtue of their solute enrichment (see *KoM* Figs. 20.4 and 20.12).
- The stability of a spherical precipitate growing by volume diffusion-control has been modeled by Mullins and Sekerka using the perturbation method. Spherical harmonic functions are used to describe shape distortions of a sphere (*KoM* Eq. 20.58; also see *Mathematica* notebook for this lecture). The effect of interface curvature on equilibrium concentration at the boundary (Gibbs–Thomson effect) is included (*KoM* Eq. 20.61), thus introducing the stabilizing influence of capillarity into the analysis. A solution to Laplace's equation (invariant-field approximation) is used to derive an expression for the diffusion field in the matrix that is in equilibrium with the perturbed sphere, *KoM* Eq. 20.69). The expression for the rate of change of the amplitude of the perturbations is given by *KoM* Eq. 20.73, and this provides the condition for morphological stability in terms of the "wavelength"  $n$  of the perturbation, the instantaneous size of the growing particle, the concentration gradient  $G$  ahead of the interface, and the capillary stabilizing forces included in the term  $\Gamma$ . The analysis indicates that for sufficiently large particles there will always be a shape instability (see *KoM* Eq. 20.77). In practice, such instabilities are rarely observed in solid/solid precipitation.

Related Exercises in *Kinetics of Materials*

Review Exercise 20.8, pp. 529–531.