

3.21 Kinetics of Materials—Spring 2006

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Lecture 17: Surface Evolution Due to Capillary Forces.

References

1. Balluffi, Allen, and Carter, *Kinetics of Materials*, Section 14.1.

Key Concepts

- If an initially planar isotropic surface of a material is roughened with parallel grooves, its surface area and hence total surface free energy will increase. The roughened surface experiences a *capillary driving force* (i.e., a diffusion potential) equal to  $\gamma(K_1 + K_2) \equiv \gamma\kappa$  such that curved regions of the surface tend to move toward their *center of curvature*. The equations governing the smoothing kinetics depend on the transport mechanism that is active.
- When transport occurs by surface diffusion, the surface flux is proportional to the gradient of curvature along the surface (approximately the third derivative of the surface height profile  $h(x, t)$  with respect to  $x$ ). The rate of change of height is equal to the divergence of the flux, and thus to the fourth derivative of  $h(x, t)$  with respect to  $x$ . The resulting evolution equation (see *KoM* Eq. 14.10) can be solved using separation of variables. For sinusoidal groove shapes the amplitude of the grooves decays as  $\exp[-t/\lambda^4]$  (see *KoM* Eq. 14.13).
- When surfaces evolve by transport through the vapor, the surface moves by a process of evaporation and/or condensation. The normal velocity of the surface is proportional to the vapor flux, and matter in the solid is not conserved. The flux in the vapor is proportional to the difference between the ambient pressure and the vapor pressure in local equilibrium with a surface element with curvature  $\kappa$  (see *KoM* Eq. 14.14). In this case the local velocity of the interface is proportional to local curvature and when the surface is gently curved (i.e., small slope approximation) the resulting evolution equation has the form of Fick's second law (*KoM* Eq. 14.20). *KoM* pp. 342–343 shows how the resulting equation can be applied to *grain boundary grooving* at the junction of a grain boundary with the surface of a polycrystal (see *KoM* Fig. 14.9 for a great micrograph!).
- The morphological stability of a cylindrical body with respect to breakup into a row of droplets is a famous problem solved by Lord Rayleigh in 1878. This is often called the *Rayleigh instability*. It is not difficult to write the equation for the surface area of a cylinder that has a sinusoidal axial perturbation of its radius with the form  $R(z, t) = R_{\text{cyl}}(t) + \epsilon(t) \cos \frac{2\pi z}{\lambda}$ . The resulting analysis shows that there is a *critical wavelength* equal to  $2\pi R_{\text{cyl}}$  below which the area of the cylinder is increased, and above which it is decreased. The cylinder is stable with respect to short-wavelength perturbations, and unstable with respect to long-wavelength perturbations.
- *KoM* Sections 14.1.3 and 14.1.4 develop expressions for the time evolution of the amplitude of sinusoidal perturbations of a cylinder for the vapor transport and surface diffusion mechanisms, respectively. The results of the surface diffusion case are particularly interesting in that there is a *kinetic wavelength* at which the breakup of an unstable cylinder will occur the fastest (see *KoM* Fig. 14.5).

Related Exercises in *Kinetics of Materials*

Review Exercises 14.1–14.3, pp. 354–359.