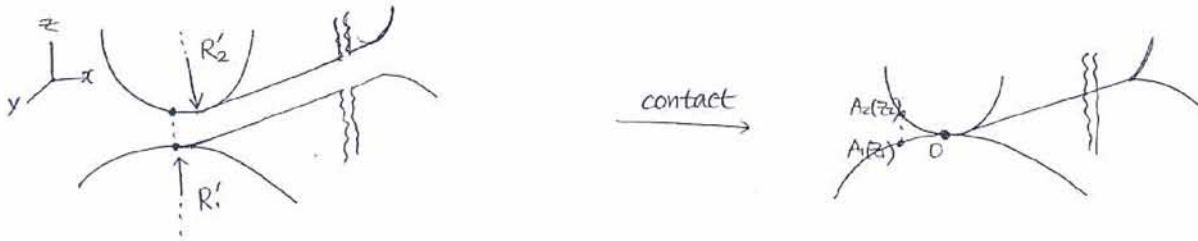


MIT OpenCourseWare
<http://ocw.mit.edu>

3.22 Mechanical Properties of Materials Spring 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

(C) 2-D indentation



In the derivation from PS 2, b in an elliptical contact becomes much larger than a . Thus it becomes a 2-D problem regardless of y .

① • separation distance between A_1 and A_2 is

$$h = z_1 + z_2 = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) x^2 = \frac{1}{2R} x^2 \quad \text{where } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

• then sum of plastic displacement is

$$\bar{u}_{z1} + \bar{u}_{z2} = \delta - h = \delta - \frac{x^2}{2R} \quad \dots \text{ (a)}$$

② Local contact stresses

$$\frac{\partial \bar{u}_{z1}}{\partial x} + \frac{\partial \bar{u}_{z2}}{\partial x} = -\frac{x}{R} = -\frac{2(1-\nu_1^2)}{\pi E_1} \int_{-a}^a \frac{p(s)}{x-s} ds - \frac{2(1-\nu_2^2)}{\pi E_2} \int_{-a}^a \frac{p(s)}{x-s} ds \quad \dots \text{ (b)}$$

where $p(x)$ is a ^{normal} pressure acting on the strip $-a \leq x \leq a$.

$$\text{When } \frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2},$$

$$-\frac{x}{R} = -\frac{2}{\pi E^*} \int_{-a}^a \frac{p(s)}{x-s} ds \quad \therefore \int_{-a}^a \frac{p(s)}{x-s} ds = \frac{\pi E^*}{2R} x \quad \dots \text{ (c)}$$

Taken the distribution of pressure for granted,

$$\text{then } p(x) = -\frac{\pi E^*}{2R} \frac{x^2 - a^2/2}{\pi \sqrt{a^2 - x^2}} + \frac{P}{\pi \sqrt{a^2 - x^2}} \quad \dots \text{ (d)}$$

To get uniqueness of p , use boundary condition of $p(x=\pm a) = 0$.

$$\therefore p(x=\pm a) = \lim_{x \rightarrow \pm a} \frac{1}{\pi \sqrt{a^2 - x^2}} \left(-\frac{\pi E^*}{4R} a^2 + P \right) = 0$$

$$\therefore P = \frac{\pi a^2 E^*}{4R} \quad \text{or} \quad a^2 = \frac{4PR}{\pi E^*} \quad \dots \textcircled{e}$$

$$p(x) = \frac{2P}{\pi a^2} \sqrt{a^2 - x^2} \quad \dots \textcircled{f} \quad (\text{plug } \textcircled{e} \text{ into } \textcircled{d})$$

• stresses within two solids

$$\begin{cases} \sigma_x = -\frac{2z}{\pi} \int_{-a}^a \frac{p(s)(x-s)^2}{\{(x-s)^2 + z^2\}^2} ds \\ \sigma_z = -\frac{2z^3}{\pi} \int_{-a}^a \frac{p(s)}{\{(x-s)^2 + z^2\}^2} ds \end{cases} \quad \dots \text{principal stresses } \textcircled{g}'$$

\(\therefore\) in this case

$$\begin{cases} \sigma_x = -\frac{p_0}{a} \left\{ \frac{a^2 + 2z^2}{\sqrt{a^2 + z^2}} - 2z \right\} \\ \sigma_z = -\frac{p_0}{a} \frac{1}{\sqrt{a^2 + z^2}} \end{cases} \quad \dots \text{principal stresses } \textcircled{g}$$

where p_0 (maximum pressure where $x=0$) = $\frac{2P}{\pi a} = \frac{4}{\pi} p_m = \sqrt{\frac{PE^*}{\pi R}}$
 \overline{p} mean pressure

and principal shear stress

$$\tau_1 = -\frac{2z^2}{\pi} \int_{-a}^a \frac{p(s)(x-s)}{\{(x-s)^2 + z^2\}^2} ds = -\frac{p_0}{a} \left(z - \frac{z^2}{\sqrt{a^2 + z^2}} \right) \quad \dots \textcircled{h}$$

• the maximum shear stress

$$\frac{\partial \tau_1}{\partial z} = 0 \quad \rightarrow \quad \text{then } (\tau_1)_{\max} = 0.30 p_0 \quad \text{at } \underline{z = 0.707a}$$

↓
depth for maximum shear stress