

Your Name: \_\_\_\_\_

### 3.225 Quiz 2005

$$e=1.602 \times 10^{-19} \text{ C} \quad m_0=9.11 \times 10^{-31} \text{ kg} \quad c=2.998 \times 10^8 \text{ m/sec} \quad \epsilon_0=8.854 \times 10^{-12} \text{ F/m}$$

$$k_B=1.38 \times 10^{-23} \text{ J/K} \quad h=6.626 \times 10^{-34} \text{ J-sec} \quad \hbar=1.054 \times 10^{-34} \text{ J-sec} \quad A=6.022 \times 10^{23} \text{ mole}^{-1}$$

#### **1. 2-D Material**

Consider a 2-D crystal, size  $L \times L$ . The lattice constant is  $a$ .

(a) Sketch the first zone of the reciprocal lattice, which is a square, and draw the Fermi surface for valence 1 and valence 2.

(b) Now draw a separate sketch of  $E$  vs.  $k$  along the  $[10]$  and  $[11]$  directions. Sketch only the first and second bands. Indicate  $k_F$  and  $E_F$ , for valence ( $Z$ ) equal to 1 and valence equal 2 cases. Assume  $U$  is about 25% of the  $E_F(Z=1)$ .

(c) Describe the expected conductivity in  $Z=1$  and  $Z=2$  material at room temperature and  $T=0$  K.

(d) When the size of the crystal shrinks to  $L=8a$ , derive an equation (for a general valence  $Z$ ) that defines the set of points on the Fermi surface that remain valid conducting states in the new smaller material.

## **2. Free electron model in aluminum**

(a) Determine the conductivity, scattering length, and reflection spectra (reflection vs. wavelength) of Al using the Drude/free electron model assuming a scattering time of  $10^{-14}$  seconds. Al is valence 3.

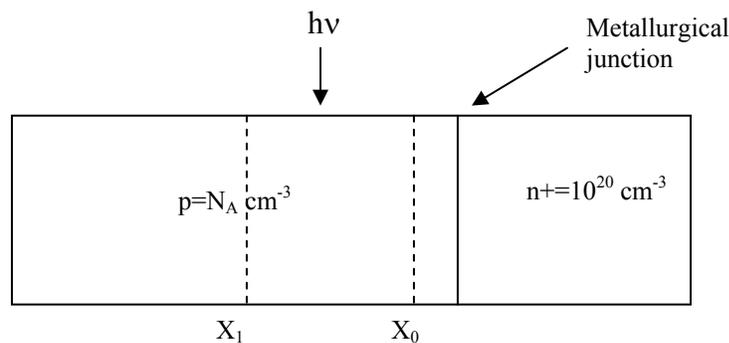
(b) How does the scattering length compare with inter-atomic distance? What dislocation density would be needed to decrease the conductivity by a factor of 10? What effective size of dislocation would be necessary to achieve this density? Is this possible?

(c) Determine the conductivity of Si (atomic number 14) using the Drude model and compare it to the value you expect for Al. Does this value match experiment? Explain.

### 3. Steering Photons

Methods of switching propagating directions of photons are an active area of research, since optical switching in the network is a desirable attribute. One way to do this is to change the carrier concentration in a semiconductor through an applied potential. The index of refraction is a function of the free carrier concentration, and therefore the index can be changed through application of an electrical potential. This can lead to photon switching in optimized device geometry.

A simplified concept of this device is simply a p-n junction, shown schematically below.



Note this is a n<sup>+</sup>-p junction. Without reverse bias, the depletion region on the p-side is labeled X<sub>0</sub>, whereas when a reverse bias V<sub>R</sub> is applied, it becomes X<sub>1</sub>. Light is entering as shown by the arrow.

A. Describe how this device can affect the propagation of the light in the material by introducing an index change.

B. Assume that we need at least one micron between  $X_0$  and  $X_1$  in order to have the light propagate sufficiently. For a given reverse bias  $V_A$ , determine an expression that relates  $N_A$  to  $V_R$ .

#### **4. Dielectrics**

Describe why non-conducting materials have an index of refraction that is greater than the value 1, and also why such materials tend to be transparent. For such a material, describe the expected frequency dependence of the dielectric constant.