

**PROBLEM SET – 3**Tuesday, November 27th, 2007

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Problem 1.

If  $E(k)$  in a 2D square lattice crystal equals  $4C - 2C \cos(k_x a) - 2C \cos(k_y a)$ , calculate the values of  $k$  along the  $\pm k_x$  axis ( $k_y = 0$ ) corresponding to  $E = C, 2C, 3C$ , and  $4C$  and along the  $k_x = k_y$  line corresponding to these four values plus  $E = 5C, 6C, 7C$ , and  $8C$ . Put the resulting data on an  $E(k)$  plot and draw smooth lines indicating  $E(k)$  for the [10] and [11] directions.

Problem 2.

In the 2D square lattice of problem 1, what number of electrons per atom corresponds to the energy  $E = 4C$  at which the Fermi surface first touches the boundaries of the 1<sup>st</sup> Brillouin zone? At that energy, what is the group velocity of electrons in the  $\langle 10 \rangle$  and  $\langle 11 \rangle$  directions?

Problem 3.

In the 2D square lattice of problem 1, calculate the group velocity of electrons in the [11] direction as a function of  $k$ .

Problem 4.

The density of occupied states in the conduction band goes through a maximum slightly above the bottom of the band. Calculate the energy separation (in  $eV$ ) between the position of this maximum and the bottom of the band at  $T = 300$  K.

Problem 5.

A group IV semiconductor is doped with  $10^{19}$  donor atoms per  $\text{cm}^3$ . The intrinsic carrier concentration at  $T = 300$  K is  $2.3 \times 10^{13}$  per  $\text{cm}^3$ , and  $N_c = N_v = 7 \times 10^{16}$  per  $\text{cm}^3$ . (a) At 300 K, the donor atoms are essentially fully ionized. What is the hole concentration at 300 K? (b) This semiconductor has a dielectric constant of 3. What is its plasma frequency at 300 K? (c) Calculate the position of the absorption edge corresponding to interband transitions? (d) Sketch the light absorption of this semiconductor as a function of photon energy, identifying quantitatively the energies of any opaque-transparent and/or transparent-opaque transitions.

Problem 6.

In metals and intrinsic semiconductors, the Fermi energy is independent of temperature. In extrinsic semiconductors, however, the Fermi energy varies with temperature in the low-temperature range in which the donor or acceptor atoms become ionized. In an n-type semiconductor, what is the position of  $E_F$  with respect to  $E_D$  at the following temperatures: (a) temperature,  $T_1$ , where only 10% of the donor atoms are ionized, (b) temperature,  $T_2$ , where 50% are ionized, and (c) temperature,  $T_3$ , where 90% are ionized?