Metals and Insulators

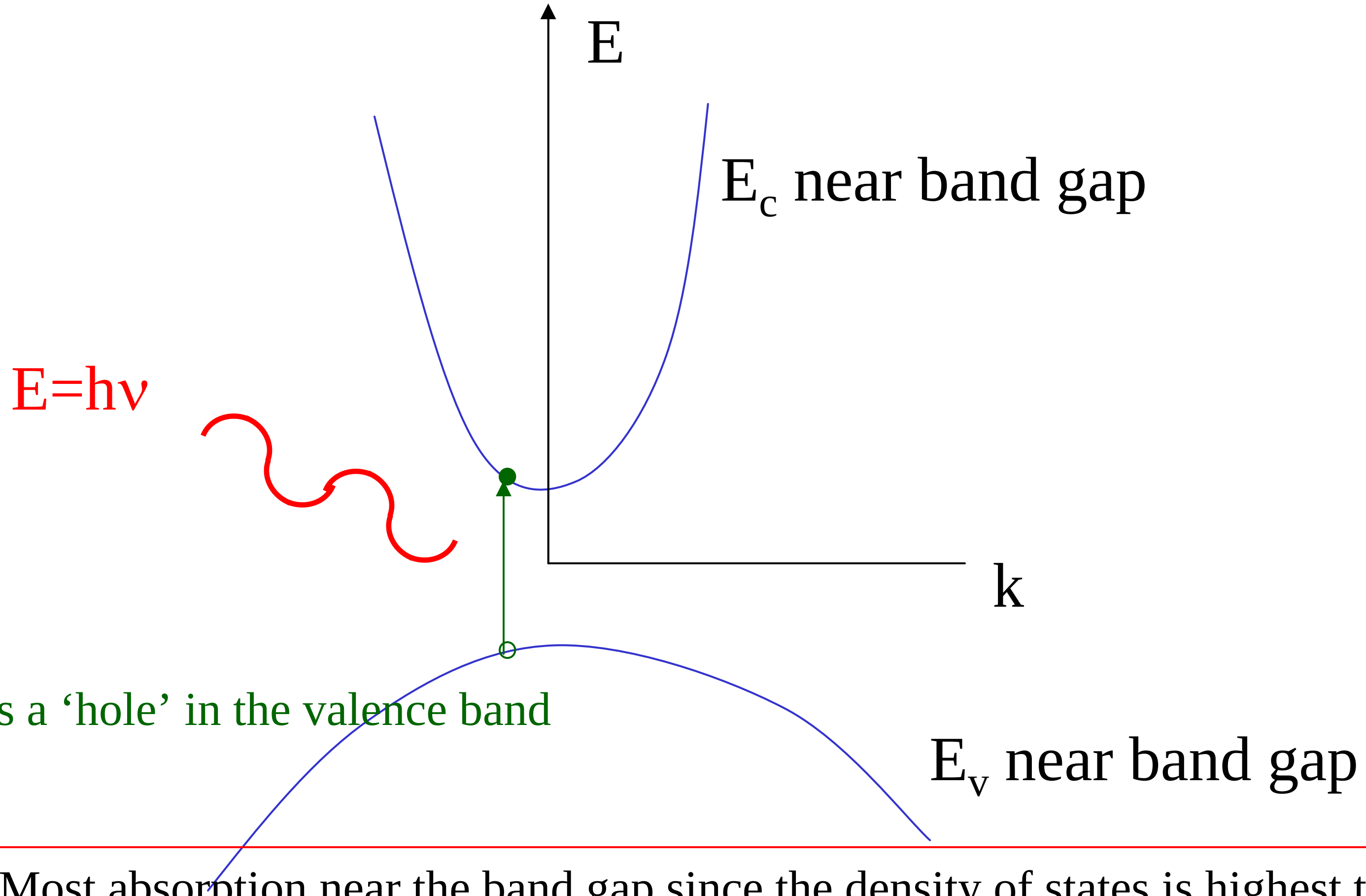
- Covalent bonds, weak U seen by e-, with E_F being in mid-band area: free e-, *metallic*
- Covalent or slightly ionic bonds, weak U to medium U, with E_F near band edge
  - E_F in or near kT of band edge: *semimetal*
  - E_F in gap: *semiconductor*
- More ionic bonds, large U, E_F in very large gap, *insulator*

Insulators

- Very large band gaps=no conduction electrons at reasonable temperatures
- All electrons are bound
- Optical properties of insulators are derived from the electric field being able to temporarily move electrons: polarization
- We will return to the interaction of E-field with bound electrons in Dielectrics Section

Semiconductors: Photon Absorption

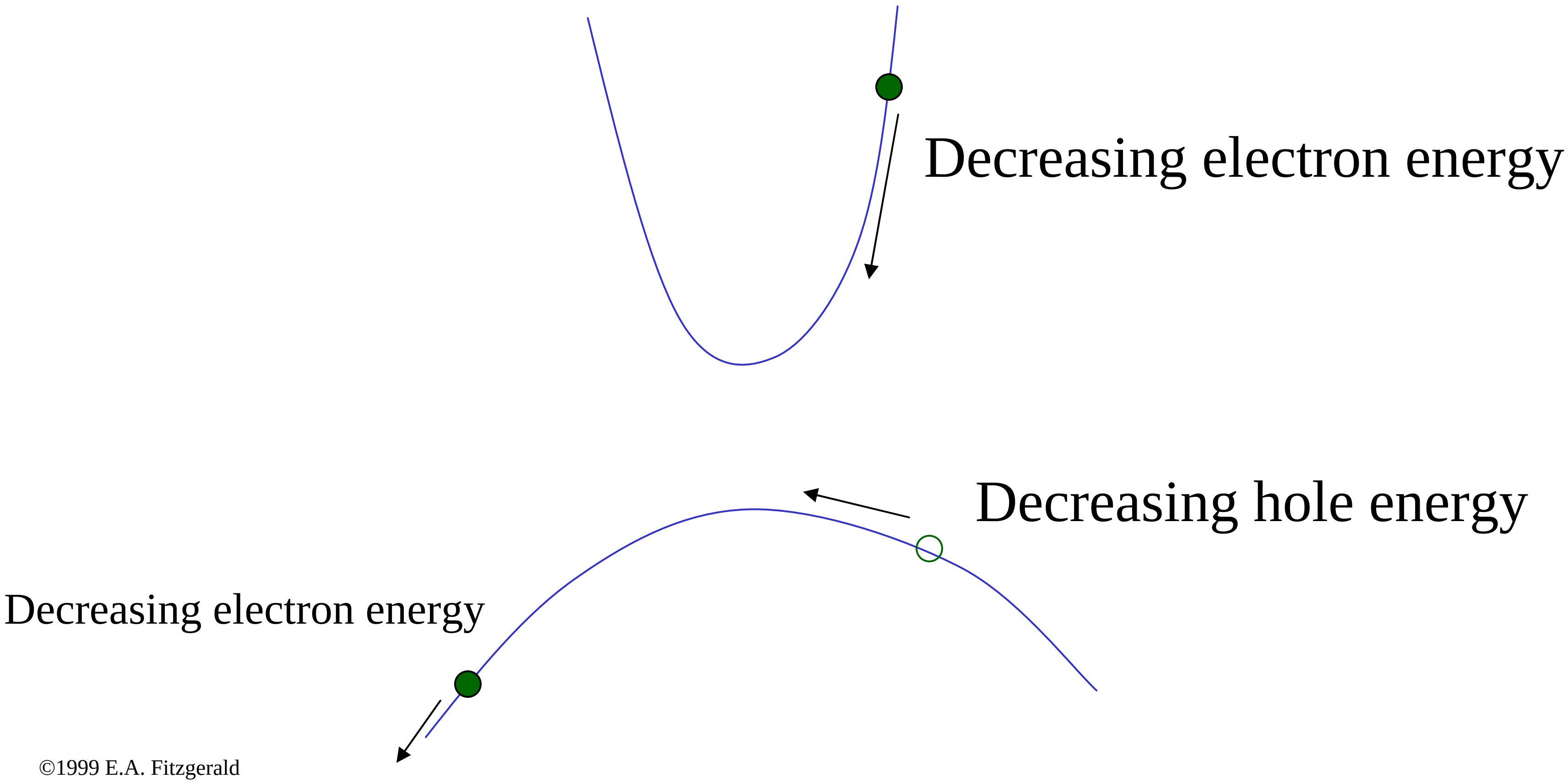
- When $E_{light}=h\nu>E_g$, an electron can be promoted from the valence band to the conduction band



Note: Most absorption near the band gap since the density of states is highest there

Holes and Electrons

- Instead of tracking electrons in valence band, more convenient to track vacancies of electrons, or ‘holes’
- Also removes problem with negative electron mass: since hole energy increases as holes ‘sink’, the mass of the hole is positive as long as it has a positive charge
- Both carriers at the band edge can be thought of as classical free carriers like the Drude model had, as we shall see



Decreasing electron energy

Decreasing hole energy

Decreasing electron energy

Conductivity of Semiconductors

- Need to include both electrons and holes in the conductivity expression

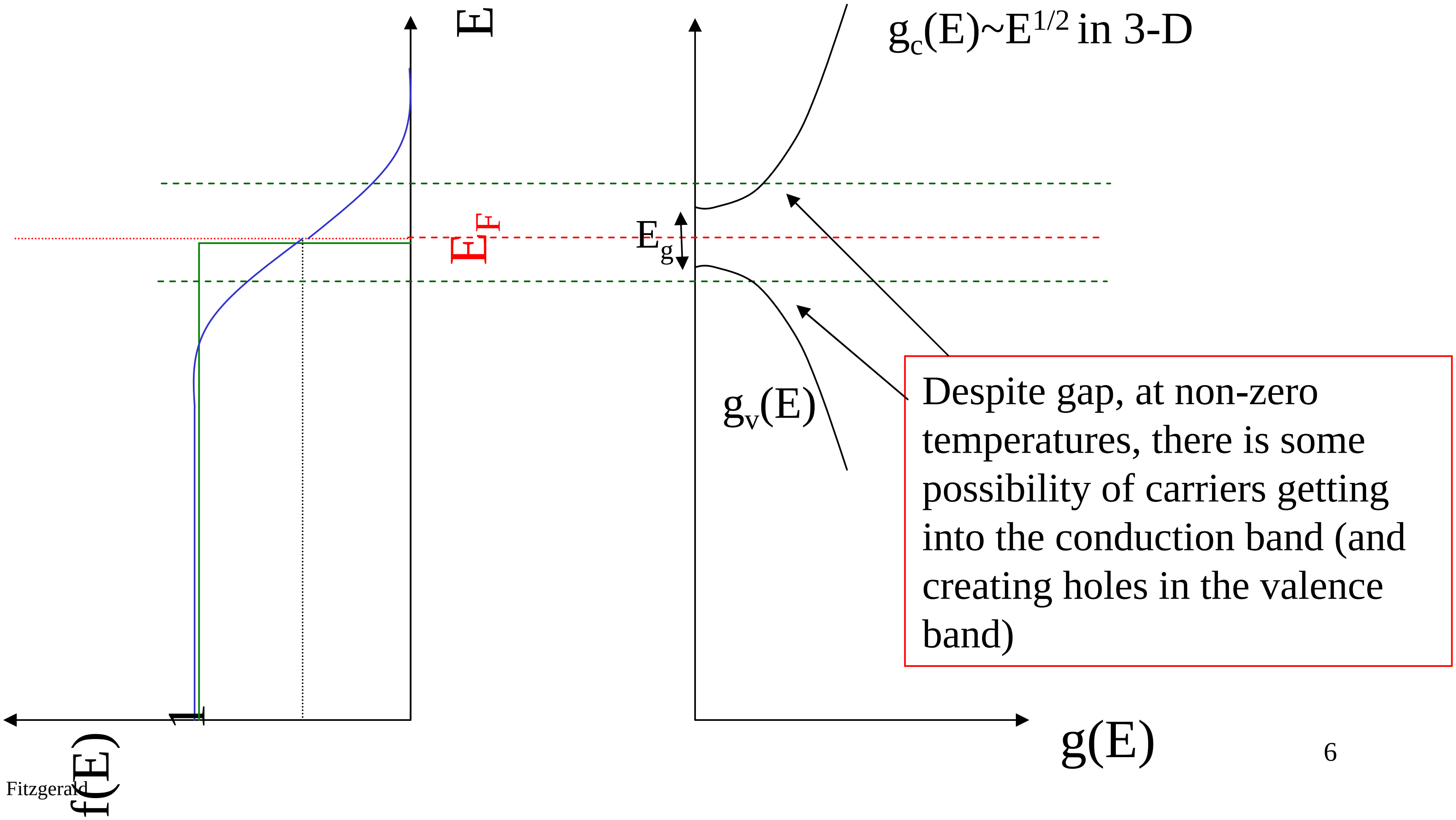
$$\sigma = ne\mu_e + pe\mu_h = \frac{ne^2\tau_e}{m_e^*} + \frac{pe^2\tau_h}{m_h^*}$$

p is analogous to n for holes, and so are τ and m*

Note that in both photon stimulated promotion as well as thermal promotion, an equal number of holes and electrons are produced, i.e. n=p

Thermal Promotion of Carriers

- We have already developed how electrons are promoted in energy with T: Fermi-Dirac distribution
- Just need to fold this into picture with a band-gap



Despite gap, at non-zero temperatures, there is some possibility of carriers getting into the conduction band (and creating holes in the valence band)

Density of Thermally Promoted of Carriers

Number of electrons per volume in conduction band

$$n = \int_{E_c}^{\infty} \underbrace{f(E)}\underbrace{g(E)}dE$$

Density of electron states per volume per dE

Fraction of states occupied at a particular temperature

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{k_bT}} + 1} \approx e^{-\frac{(E-E_F)}{k_bT}} \quad \text{when} \quad (E - E_F) >> k_bT$$

$$g_c(E) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{\frac{3}{2}}\left(E - E_g\right)^{\frac{1}{2}} \longrightarrow n = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{\frac{3}{2}} e^{\frac{E_F}{k_bT}} \int_{E_g}^{\infty}\left(E - E_g\right)^{\frac{1}{2}} e^{\frac{-E}{k_bT}} dE$$

Since $\int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx = \frac{\sqrt{\pi}}{2}$, then $n = \underbrace{2\left(\frac{m_e^* k_bT}{2\pi\hbar^2}\right)^{\frac{3}{2}}}_{N_C} e^{\frac{E_F}{k_bT}} e^{\frac{-E_g}{k_bT}}$

$$\boxed{n = N_C e^{\frac{E_F - E_g}{k_bT}}}$$

Density of Thermally Promoted of Carriers

- A similar derivation can be done for holes, except the density of states for holes is used
- Even though we know that n=p, we will derive a separate expression anyway since it will be useful in deriving other expressions

$$g_v(E) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{\frac{3}{2}}(-E)^{\frac{1}{2}} \longrightarrow p = \int_{-\infty}^{0} f_h(E)g_v(E)dE, \text{ where } \quad f_h = 1 - f(E)$$

$$p = 2\left(\frac{m_h^* k_b T}{2\pi\hbar^2}\right)^{\frac{3}{2}} e^{\frac{-E_F}{k_b T}}$$

$$\boxed{p = N_v e^{\frac{-E_F}{k_b T}}}$$

Thermal Promotion

- Because electron-hole pairs are generated, the Fermi level is approximately in the middle of the band gap
- The law of mass action describes the electron and hole populations, since the total number of electron states is fixed in the system

$$n = p \text{ gives } E_F = \frac{E_g}{2} + \frac{3}{4} k_b T \ln\left(\frac{m_h^*}{m_e^*}\right)$$

Since m_e* and m_h* are close and in the ln term, the Fermi level sits about in the center of the band gap

$$p \text{ or } n = n_i = 2\left(\frac{k_b T}{2\pi\hbar^2}\right)^{\frac{3}{2}} \left(m_e^* m_v^*\right)^{\frac{3}{4}} e^{\frac{-E_g}{2k_b T}}$$

Law of Mass Action for Carrier Promotion

$$n_i^2 = np = 4\left(\frac{k_bT}{2\pi\hbar^2}\right)^3 \left(m_e^* m_h^*\right)^{\frac{3}{2}} e^{\frac{-E_g}{k_bT}} \quad ; \qquad n_i^2 = N_C N_V e^{\frac{-E_g}{k_bT}}$$

- Note that re-arranging the right equation leads to an expression similar to a chemical reaction, where E_g is the barrier
- $N_C N_V$ is the density of the reactants, and n and p are the products

$$\left[N_C N_V\right] \xrightarrow{E_g} [n] + [p]$$

$$\frac{[n][p]}{\left[N_C N_V\right]} = e^{\frac{-E_g}{k_bT}} = \frac{[n_i]^2}{\left[N_C N_V\right]}$$

- Thus, a method of changing the electron or hole population without increasing the population of the other carrier will lead to a dominant carrier type in the material
- Photon absorption and thermal excitation produce only pairs of carriers: *intrinsic semiconductor*
- Increasing one carrier concentration without the other can only be achieved with impurities, also called doping: *extrinsic semiconductors*

Intrinsic Semiconductors

- Conductivity at any temperature is determined mostly by the size of the band gap
- All intrinsic semiconductors are insulating at very low temperatures

Recall: $$\sigma = ne\mu_e + pe\mu_h = \frac{ne^2\tau_e}{m_e^*} + \frac{pe^2\tau_h}{m_h^*}$$

$$\sigma_{\text{int}} = n_i e\left(\mu_e + \mu_h\right) \propto e^{\frac{-E_g}{2k_bT}}$$

This can be a measurement for E_g

For Si, Eg=1.1eV, and let μ_e and μ_h be approximately equal at 1000cm^2/V-sec (very good Si!)

$\sigma \sim 10^{10}$cm^{-3}*1.602x10^{-19}*1000cm^2/V-sec=1.6x10^{-6} S/m, or a resistivity ρ of about 10^6 ohm-m max

- One important note: No matter how pure Si is, the material will always be a poor insulator at room T
- As more analog wireless applications are brought on Si, this is a major issue for system-on-chip applications
  - E-M waves lose strength since e- are responding to wave: loss and low Q resonant circuits

Extrinsic Semiconductors

- Adding ‘correct’ impurities can lead to controlled domination of one carrier type
  - n-type is dominated by electrons
  - p-type if dominated by holes
- Adding other impurities can degrade electrical properties

Impurities with close electronic structure to host

Impurities with very different electronic structure to host