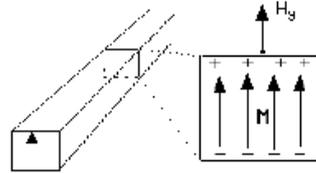


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EXAM Solutions
April 5, 2000

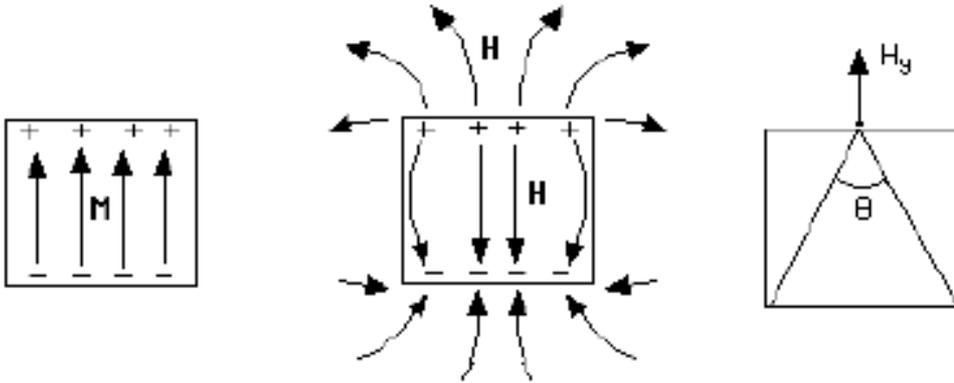
1. (25 points) A long permanent magnet has a remanent induction of $\mu_0 M_r = 0.64\text{T}$ in the y direction. Its cross section in the x - y plane is square measuring $a \times a$ and its length is very large in the z direction.

- a) Sketch the \mathbf{H} vector fields near the top and bottom surfaces. (10)
- b) Calculate the y component of the field $\mu_0 H_y$ at the midpoint just outside the top surface.



Show your equations and get a numerical result (the numbers were chosen so a calculator would not be necessary). (15)

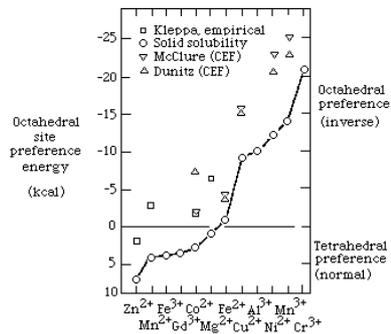
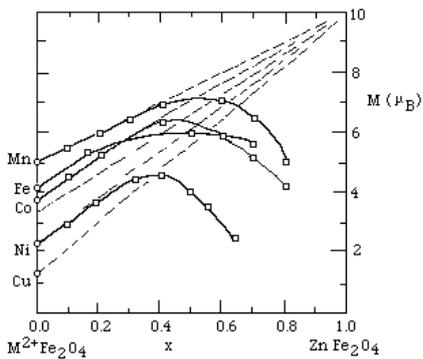
Ans. 1 a) field emanates from + (N) poles and terminates on - (S) poles as sketched.



b) From top surface ($\sigma > 0$) where $\theta = \pi$: $\mu_0 H_y = \frac{\mu_0 \sigma}{2\pi} \theta = \frac{0.64}{2\pi} \pi = 0.32\text{T}$

From bottom surface ($\sigma < 0$), $\theta/2 = \arctan(0.5)$, $\theta = 53^\circ = 0.93 \text{ rad}$, $\mu_0 H_y = -0.94 \text{ T}$.
Total field just outside center of top surface is $\mu_0 H_y = +0.23 \text{ T}$.

2. (25) Consider the data below.



- a) Explain why the magnetic moment per formula unit (FU) increases when small amounts of ZnO are substituted for MnO in the ferrite MnO-Fe₂O₃. (10)
- b) Why does the moment per FU then start decreasing when the Zn concentration exceeds about 50 %? (10)

Your answers should include one sentence about each of the following: cation site types, moment per transition metal ion, site selection, the nature of the exchange coupling in this system, and the nature of the exchange couple between magnetic species on different or similar sites.

- c) Speculate about why the peak in moment per FU occurs in Ni-Zn-ferrite at a smaller Zn concentration than it does in Mn-Zn-ferrite. (Ans. not in text) (5)

Ans. 2 a) There are two cation sites in the spinel structure, tetrahedral (A) and octahedral (B). One cation per FU occupies an A site and two occupy B sites. The different transition metal ions prefer these sites as indicated in the figure. The moments on the ions of interest are $\text{Fe}^{2+} = 4$, $\text{Fe}^{3+} = 5$, $\text{Mn}^{2+} = 5$, $\text{Zn}^{2+} = 0$ and $\text{Ni}^{2+} = 2$ Bohr magnetons. The exchange interactions are of the antiferromagnetic (AF) superexchange type, i.e. cation moments coupled through the p orbitals of an intervening anion, in this case, oxygen. The exchange interactions, in order of decreasing strength, are A-B, B-B, A-A no interaction.

In MnOFe_2O_3 :	A	B	
	Mn^{2+}	2Fe^{3+}	
	-5	+10 μ_B	net moment is +5.

Introduction of Zn^{2+} displaces Mn^{2+} from A site to B site, increasing the net moment.

b) At some Zn concentration, the A-B AF exchange interaction, proportional to $J_{A-B}S_A S_B$ becomes weaker than the B-B AF exchange interaction because of the smallness of S_A . At that point, the B-site moment decreases and the net moment begins to decrease.

c) In Ni ferrite, the Ni ion occupies the B site with one of the Fe^{3+} ions; the other Fe^{3+} is on the A site. The A-B exchange interaction energy is weaker than that in Mn-ferrite because of the smaller Ni moment. Thus, it gives way to the antiferromagnetic A-A interaction at a lower Zn concentration.

3. (25) You make some HCP cobalt and some HCP cobalt-rich magnetic alloys. You measure their anisotropy constants to be as follows:

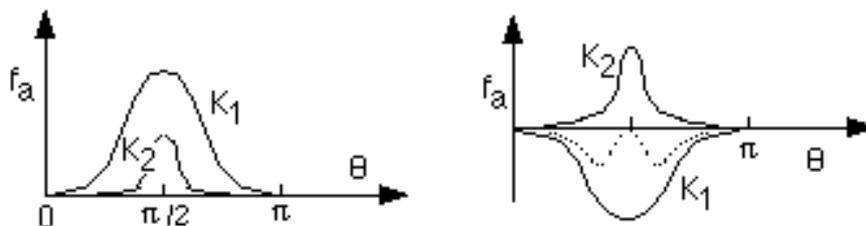
Co:	$K_{u1} = 4.1 \times 10^5 \text{ J/m}^3$, $K_{u2} = 1.5 \times 10^5 \text{ J/m}^3$.
Co-rich alloy A:	$K_{u1} = -5.0 \times 10^5 \text{ J/m}^3$, $K_{u2} = +5 \times 10^5 \text{ J/m}^3$.

Your data were analyzed using the form $f_a = K_0 + K_{u1} \sin^2\theta + K_{u2} \sin^4\theta + \dots$

- What is the stable direction (in $H = 0$) of magnetization in the Co? (5)
- What is the stable direction (in $H = 0$) of magnetization in Co-rich alloy A? (show your work) (10)
- In alloy A, you measure $\lambda_{0001} = +100 \times 10^{-6}$. Give an expression and a numerical result for the stress along the c axis that is just sufficient to bring \mathbf{M} into the base plane ($\theta = 90^\circ$). Identify the stress as tensile or compressive. Use (3/2) $\lambda\sigma$ form not $B\epsilon\alpha\alpha\dots$) (10)

Ans. 3 a) \mathbf{M} is stable at $\theta = 0, \pi, \dots$ for Co from the minima in f_a .

b) For alloy A, see sketch below, right.

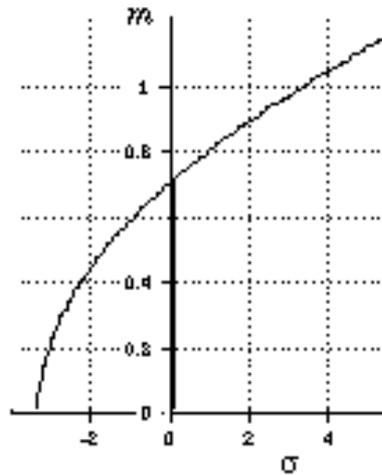


- b) In the case of alloy A, $\frac{df}{d\theta} = 0 = 2K_{u1} \cos\theta \sin\theta + 4K_{u2} \cos\theta \sin^3\theta$. You can divide by $\sin\theta \cos\theta$ because it is obvious from the figure that it is not zero near the energy minimum. Thus, $\sin^2\theta = -K_{u1}/2K_{u2} = 0.5$, $\theta = 45^\circ$.
- c) Putting $\theta = 45^\circ$ into f_a gives for the depth of the energy well in alloy A $f_a(45) - f_a(0) = -1.25 \times 10^5 \text{ J/m}^3$. Given the positive magnetostriction, it should be clear that a compressive stress along the c axis will tend to stabilize \mathbf{M} in the base plane. The stress energy stabilizing \mathbf{M} at $\theta = 90^\circ$ is $(3/2)\sigma\lambda_c$ which could be equated to $f_a(45) - f_a(0)$. Thus, $\sigma = -1.25 \times 10^5 / (1.5 \times 10^{-4})$, $\sigma = 0.83 \times 10^9 \text{ Nm}^2 = 0.83 \text{ GPa}$.

More accurately, we should minimize the anisotropy plus magnetoelastic energies and get the full solution. We express the angular dependence of the ME energy as $-(3/2)\lambda\sigma \sin^2\theta$ because a compressive stress stabilizes $\theta = 90^\circ$. Energy minimization gives

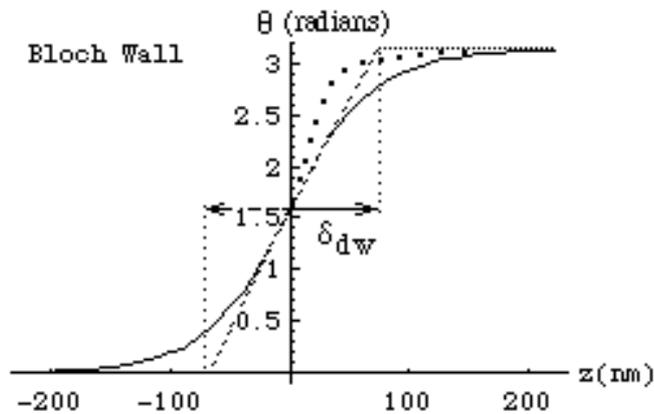
$$\cos\theta \left[\sin^2\theta - \frac{3\lambda\sigma - 2K_{u1}}{4K_{u2}} \right] = 0.$$

Equating the square bracket term to zero shows that the stress dependence of the in-plane component of magnetization, namely $m = \sin\theta$, has a zero-stress value of 0.707 as we expect from its 45° equilibrium orientation. Upon increasing the stress, $\sin\theta$ increases like the square root of σ , reaching unity (saturation) at $\sigma = (4K_{u2} + 2K_{u1}) / (3\lambda) = \underline{3.3 \text{ GPa}}$.



4) (25) On your answer sheet, sketch (as a solid line) the Bloch domain wall profile below which applies for a material with $f_a = K_{u1} \sin^2\theta$.

a) Use your knowledge of the equilibrium torque balance at every point in a domain wall to sketch (as a dashed line) what happens to the domain wall profile if $K_{u1} = 0$ and $K_{u2} > 0$. Explain in one or two sentences. (10).



- b) Use order-of magnitude estimates and scaling arguments to determine which defect would have a greater pinning effect on a domain wall in cobalt and why (you'll need to calculate the domain wall width in Co - use $A = 10^{-11}$ J/m - and look up a constant in Ch. 7):
- an interstitial impurity that produces a strain field dropping from $e = 1\%$ and extending for about 60 nm about its site, or an Fe substitutional defect that has a magnetic moment 30% greater than that of the Co atom it replaced? (10)
 - A region about 60 nm in diameter in which the stacking faults changed the cobalt structure to FCC (and thereby reduces the magnetocrystalline anisotropy by an order of magnitude in that region) or the stress field above in i). (5)

Ans. 4. a) The zero torque condition is $\frac{\partial f_a}{\partial \theta} = A \frac{\partial^2 \theta}{\partial z^2}$. The second order anisotropy function is flatter (smaller slope) at small angles and steeper near 70° than is the first order anisotropy function. Therefore, the *curvature* of the second-order function is greater between these limits. The shape of the second order anisotropy Bloch wall is suggested by the bold dots on part of the figure above.

b) Wall width is $\delta_{dw} = \pi(1 \times 10^{-11} / 4.1 \times 10^5)^{0.5} = 1.5 \times 10^{-8}$ m.

i) Strain field produces a magnetoelastic defect of approximate strength $B_1 e = 6 \times 10^6 \times 0.01$ J/m³ with a range of order 60 nm. In this large fuzzy defect case H_c is proportional to $(\delta_{dw}/D)(\Delta K_\sigma/K_u)H_a = (1.5 \times 10^{-8} / 6 \times 10^{-8})(6 \times 10^4 / 4 \times 10^5) = 0.04H_a$. Fe substitutional defect is smaller than the wall width so H_c is proportional to $(D/\delta)(\Delta M/M)H_a = (6 / 1.5)(1.3/1)H_a = 5.3H_a$. Fe defect is more potent.

ii) Large stacking fault presents an abrupt or sharp defect. The coercivity scales with $(\Delta K/K)H_a$ and is independent of defect size (Paul model). H_c is proportional to $(\Delta K/K)H_a = 0.9H_a$, giving a much larger factor than either defect in part i). This is the most potent defect.

Suppl. 2c. The dominant antiferromagnetic A-B exchange goes as $J_{AB}S_A \cdot S_B$. For Mn ferrite and Ni ferrite, the magnitude of the sublattice spins go as $S_A = 5(1-x)$ in both cases and $S_B = 5(2+x)$ and $5(1+x) + 2(1-x)$, respectively. We compare the products of the sublattice magnetizations in the two cases and find that after a common term they vary as $25(1-x)$ for Mn ferrite and $10(1-2x)$ for Ni ferrite. Clearly, the latter is weaker and drops off faster with increasing x than the former. Hence, in Ni-ferrite the AF B-B exchange sets in at smaller x than it does in Mn ferrite.