

## 2. MAGNETOSTATICS

### Problems

2.1 Assume that an interface, similar to that sketched in Fig. 2.7 b but with  $\mu_1/10 = \mu_2 = \mu_0$ , is carrying a current  $(0, 0, J(y)) = 2J_0 (y/h + 1/2)$  and  $h$  is not necessarily small.

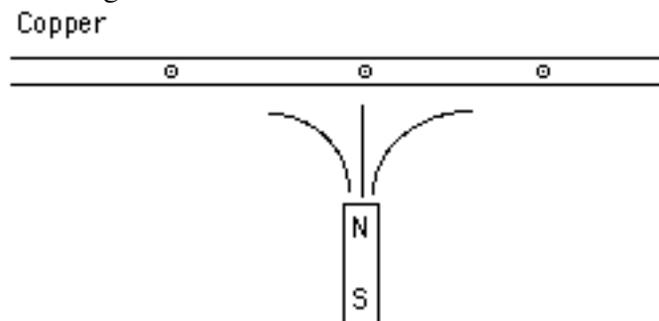
a) Sketch the dependence of the current density on  $y$ . b) What are the boundary conditions on an  $\mathbf{H}$  field  $(H_x(y), H_y(y), 0)$  across the interface. c) Sketch your result and justify it in terms of the field generated by the current distribution. Make sure your result behaves properly as  $h$  approaches zero.

2.2. You need to make an electromagnet that produces a field of 0.6 T in a 1 cm gap. You use a rectangular core design (20 cm  $\times$  10 cm with the gap on one of the 20 cm legs). You make the core out of soft iron with  $\mu_r \approx 100$ . The yoke has cross sectional area = 10 cm<sup>2</sup>.

a) If you are able to conveniently wind 600 turns of copper wire on the 20 cm leg, how much current will you need to achieve  $B = 0.6$  T in the gap. (Neglect edge effects).

b) What is the flux density in the iron core at this current?

2.3 Consider a permanent dipole magnet with its north pole a distance  $h$  away from a sheet of copper. Calculate the magnetic field in the copper and sketch the field in and near the copper sheet when it carries no current and when it carries a uniform current  $I_0$  out of the plane of the figure as sketched below.



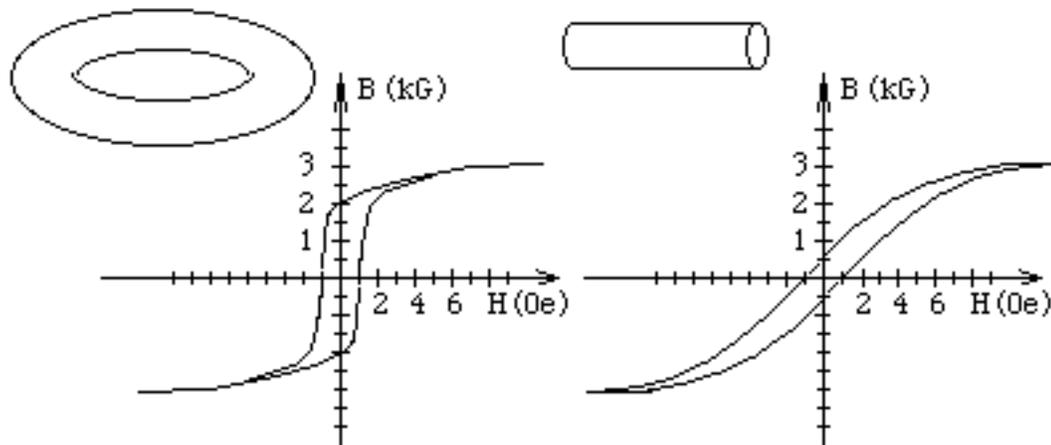
2.4 Describe the energy required to move a *demagnetized* ferromagnetic sample from negative infinity toward a permanent magnetic dipole at the origin having its north pole pointed toward the ferromagnet. Include the effects of magnetization of the sample and its energy in the dipole field. Then, assuming you can hold the magnetization fixed, remove the magnetized sample back to negative infinity considering only its energy in

the dipole field is changing. What is the energy change in this second process? How do these energies relate to  $A_1$  and  $A_2$ ?

2.5 Derive Eq. 2.30 by eliminating  $H_i$  from the two expressions that led to Eq. 2.16.

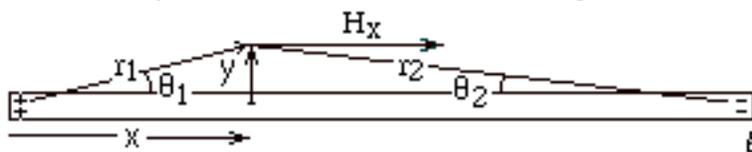
2.6 Verify that Eq. 2.45 satisfies Laplace's Eq., 2.48.

2.7 You formulate a new soft ferrite composition and want to test its low-field magnetic properties. You sinter a toroid and a rod from your new proprietary composition. The figures below depict the  $B$ - $H$  loops you observe on each.



- What is the remanent induction in each case?
- What is the coercivity in each case?
- Approximate the effective permeability in each case for a drive field of 2 Oe.
- Assume  $B = H + 4\pi M \approx 4\pi M$  and use  $M = \chi H$  with Eq. 2.29 to estimate the demagnetization factor,  $N$ , for the rod.
- Use Table 2.1 to estimate the demagnetization factor of the rod-shaped sample which you measure to have an aspect ratio of 10:1.

2.8 You need to calculate the magnetic field parallel to and outside a strip of a permanent magnet of length  $l$ . Assume that  $M$  is uniform inside the magnet. The strip is  $l/5$  wide (perpendicular to the plane of the figure below) and  $l/200$  thick (in the  $y$ -direction, below) and you need to know the field at heights of  $0.05l$  and  $0.3l$  for  $0 < x < l$ .



Use a three-dimensional formula (Eq. 2.45) for the field assuming that the width is small relative to the length (i.e., the strip acts like a rod) and also use Eq. 2.3 for a two-dimensional approximation (ignoring the field spread perpendicular to the plane of the paper). The form of the  $x$  dependence of  $H(x)$  is as important as the magnitude of the field.