

Solutions, Chapter 7

7.1 The potential energy of two dipoles, μ_m at the origin and μ'_m a distance r away, is given by Eqs. 1.9 and 1.8. When ferromagnets are of interest, the two dipoles have the same orientation and the second dipole moment can be expressed in polar coordinates of its position as $\boldsymbol{\mu}'_m = \mu'_m (\cos\theta \mathbf{e}_r - \sin\theta \mathbf{e}_\theta)$. The radial force between the two dipoles (now $\mu_m = \mu'_m$) is given by

$$\mathbf{F}_r = -\frac{\partial U}{\partial r} \mathbf{e}_r - \frac{\partial U}{r \partial \theta} \mathbf{e}_\theta = -\frac{\mu_o}{4\pi} \frac{3\mu_m^2}{r^4} \left[2 \cos^2(\theta) \mathbf{e}_r - \sin^2(\theta) \mathbf{e}_\theta \right]$$

F is attractive for collinear moments ($\theta = 0$) and repulsive for side-by-side moments ($\theta = \pi/2$). Thus one would expect strain $e_1 < 0$ for any direction of magnetization and $e_2 > 0$ perpendicular to *any direction of magnetization*. Applying Equation 9.1 to these dipole strains implies $\lambda_s < 0$. But Fe and Ni have very different λ 's in different directions and λ_{100} for iron is positive not negative.

7.3 The six Eqs. 7.6 contain six unknowns, the e_{ij} . The coefficients c_{ij} and constants $B_1 \alpha_i^2$ and $B_2 \alpha_i \alpha_j$ are assumed known. Solve the first three for e_{ij} ($i = j$) and the last three for e_{ij} ($i \neq j$) to get Eq. 7.7.

7.4 Eq. 7.16 then gives

$$\begin{aligned} e &= (3/2) \lambda_s (\alpha_1^2 \beta_1^2 + \alpha_2^2 \beta_2^2 + \alpha_3^2 \beta_3^2) + \\ &\quad 2 [\alpha_1 \alpha_2 \beta_1 \beta_2 + \alpha_2 \alpha_3 \beta_2 \beta_3 + \alpha_3 \alpha_1 \beta_3 \beta_1] - 1/3) \\ &= (3/2) \lambda_s [(\alpha_1 \beta_1 + \alpha_2 \beta_2 + \alpha_3 \beta_3)^2 - 1/3] \end{aligned}$$

Since the principal axes are no longer tied to the crystal (which is isotropic), the coordinates can be rotated so that the z axis coincides with the direction in which the strain is measured. Then only one direction cosine survives, $\alpha_3^2 = \cos^2\theta$ where θ is the angle between M and the strain direction.

7.5. Misfit, $(a_{\text{Cu}} - a_{\text{Ni}})/a_{\text{Cu}}$ is $\eta = 2.5\%$ from lattice constants given.

- a) $K_1 = -4.5 \times 10^3 \text{ J/m}^3$ and from Eq. 7.20, $B_1 = -(3/2) \lambda_{100}(c_{11} - c_{12}) = 6.2 \times 10^6 \text{ N/m}^2$. For a biaxial misfit strain, the tensor components are:

$$e = \eta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{-2\nu}{1-\nu} \end{pmatrix} \approx \eta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

and $f_{\text{ME}} = B_1(e_{11}\alpha_1^2 + e_{22}\alpha_2^2 + e_{33}\alpha_3^2) + B_2(e_{12}\alpha_1\alpha_2 + e_{23}\alpha_2\alpha_3 + e_{31}\alpha_3\alpha_1)$ becomes $f_{\text{ME}} = 1.55 \times 10^5 ([\cos^2\phi + \sin^2\phi]\sin^2\theta - \cos^2\theta)$ ignoring any shear strain in the film.

Now the first question is what is the relative magnitude of the energies involved, i.e. can the magnetization point out of plane? The magnetostatic energy of Ni, $\mu_0 M_s^2/2$, (favoring in plane magnetization) is of order $1.5 \times 10^5 \text{ J/m}^3$ which is much greater than the magnetocrystalline anisotropy (which favors $\langle 111 \rangle$ easy axes), but is comparable to the ME energy $B_1 e \approx 1.55 \times 10^5 \text{ J/m}^3$ (whose orientation preference has yet to determine).

$$\begin{aligned} f_{\text{ME}} &= 1.55 \times 10^5 \cdot ([\cos^2\phi + \sin^2\phi]\sin^2\theta - \cos^2\theta) \\ &= 3.1 \times 10^5 \cdot \sin^2\theta + \text{const.} \end{aligned}$$

$$f_a = -4.5 \times 10^3 (\cos^2\phi \sin^2\phi \sin^4\theta + \sin^2\theta \cos^2\theta)$$

$$f_{\text{MS}} = 1.5 \times 10^5 \cos^2\theta$$

Clearly the ME energy density dominates (as long as the strain in the film has the full misfit value). The magnetostatic energy is a close second and the crystal anisotropy is but 1% of the other two.

- b) First, f_a is the only term that contains the angle ϕ , so even though it is the weakest term, it should be minimized with respect to ϕ to find, as expected, $\phi = \pm 45^\circ, \pm 135^\circ$..., i.e. the azimuths containing the $\langle 111 \rangle$ directions are favored. Now because f_a is so small, it is necessary to consider only the ME and MS terms in minimization with respect to θ . $f_{tot} = (3.1 \times 10^5 - 1.5 \times 10^5 \text{ J/m}^3) \sin^2 \theta + \text{const.}$ obviously minimizes for $\theta = 0^\circ$. This reflects the fact that the ME energy dominates the MS energy and the nature of the strain and ME coefficient is such that perpendicular magnetization is favored. So it no longer matters what ϕ is.
- c) As the film grows, the in-plane biaxial strain decreases due to misfit dislocation formation. The shear strain e_{xy} probably remains negligible, but the biaxial, x - y plane strain may take on a z -dependence from the Cu/Ni interface to the top of the film, i.e. there may be shear components $e_{yz}, e_{zx} \neq 0$. In this case it is necessary to consider the terms $B_2(e_{23}\alpha_2\alpha_3 + e_{31}\alpha_3\alpha_1)$. Because $B_2 = 4.3 \times 10^6 \text{ N/m}^2 < B_1$ and the shear strains are probably small compared to the biaxial strains, there should be no effect from this term. Also, unless this term is large, α_1 and α_2 will remain zero.
- d) If $e_{12} = 2e_{11} = 0.05$, then the terms $B_2(e_{12}\alpha_1\alpha_2) = 2.1 \times 10^5 \sin\phi \cos\phi \sin^2\theta$, ($B_2 = 4.3 \times 10^6 \text{ N/m}^2$) must be retained. This term is stronger than the Ni magnetocrystalline anisotropy term which favors $\phi = \pm 45^\circ, \pm 135^\circ$..., so it will dictate the equilibrium azimuth at $\phi = -45^\circ, +135^\circ$ ($-2.1 \times 10^5 \sin^2\theta$). This new shear magnetoelastic anisotropy is negative at its equilibrium values. So it combines with the magnetostatic term, $-1.5 \times 10^5 \sin^2\theta$ to compete with the ME term, $+2e_{11} \times 6.2 \times 10^6 \sin^2\theta$, and now causes the magnetization to fall back in plane assuming an orientation consistent with a uniaxial easy axis $\phi = -45^\circ, +135^\circ$.

7.7. For a sample being **magnetized** with $H = H_a$, it follows that $K_{tot} = K_{xtl} + K_s + \dots = (1/2)H_a M_s$ which is the effective anisotropy energy density, i.e. it includes ΔK_1 effects as in Eq. 7.18.

For $H \geq H_a$, a uniaxial material strains by anywhere from $e = \lambda_s$ at $\theta = 0$ to $e = -\lambda_s/2$ at $\theta = \pi/2$ so the magnetoelastic energy density $f_1 = F/V = Be$ is of order:

$$f_1 = (3/2) Be = (3/2) B\lambda_s \approx (3/2) E \lambda_s^2 \quad f_1 = (3/2) E\lambda_s^2 \quad (\text{see Eq. 9.19})$$

Now if you **impose a stress** s which is great enough to give $e = \lambda_s$, then,

$$e = \sigma/E = \lambda_s \quad \text{and}$$

$$f_2 = K_s = (3/2)\lambda_s \sigma \approx (3/2)\lambda_s eE = (3/2)\lambda_s^2 E$$

so

$$f_2 = (3/2) E\lambda_s^2$$

which is the same energy as when the material is magnetized to saturation: $f_1 = f_2$.

But the question remains, is this energy comparable to the total anisotropy energy?

The answer is yes, only if K_{xtl} is small:

$$K_{tot} = K_s + K_{xtl} + \dots = (3/2)\lambda_s \sigma + K_{xtl} = f_1 \text{ or } f_2 \quad \text{only if } K_{xtl} \ll K_s$$

7.8 Using a 90° biaxial strain gauge, it is possible to measure the strains in [100] and [110] simultaneously for two field directions [100] and [110]. From the strain measurements with field parallel to the strain direction e_{100}^{\parallel} , e_{100}^{\parallel} , it is possible to calculate $e_{100}^{\parallel} - 4e_{110}^{\parallel}$, which gives $-4h_2$.

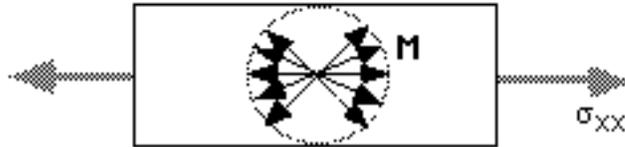
The numerical value for h_2 now gives $\lambda_{111} = 1/3 h_2$.

Using the h_2 value in $1/6 h_1 + h_2 = e_{110}^{\parallel}$ which is measured, gives us a value for h_1 and $\lambda_{100} = 2/3 h_1$. Two more independent measurements would be needed to get h_3 and h_4 .

7.9. For $\lambda_s > 0$ ($B < 0$), $\sigma_{xx} \neq 0$ and all other stresses zero, it is expected that $e_{yy} = e_{zz} = -ve_{xx}$ so the magnetoelastic free energy is

$$f^{ME} = -|B|e_{xx} [\alpha_1^2 - v(\alpha_2^2 + \alpha_3^2)] \approx -|B|e_{xx} [4\alpha_1^2 - 1]/3$$

The energy is lowered if $a_1^2 > v (\alpha_2^2 + \alpha_3^2)$ so magnetization along $\pm x$ is favored and the random distribution of moments would become:



For application of a field, only M in the field direction (not opposite it) lowers the energy. So the distribution is unidirectional:

The M - H curves are linear and for $\sigma_{xx} > 0$, M_x vs. H_x saturates at lower fields (see above, right). To plot M vs. σ_{xx} , the energy density must be considered:

$$f = fME + fK + fZ$$

$$= -|B| e_{xx} [4\alpha_1^2 - 1]/3 + (1/2) M_s H_a \alpha_1^2 - M_s H \alpha_1$$

Choosing θ between M and the x axis, solve $\partial f / \partial \theta = 0$ for the reduced magnetization along x :

$$\frac{M_x}{M_s} = \cos \theta = m = \frac{K_u}{K_u + (4/3) B e_{xx}} h$$

with $e_{xx} = \sigma_{xx}/E$, $K_u = (1/2)M_s H$ and $h = H/H_a$.

Thus M_x vs. H is linear with a slope of M_s/H_a for $\sigma_{xx} = 0$ and an increasing slope of $M_s/[H_a - (8/3) |B| \sigma / (M_s E)]$ as σ_{xx} increases ($B < 0$)

In all cases, M_x saturates at M_s when $\theta = 0$. We can write the equation for m as

$$m = h/(1-x)$$

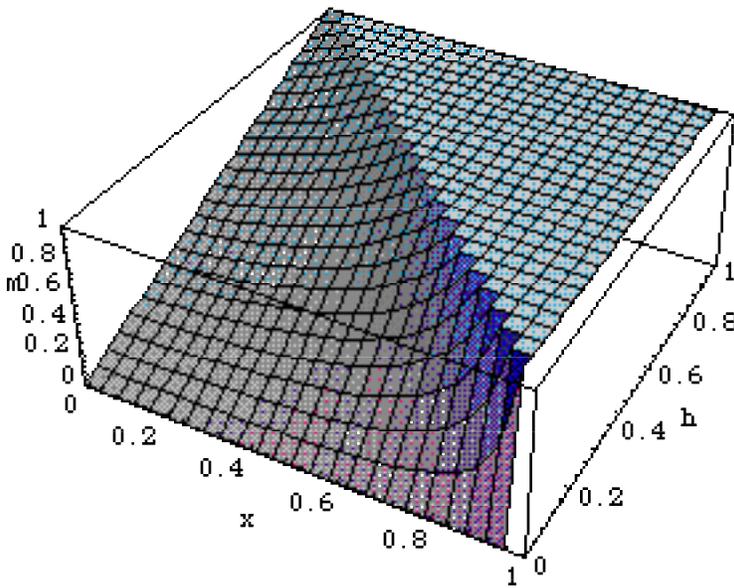
with $x = (4/3) |B| e / K_u$. The result is plotted below after the Mathematica® program that gives the plot.

$$m=h/(1-x)$$

$$\text{Plot3D}[m, \{x, 0, 1\}, \{h, 0, 1\},$$

$$\text{AxesLabel}->\{ "x", "h", "m" \}, \text{PlotRange}->\{0, 1\},$$

PlotPoints-> 25]



7.10

- a) Alloy A has an anisotropy energy surface that is a cut through Fig. 6.6a and that of B is a cut through Fig. 6.6b.
- b) For zero strain, A is easily magnetized along $\langle 100 \rangle$ directions, B and C along $\langle 111 \rangle$.
- c) The $M-H_{[100]}$ curve of A is like that of Fe in Fig. 6.1a; the remanence is close to unity. Those of alloys B and C are like that of Ni in Fig. 6.1c; the remanence is given from Eq. 6.6 and $\mathbf{m}(0) = (1,1,1)/\sqrt{3}$ as $\mathbf{m} \cdot \mathbf{H}/|\mathbf{H}| = 1/\sqrt{3} = 0.577$. Taking the field along z for convenience, the free energy is

$$f = -\mu_0 M_s \cdot H + K_1(\alpha_1^2 \alpha_2^2 + \text{cycl.}) = -\mu_0 M_s H \cos \theta + K_1(1/4 + \cos^2 \theta)$$

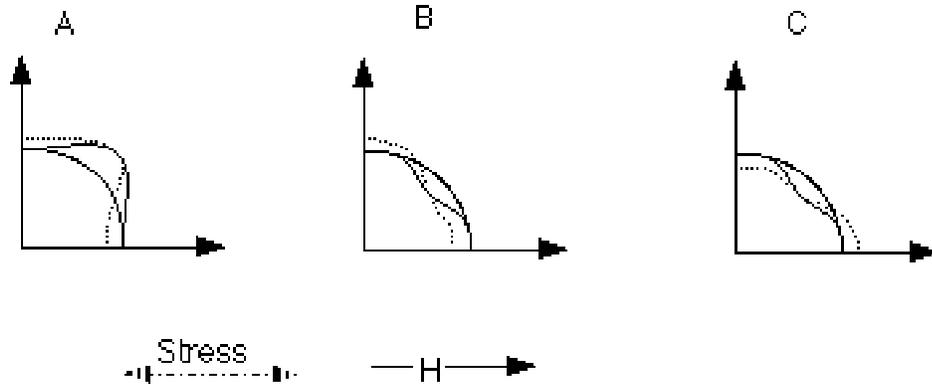
$$\text{giving } f' = 0 \Rightarrow \mu_0 M_s H = 2K_1 \cos \theta$$

$$\text{and } \cos \theta = m = \mu_0 M_s H / 2K_1.$$

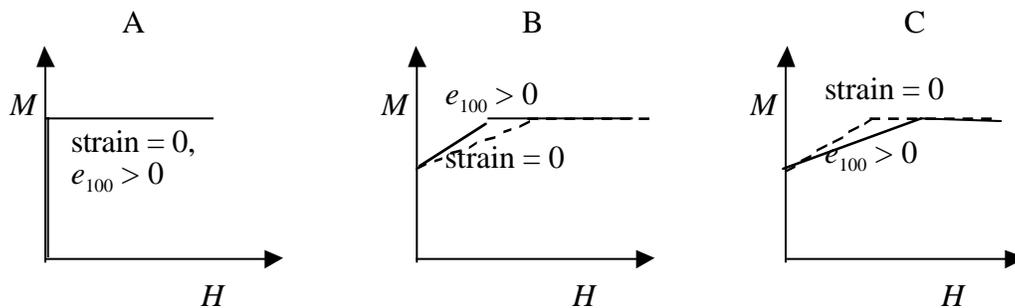
Thus saturation occurs at $H_a = 2K_1/\mu_0 M_s$.

- d) Sample B would make the best soft material because both K and λ are close to zero.

- e) With a field and tensile stress along [100], the magnetoelastic energy will favor [100] magnetization when $\lambda_{100} > 0$ (cases A and B) and favor [010] and [001] magnetization when $\lambda_{100} < 0$. That is, the energy surfaces determined in part a) will be supplemented with magnetoelastic energy terms shaped like oblate spheroids with axis along [100] for samples A and B and prolate spheroids with axis along [100] for sample C.



In the energy surfaces above, the solid lines represent the cubic magnetocrystalline anisotropy energy surfaces. The magnetoelastic contribution adds an uniaxial term with axis of symmetry along the strain direction. The dotted surface is the resultant energy surface. The changes in MH are shown by the solid lines relative to the unstrained loops (dashed).



- f) $B_1 = -(3/2) \lambda_{100} (c_{11} - c_{12}) = -7.5 \times 10^5 \text{ N/m}^2$. The strain needed to cancel the cubic anisotropy is about $\epsilon = K_1 / (1.3B_1) = 5 \times 10^3 / [1.3(-7.5 \times 10^5)] = 0.51\%$. The factor of 1.3 in the magnetoelastic energy is the $1+\nu$ term for uniaxial deformation in Fig. 7B.3.