

3.46 PHOTONIC MATERIALS AND DEVICES

Homework Assignment 4—March 1, 2006

Due: 5pm, March 8, 2006

1. Photonic Crystals

Calculate the penetration depth x and stopband width $\Delta\lambda$ for $\lambda = 1.55 \mu\text{m}$ light incident on a 1D photonic crystal (with films thickness following the $\lambda/4n$ criterion) made of:

- a) Si and SiO_2 pairs
- b) Si_3N_4 and SiO_2 pairs
- c) Si and Si_3N_4 pairs
($n_{\text{Si}} = 3.5$, $n_{\text{Si}_3\text{N}_4} = 2.0$, $n_{\text{SiO}_2} = 1.445$)

2. Resonant Cavity (read more about 'mirror loss' in *Fundamentals of Photonics*)

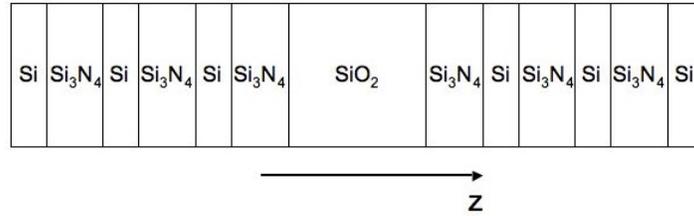
A linear microcavity resonator of size d is bounded on either end by a Bragg Reflector with reflectivity R (at $\lambda = 1.55 \mu\text{m}$). When a pulse of light transits across the microcavity and reflects off a Bragg Reflector, it loses some power. We can define a loss coefficient α_m , called the 'mirror loss,' which renormalizes reflector loss, per unit length of the cavity. Derive the following expression for mirror loss:

$$\alpha_m = \frac{1}{d} \ln\left(\frac{1}{R}\right)$$

Given a group velocity v_g for the light pulse, and assuming there are no other loss mechanisms inside the microcavity, derive an expression relating the microcavity Quality factor Q to α_m . The peak reflectivity at a Bragg Reflector stopband's central frequency can be derived as a function of reflector indices n_L , n_H and number of low/high index pairs p :

$$R = \left| \frac{(n_L/n_H)^{2p} - n_H^2}{(n_L/n_H)^{2p} + n_H^2} \right|^2$$

Using all of the above relationships, determine what the resonance linewidth $\Delta\lambda$ will be for a SiO_2 microcavity ($n = 1.445$) designed to trap the first mode of $\lambda = 1.55 \mu\text{m}$ light, entering at normal incidence (the propagation wavevector β lies fully along the z -axis direction of propagation). The cavity is bounded on either end by Bragg Reflectors made up of 3 pairs of Si/ Si_3N_4 (Si_3N_4 is the layer in physical contact with the SiO_2 microcavity on either end). The Si/ Si_3N_4 layers should be designed to meet the $\lambda/4n$ condition. State what all your assumed film thickness values are.



3. Ring Resonator (dB/cm definition given in *Fundamentals of Photonics*)

If you have a single mode microring resonator ($n_1 = 1.445$, $n_2 = 3.5$, $n_{\text{eff}} = 2.2$), with $r = 5.04 \mu\text{m}$ and operating wavelength at $1.55 \mu\text{m}$:

- What is the quality factor if the loss of the ring is 100 dB/cm? How about 10 dB/cm?
- What is the 3 dB bandwidth if the loss of the ring is 100 dB/cm? How about 10 dB/cm?

If you want to achieve a 1 nm tuning range:

- What is the radius change you need to have? What is the refractive index change you need to have?