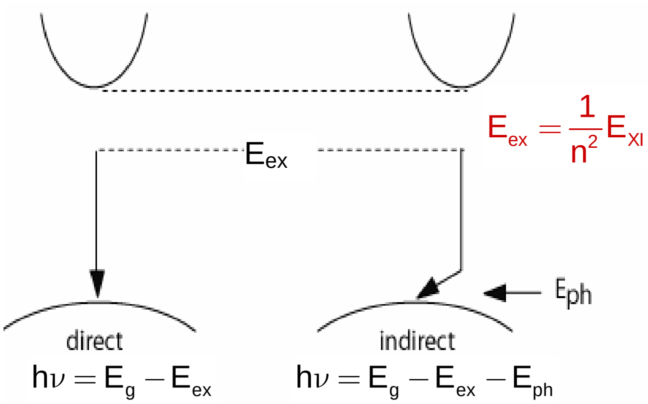
Lecture

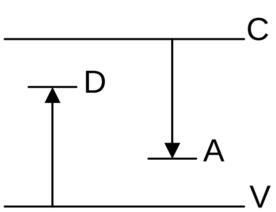
Recombination

Free excitons



E_{ex}: exciton binding energy (dielectric medium)
E_{XI}: exciton binding energy (vacuum)
E_{ph}: phonon energy

Bound excitons



$$h\nu_{direct} = E_g - E_{A/D}$$
$$h\nu_{indirect} = E_g - E_{A/D} - E_{ph}$$

Isoelectronic Traps

dopant	localized defect
C in Si	1) traps e^-
N in GaP	2) attracts hole
O in ZnTe	3) exciton bound to trap

Notes

Direct: band-to-band transitions dominate

Indirect: impurity-to-band dominates

Recombination is the process whereby an electron-hole pair mutually annihilate. If the energy lost when an electron is demoted from a higher-lying to a lower-lying electronic state is converted into a photon, the process is known as radiative recombination.

Excitons are Coulombically-bound electron-hole pairs.

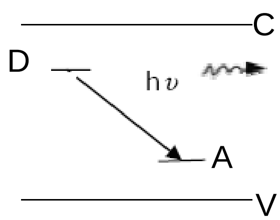
Since photons have very small momenta compared to electrons and holes, it is convenient and correct to think of the conserved momentum before and after recombination with respect to the electron and hole alone.

In a direct-bandgap semiconductor, the band extrema (valence band maximum, conduction band minimum) coincide in wavevector. There is thus an abundance of electrons and holes available with corresponding momenta available to recombine with no additional requirement of momentum. In direct bandgap materials, radiative recombination occurs efficiently and rapidly.

This is not the case in indirect-bandgap materials such as the elemental semiconductors silicon and germanium. If their charge carriers are delocalized, their lifetime is long and their radiative efficiency low. Impurity-to-band recombination processes are possible and can develop reasonable radiative efficiencies.

Lecture

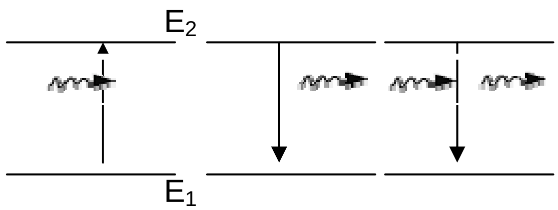
Donor-acceptor Pairs



$$h\nu = E_g - E_A - E_D + \frac{q^2}{\varepsilon r}$$

$r = D \rightarrow A$ separation

Lasers   Stimulated Emission



absorption   spontaneous   stimulated

$\propto\ B\,u(\nu)$ $\propto A = \dfrac{1}{t_{sp}}$ $\propto\ Cu(\nu)$

equilibrium   (N_i = # atoms in state i)

absorption $\equiv$ emission

$N_1 B[u(\nu)] \equiv N_2\ [A + C\,u(\nu)]$

Boltzmann

$$\frac{N_2}{N_1} = \exp\left[-\left(E_2 - E_1\right)\right]/kT$$

$$= \exp(-(h\nu)/kT)$$

$$\Rightarrow u(\nu) = \frac{A}{e^{h\nu/k_BT} - C}$$

Black Body Radiation   (Planck)

$\equiv$ equilibrium

$\therefore B = C$

$$\therefore \frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$$

Notes

1) $h\nu\uparrow$ with $P_i\uparrow$
transition prob.: $W(r) = W_{max}\, e^{-r/R}$

2) as N_A, N_D $\uparrow$, $h\nu\uparrow$ (closer pairs)

3) @ long times after injection $h\nu\downarrow$

$u(\nu) =$ energy density

Einstein postulate:

excited state driven to ground state by EM wave with $h\nu = E_2 - E_1$

<u>same coefficient</u> governs absorption and stimulated emission. By postulating stimulated emission, Einstein showed that $C \neq O$
$\Rightarrow$ Planck's factor of (-1) in the denominator, actually corresponds to stimulated emission!

Lecture

Notes

Separate electron and hole traps can result in radiative recombination. The process is referred to as spatially indirect (in contrast with the wavevector-indirect transition inside indirect-bandgap materials). The transition probability decays exponentially with the spatial separation among the donor-accept component.

Lasing and stimulated emission

Lasing and the intimately-connected notion of population inversion are conceptualized most readily through the development of a set of rate equations which account for three processes:

1) absorption, whereby an incident photon excites an electron-hole pair
2) stimulated emission, whereby an incident photon deexcites an electron-hole and, in the process, stimulates the production of an *identical* emitted photon.
3) spontaneous emission, which results in the production of a photon but does not rely on the presence of a stimulating field.

The rates of absorption and stimulated emission are proportional to the energy density associated with the stimulating field.

In the steady-state, the rate of excitation must equal the rate of deexcitation, an equality which leads to $N_1 B\left[u(\nu)\right] \equiv N_2\left[A + Cu(\nu)\right]$

Neglecting Fermi-Diract statistics but instead employing classical Maxwell-Boltzmann statistics, which work well in non-degenerate media, it is possible to solve for the steady-state energy density $u(\nu)$ – and thus for the spectrum of blackbody radiation, i.e. of a medium in equilibrium.

Lecture	Notes

Stimulated emission

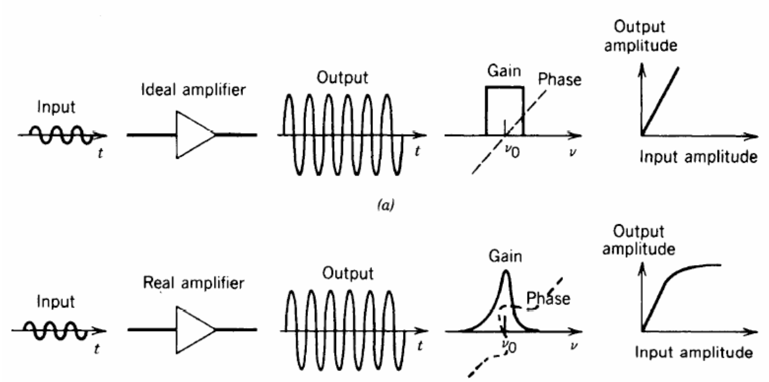
- wave emitted with <u>same phase</u> and in <u>same direction</u> as the driving wave.

(equilibrium black body)

$$\frac{B \cdot u(\nu)}{A} = \frac{1}{e^{h\nu/kT} - 1}$$

$$\frac{\text{stimulated}}{\text{spontaneous}} = 10^{-2} \qquad (5000\ \text{K})$$

Laser = Light Amplification by Stimulated Emission of Radiation



Figures of Merit

- gain
- bandwidth
- phase shift
- power source
- non linearity (saturation)
- noise

A. Discrete states (atoms in gas)

W_{21}, W_{12}: rate of interaction $= \underbrace{\phi\sigma_{21}(\nu),\ \phi\sigma_{12}(\nu)}$ $\left(s^{-1}\right)$

prob. rate for stimulated emission/absorption

$$\sigma(\nu) = \left(\frac{\lambda^2}{8\pi\tau_r}\right)\underbrace{L(\nu)}$$

Lorentzian line shape function $\int L(\nu)\,d\nu = 1$

$W_{21} = W_{12}$ same rate for absorption & stimulated emission

Lecture

N_1W_{12} = # photons absorbed/cm^3/s
N_2W_{21} = # 'cloned', stimulated emission photons/cm^3/s

Notes

$\Delta N = N_2 - N_1$

$\Delta N > 0 \rightarrow$ gain

$\Delta N < 0 \rightarrow$ attenuation

$\Delta N = 0 \rightarrow$ transparent

The stimulated emission process lies at the heart of lasing and determines the properties of laser light. Stimulated emission results in production of a photon which is coherent with the stimulating photon: this is to say that it has the same direction, wavelength, and phase. When combined with feedback inside a cavity, the coherence of the stimulated emission process will ensure that laser light is monochromatic, directional, and phase-coherent.

Optical gain, or amplification, is the result of the stimulated process. The rates of absorption and stimulated emission will be a product of the probability of the process per unit time with the number of discrete states available to make the transition.

The probability, in turn, is the product of the photon flux $\phi\left(cm^{-2}s^{-1}\right)$ and the cross-section $\sigma\left(cm^{2}\right)$ associated with the process.

The absorption process takes place from the lower state (1) and thus its rate is proportional to the number of states in the lower state, N_1; whereas the stimulated emission process begins from the higher-lying electronic state and is proportional to N_2.

Lecture

$$g(\nu) = \Delta N \sigma(\nu) = \Delta N \frac{\lambda^2}{8\pi\tau_{sp}} L(\nu)$$

$$= \text{gain coefficient } (\text{cm}^{-1})$$

$$\Delta N = N_2 - N_1$$

$$\text{Gain} = \exp[g(\nu)d]$$

B. Semiconductors

$$\sigma(\nu) \to \sigma(\nu)\rho(\nu) = \sigma(\nu)\frac{2m_r^{3/2}}{\pi h^2}(h\nu - E_g)^{1/2}$$

$$\Delta N \to f_g = f_e(\nu) - f_a(\nu)$$

$$= f_C(E_2) - f_V(E_1)$$

$$\Rightarrow g(\nu) = \sigma(\nu)\rho(\nu)f_g(\nu) = \frac{\lambda^2}{8\pi\tau_{sp}} L(\nu)\rho(\nu)f_g(\nu)$$

Bandwidth

- gain bandwidth

$$\frac{E_g}{h} < \nu < \frac{E_{FC} - E_{FV}}{h}$$

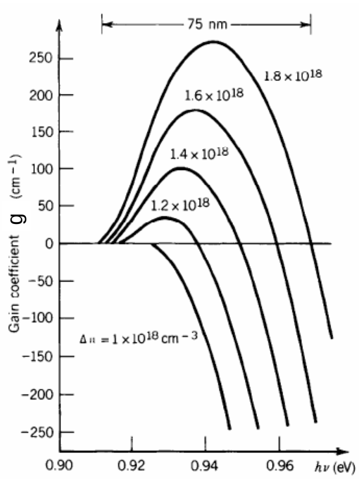
transparent $\leftarrow$ gain $\rightarrow$ absorbing

$\therefore$ as R (pump rate) $\uparrow$

$g(\nu)\uparrow$, bandwidth $\uparrow$

$f_g(\nu)\uparrow (E_{FC} - E_{FV})\uparrow$

Example: $In_{0.72}Ga_{0.28}As_{0.6}P_{0.4}$



Notes

d = cavity path length

optical joint
density of states

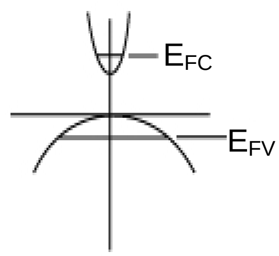
population difference

difference $\rightarrow$ Fermi inversion

factor defined by quasi E_F

prob CB state @ E_2 filled

prob VB state @ E_1 filled



E_g = 0.95eV, $\lambda_g = 1.3\mu m$

$\tau_r = 2.5$ ns

$N_A = N_D \simeq 2\times10^{17}\text{cm}^{-3}$

$m_C = 0.06\ m_0$

$m_V = 0.4\ m_0$

$n = 3.5$ (index)

Lecture

Notes

The gain per unit length experienced by the field is given by $g(\nu)$, and the single-pass gain G (relative to the transparency condition = unity) experienced during propagation over distance d is given by

$$G = e^{g(\nu)d}$$

B. Semiconductors

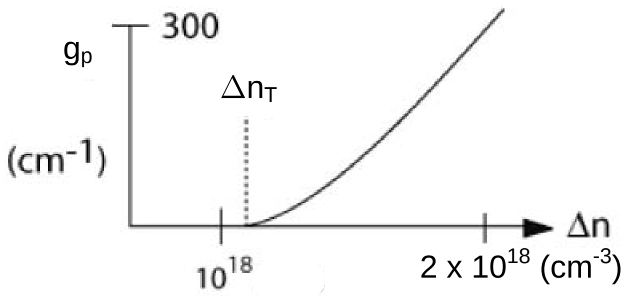
In semiconductors, instead of summing over sets of identical states as in a dilute gas, one integrates over the continuum that is the energy density of states. A joint density of states is used to account for the density of *transitions*, which is a consequence of the number of momentum-matched state pairs, which can participate in momentum-conserving absorption and emission processes.

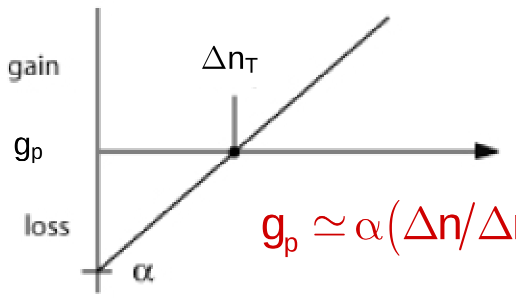
The occupation of the states in the conduction and valence band are of critical importance, and allow the semiconductor-based description of the phenomenon of population inversion needed for lasing. Whereas in a semiconductor in equilibrium, there is a single Fermi level, in a strongly-pumped semiconductor, the energy distribution in the conduction and valence bands are represented using two distinct quasi-Fermi levels.

The condition for gain in a semiconductor is known as the Bernard-Duraffourg condition: this stipulates that the quasi Fermi level separation exceed the energy of photons to be produced. This same logic leads to the gain bandwidth:

$$\frac{E_g}{h} < \nu < \frac{E_{FC} - E_{FV}}{h}$$

Lecture



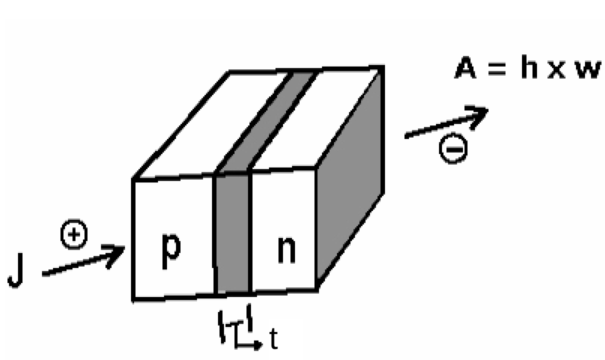


$$g_p \simeq \alpha\left(\Delta n/\Delta n_T - 1\right)$$

Power electronic pumping

$$g_P = \alpha\left(\frac{J}{J_T} - 1\right)$$

$$J_T = \frac{et}{\eta_i \tau_r}\Delta n_T$$



$J_T = 3.2 \times 10^4\ \mathrm{A/c}\quad{}^{2}$

Notes

$\Leftarrow \Delta n_T = 1.25 \times 10^{18}\,\mathrm{cm}^{-3}$

$\alpha = 600\ \mathrm{cm}^{-1}$

$@\,\Delta n = 1.75 \times 10^{18}\ \mathrm{cm}^{-3}$

$g_P = 240\ \mathrm{c}\quad{}^{-1}$

for a 350 μm long cavity:

$G = \exp\left[g_P d\right] = 4447 = 36.5\ \mathrm{dB}$

The first graph depicts the gain, in units of cm^{-1}, for increasing levels of non-equilibrium carrier concentration Δn.

Δn_T = transparency condition

α = absorption coefficient @ Δn = 0

The second plot shows loss for pump levels below transparency, and gain for pump levels exceeding the transparency carrier density.

In electrically-driven devices, a current density J supplies the excess carriers associated with quasi-Fermi level separation and gain. A transparency carrier density J_T may be found which provides a pumping level on the threshold between loss and gain.

J = injection current density

homojunction

$\tau_r = 2.5$ ns $\qquad \alpha = 600\ \mathrm{c}\quad{}^{-1}$

$\eta_i = 0.5 \qquad t = 2\ \mu\mathrm{m}$

$\Delta n_T = 1.25 \times 1\quad{}^{18}\mathrm{cm}^{-3}$

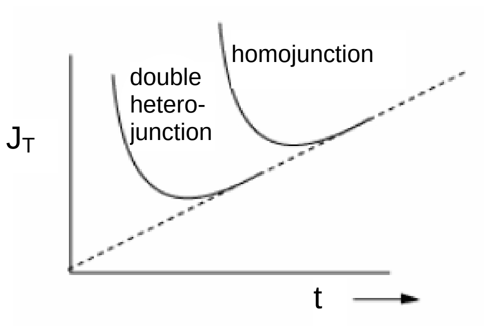
$h = 200\ \mu\mathrm{m}$

$w = 100\ \mu\mathrm{m}$

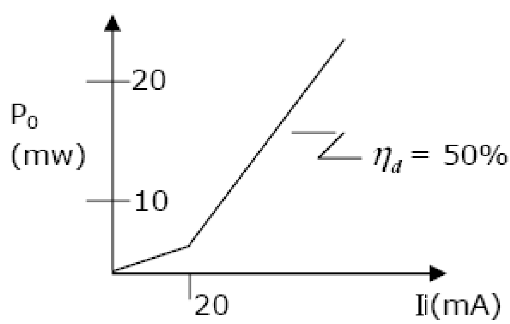
Lecture

large $J_T \Rightarrow$ heat dissipation problem

1) reduce active volume [t]
2) confine carriers
3) confine light



L-I Characteristic



$J_{th} = I_{th}/A$

J_{th} = threshold current for lasing

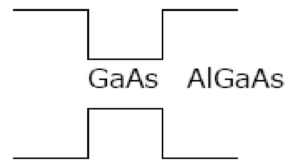
$J > J_{th}$ when $g_P > \alpha_r$

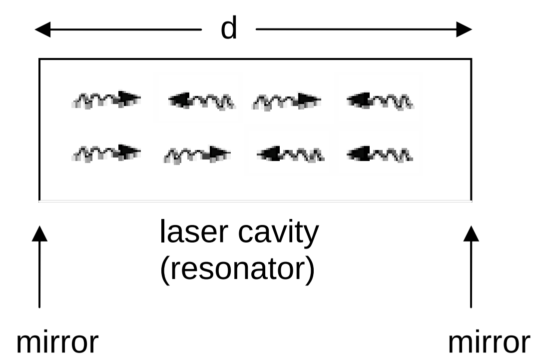
α_r = total losses

$$= \underbrace{\text{absorption}}_{\text{free carrier}} + \underbrace{\text{mirror loss}}_{\text{emitted } [\alpha_m]}$$

Notes

for homojunction $t \simeq \sqrt{D\tau_{sp}}$





Lecture

Notes

The current densities needed to pump semiconductor lasers can be quite high. Shrinking the active region volume, confining the carriers, and confining the waveguide mode to the active region all help to reduce the total current needed.

L-I Characteristic

The L-I characteristic is the output optical power, typically in mW, as a function of the pump current I, typically in mA. Below the threshold condition, only spontaneous emission occurs, and its external efficiency of emission is low, yielding a weakly-sloped L-I. Above the threshold condition, stimulated emission dominates as is efficiently coupled into the lasing mode, resulting in a high slope efficiency of Power vs. Current.

The threshold condition necessitates that all sources of round-trip loss be fully compensate by round-trip gain. Loss includes scattering per unit length along the device; loss through the mirrors which occurs upon partial reflection (partial escape of light to outside world).

Lecture	Notes

Internal

$\phi =$ flux internal to laser cavity @ steady state,
$\phi \propto$ j as determined by loss of population inversion by stimulated emission

$$\phi = \eta_i \frac{j - j_{th}}{e} \left(\text{photons/cm}^2 \cdot s\right)$$

internal photon flux

$$P = \eta_i \left(I - I_{th}\right) \frac{1.24}{\lambda_0} \left(\frac{eV}{s}\right)$$

internal laser power

$I(A)$, $\lambda_0(\mu m)$, $P(W)$

Output

$$\eta_e = \text{emission efficiency}$$
$$= \alpha_m / \alpha_{total}$$
$$= \frac{\text{Loss by emission}}{\text{Total loss in resonator}}$$

$$= \frac{\text{Photon lifetime} \times \ln\text{(mirror reflectance)}^{-1}}{\text{round trip travel time}}$$

$$= \Bigg[\underbrace{\frac{1}{d}}_{\text{cavity length}}\underbrace{\ln\left(\frac{1}{R}\right)}_{\text{mirror reflectance}}\Bigg] \times \frac{1}{\underbrace{\alpha_{total}}_{\text{cavity loss}}}$$

The external efficiency of a laser is given by the ratio of mirror loss – 'desirable loss' since it results in escape of stimulated photons into the lasing mode – divided by total loss which includes mirror + all other sources of loss.

It also includes an internal efficiency η_i that represents the fraction of carriers injected into the device which are captured into the active region for the production of optical gain.

Record-high external quantum efficiencies for semiconductor lasers are as high as 90%, with 50% being typical.

$\phi_0 =$ external photon flux

$\eta_d =$ differential quantum efficiency

$$= \eta_e \eta_i \qquad = \frac{d\phi_0}{d(j/e)}$$

$$\phi_0 = \eta_e \eta_i \frac{j - j_{th}}{e}$$

$$P_0 = \eta_d \left(I - I_{th}\right) \frac{1.24}{\lambda_0}$$

$\eta =$ power conversion efficiency

$$\eta = \eta_d \left(I - \frac{I_0}{I}\right) \frac{h\nu}{eV}$$

for emission @ $h\nu = 1eV$

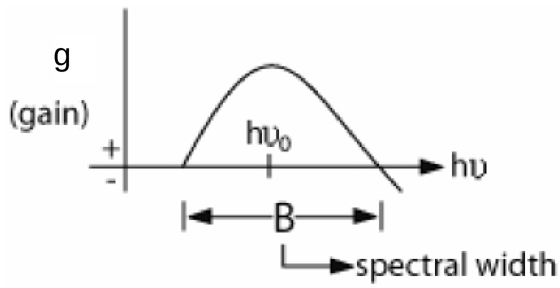
$$\eta = \eta_d = 50\%$$

Lecture

$P_0 = \eta_d (I - I_{th}) \dfrac{h\nu}{eV}$

$\eta_d =$ slope of L (or P_0) vs. I plot

Cavity Modes



modal spacing (longitudinal) $\Delta\nu \equiv FSR_\nu = c/2d$

cavity modes $M = B/FSR_\nu$

Example: InGaAsP

$n = 3.5,\ \lambda_0 = 1.3\ \mu m,\ d = 400\ \mu m$

$FSR_\nu = \dfrac{c_0}{2nd} = 107\ \text{GHz}$

$FSR_\lambda = \dfrac{\lambda_0^2}{2nd} = 0.6\ \text{nm} = 6\ \text{Å}$

$B \simeq 70\ \text{Å} \Rightarrow 11$ modes

Single mode condition: $B < \dfrac{c_0}{2nd} \Rightarrow d < 36\ \mu m$

Transverse modes

$t < \lambda_0 \Rightarrow$ single mode junction

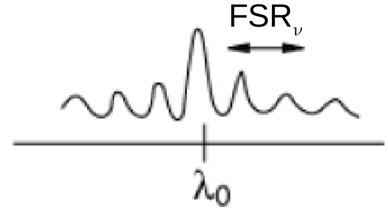
$w > \lambda_0 \Rightarrow$ lateral multimode

Notes

In a Fabry-Perot laser, feedback is provided by mirrors separated by length d. This results in the modal spacing in frequency of $FSR\nu = c/2d$.

For a gain bandwidth B, this implies that $B/FSR\nu$ cavity modes can potentially be candidates for lasing.

There can in addition be multiple transverse modes which, because they have different propagation constants, will be non-degenerate, further adding to the spectrum of modes possible. A transversely-single moded device can be made by making the waveguide sufficiently narrow.

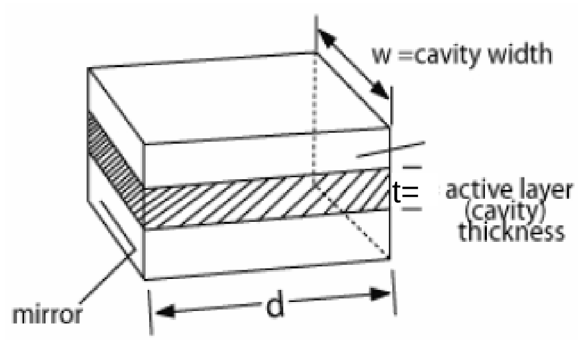


semiconductor lasers $\rightarrow$ small d

$\Rightarrow$ large FSR_ν

$\Rightarrow$ small M

possible



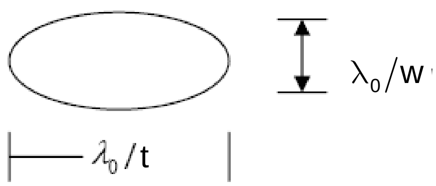
Lecture

Single frequency operation

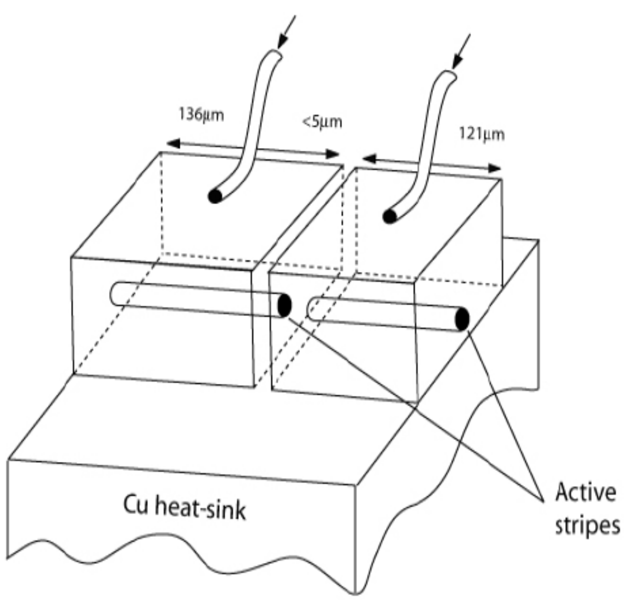
1) reduce dimensions to single mode
2) two cavities
3) DBR } gratings
4) DFB } gratings

Angular divergence

- Far field radiation pattern



$\Rightarrow$ smaller dimensions $\Rightarrow$ single mode, but with large divergence
$\Rightarrow$ harder to couple to fiber



Notes

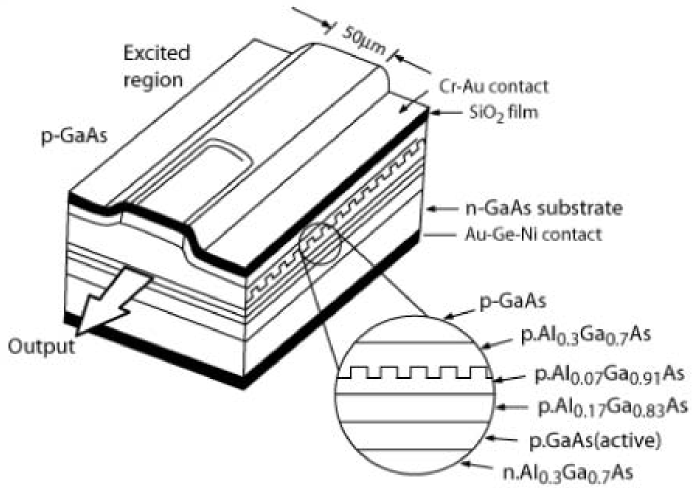
Single-frequency lasers can be realized by:

1) Reducing the transverse dimensions to provide a single lateral mode
2) Employing two cavities, known as an external cavity configuration, which provides an added level of mode selection
3) Creating a distributed Bragg reflector (DBR) grating
4) Creating a distributed feedback (DFB) grating

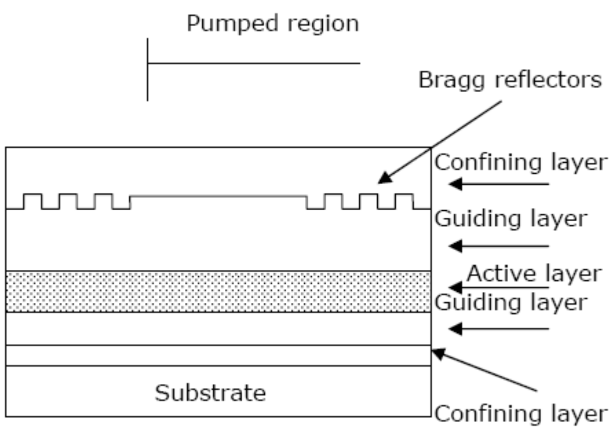
The angular divergence of the beam is important in coupling with subsequent optics, including fibers. A transversely narrow waveguide structure gives an angularly broader far-field pattern.

A two-cavity device with independently-pumped gain segments.

Lecture



(a)



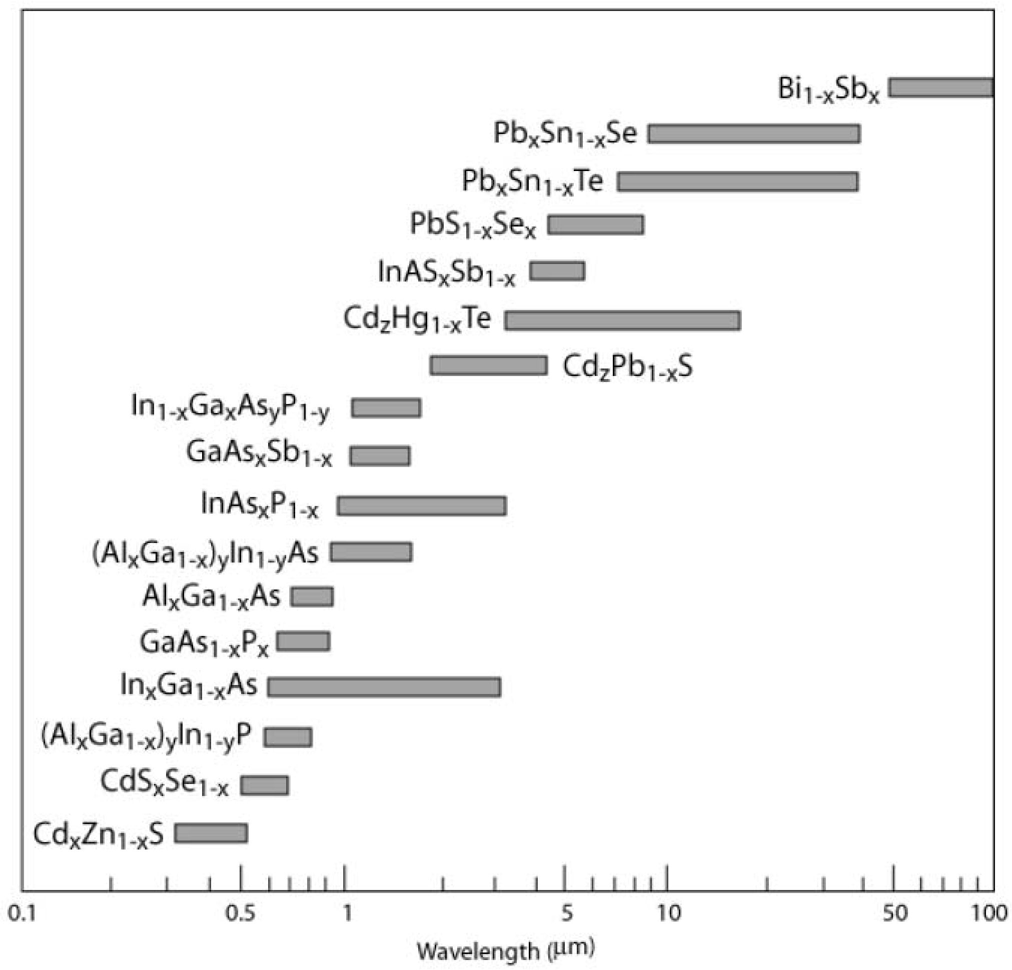
(b)

Notes

A typical p-side-up laser structure with Distributed Feedback grating for selection of a single, stable longitudinal mode. The grating, and hence the feedback, is distributed along the entire length of the device.

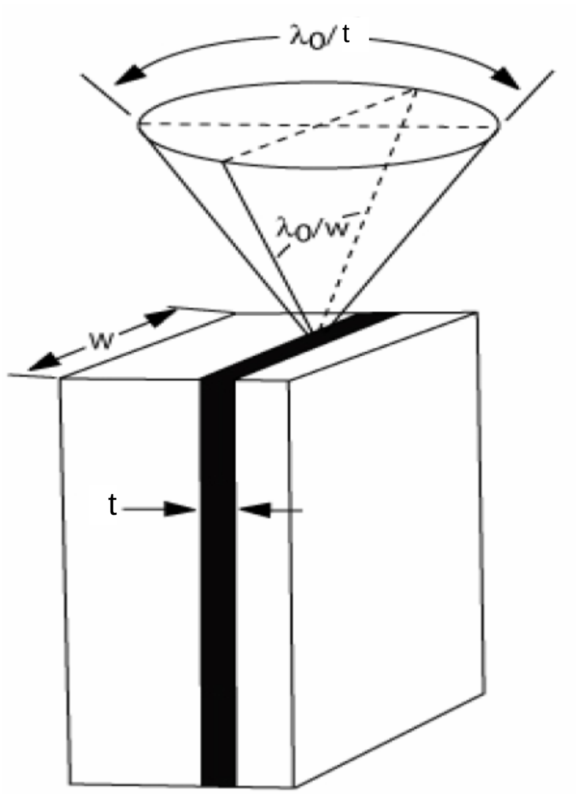
A Distributed Bragg Reflector (DBR) laser with mirrors made using Bragg reflectors.

Lecture

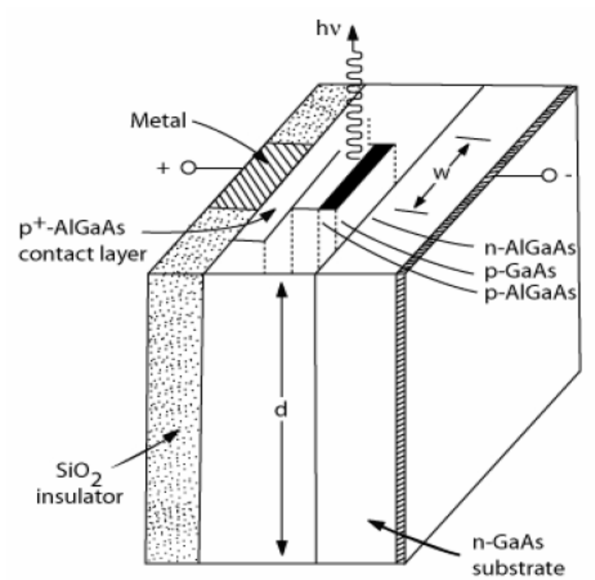


$Bi_{1-x}Sb_x$
$Pb_xSn_{1-x}Se$
$Pb_xSn_{1-x}Te$
$PbS_{1-x}Se_x$
$InAS_xSb_{1-x}$
$Cd_zHg_{1-x}Te$
$Cd_zPb_{1-x}S$
$In_{1-x}Ga_xAs_yP_{1-y}$
$GaAs_xSb_{1-x}$
$InAs_xP_{1-x}$
$(Al_xGa_{1-x})_yIn_{1-y}As$
$Al_xGa_{1-x}As$
$GaAs_{1-x}P_x$
$In_xGa_{1-x}As$
$(Al_xGa_{1-x})_yIn_{1-y}P$
CdS_xSe_{1-x}
$Cd_xZn_{1-x}S$
0.1
0.5
1
5
10
50
100
Wavelength (μm)

Lecture	Notes



Angular divergence of a laser of transverse dimensions w and t showing the inverse proportionality of divergence with transverse dimension.



An AlGaAs-GaAs semiconductor laser grown on a GaAs substrate.

Compound semiconductors, through their composition, allow a wide range in choice of bandgaps and hence emission energies.