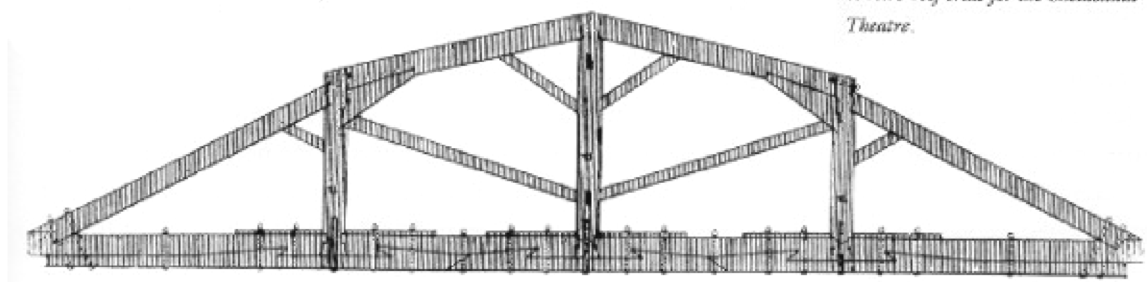
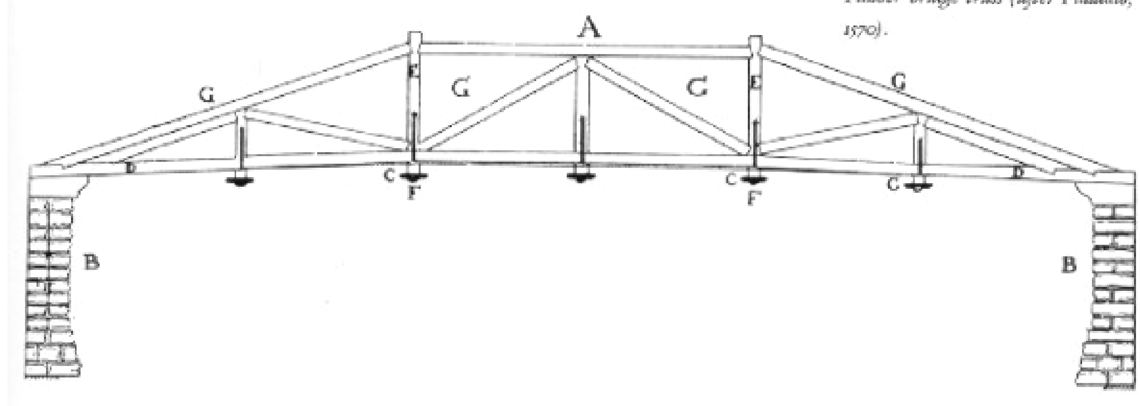
Historic Timber Structures II



5.5
Wren's roof truss for the Sheldonian Theatre.



5.6
Timber bridge truss (after Palladio, 1570).

Today's Lecture

1. **Assumptions for Analysis of Timber**

2. **Static Indeterminacy**

3. **Two case studies**
   - **-Wooden stool**
   - **-Hammerbeam roof**

4. **Conclusions**

Historical Development of Timber Structures

- Roman theatres
- Gothic roof systems
- 16th C bridges – Palladio
- 17th C roof trusses – Wren
- 18th C bridges – Grubenmann
- 19th C bridges – USA

Analysis of Timber Structures

- **Static equilibrium is the guiding principle (stresses are low)**
- **Assumptions greatly influence the results (joints and supports)**
- **Statically determinate or indeterminate structures behave in fundamentally different ways. Be clear about which type of structure you are dealing with.**

Forces in the Legs of a Stool

Three-Legged Stool

Statically determinate

One solution for the axial force in each leg

Why?

3 unknowns

3 equations of equilibrium

Uneven floor has no effect

Four-Legged Stool

Statically indeterminate

A four legged table on an uneven surface will rock back and forth

Why?

It is *hyperstatic:*

4 unknowns

3 equations of equilibrium

(or statically *indeterminate*)

Four-Legged Stool

- Infinite solutions exist
- Depends on unknowable support conditions
- A four legged table on an uneven surface will rock back and forth
- The forces in each leg are constantly changing

Fundamental difference between hyperstatic (indeterminate) and static structures

Forces in the Leg of a Stool

Statically determinate

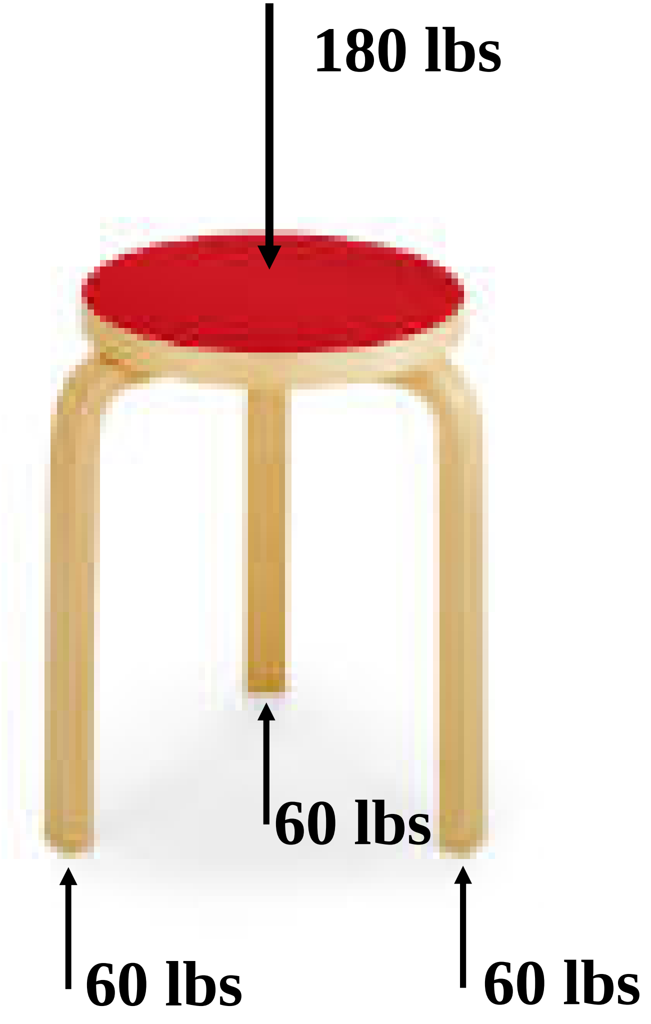
Statically Indeterminate (hyperstatic)

Three-Legged Stool

Design for a person weighing 180 pounds

→ 60 pounds/leg

Regardless of uneven floor



Collapse of a Three-Legged Stool

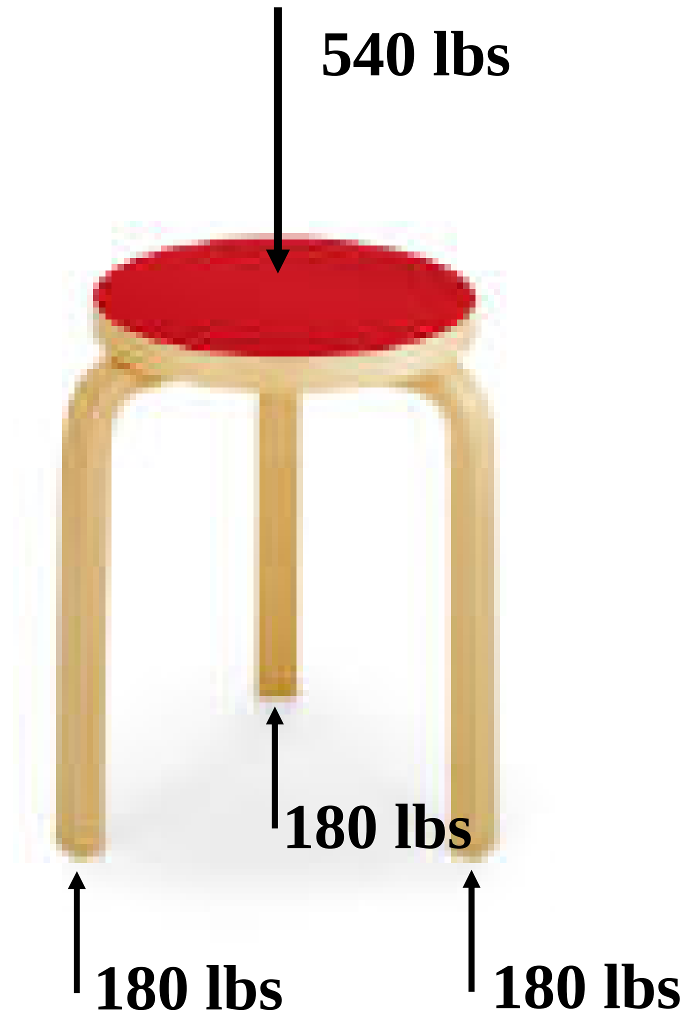
Design for a person weighing 180 pounds

If the safety factor is 3:

$P_{cr} = 3(60) = 180$ lbs

And each leg would be designed to fail at a load of 180 pounds

The stool would carry a total load of 540 pounds

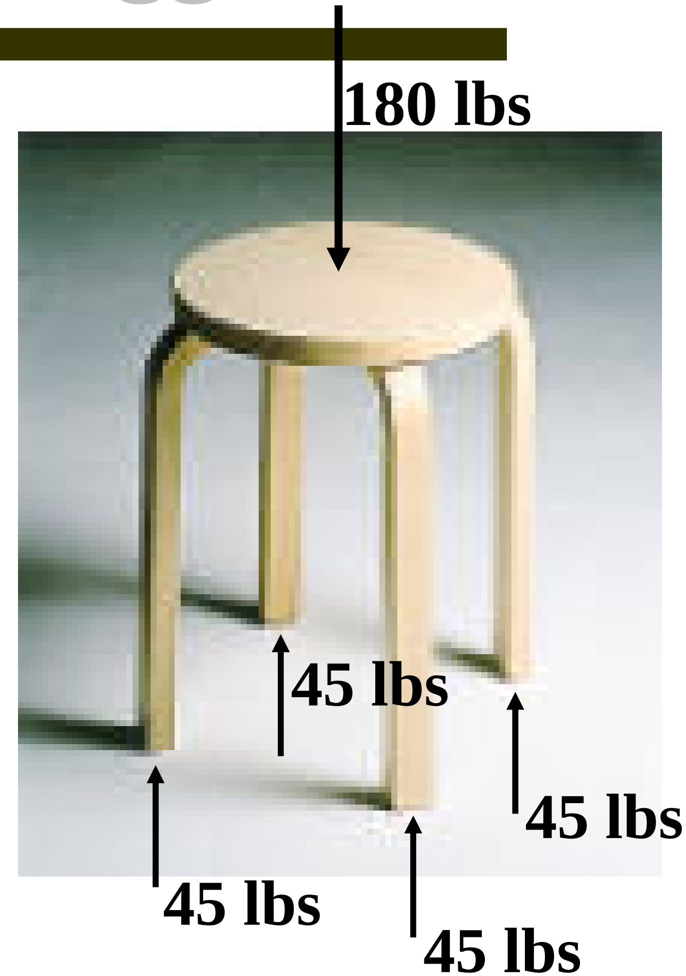


Elastic Solution for 4-Legged Stool

Design for a person weighing 180 pounds

→ 45 pounds/leg

But if one leg does not touch the floor…



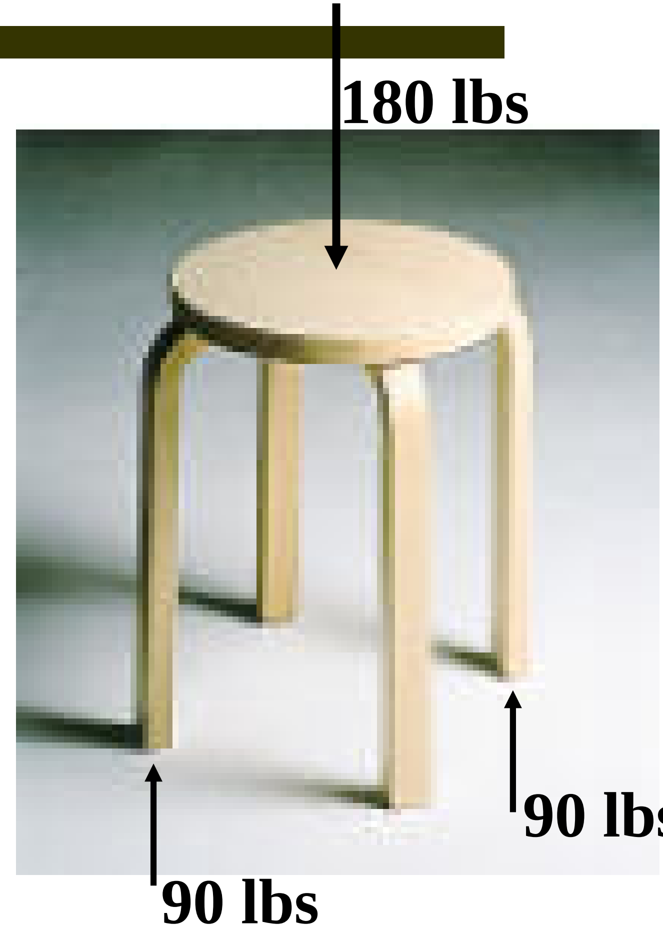
Four-Legged Stool

If one leg doesn’t touch the floor, the force in it is zero.

If one leg is zero, then the opposite leg is also zero by moment equilibrium.

The two remaining legs carry all of the load:

→ <u>**90 pounds/leg**</u>



Four-Legged Stool

Therefore…

All four legs must be *designed* to carry the 90 pounds (since any two legs could be loaded)

Four-Legged Stool

If the elastic solution is accepted, with a load in each leg of 45 pounds, then assuming a safety factor of 3 gives:

P_{cr} = 3(45 lbs) = 135 lbs

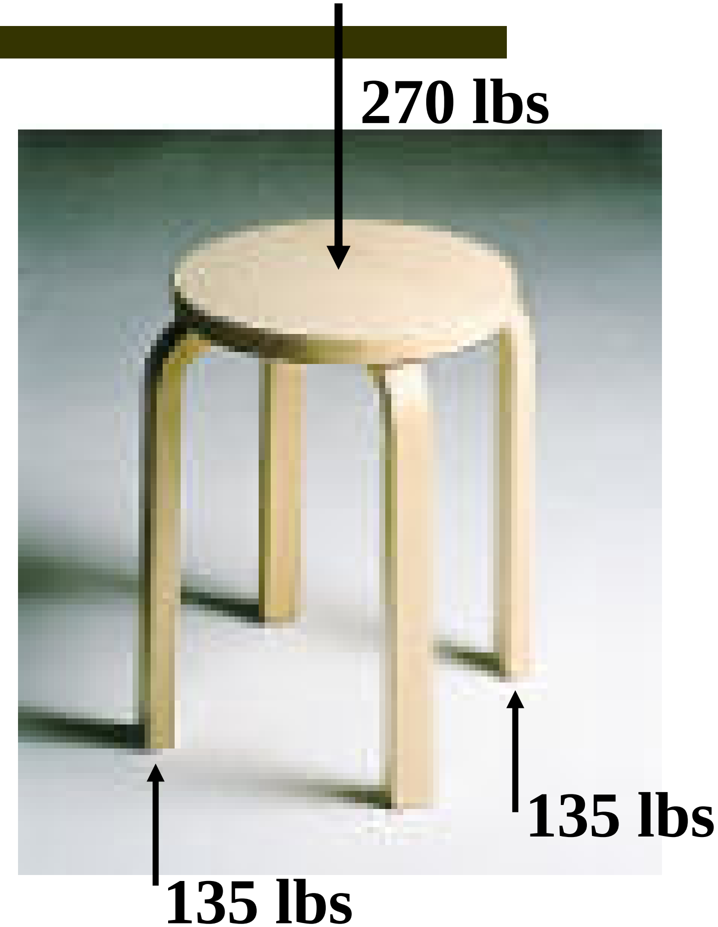
And each leg would be designed to fail at a load of 135 pounds

Four-Legged Stool

Now imagine the load is increased to cause failure

When load is 270 lbs, the two legs will begin to fail

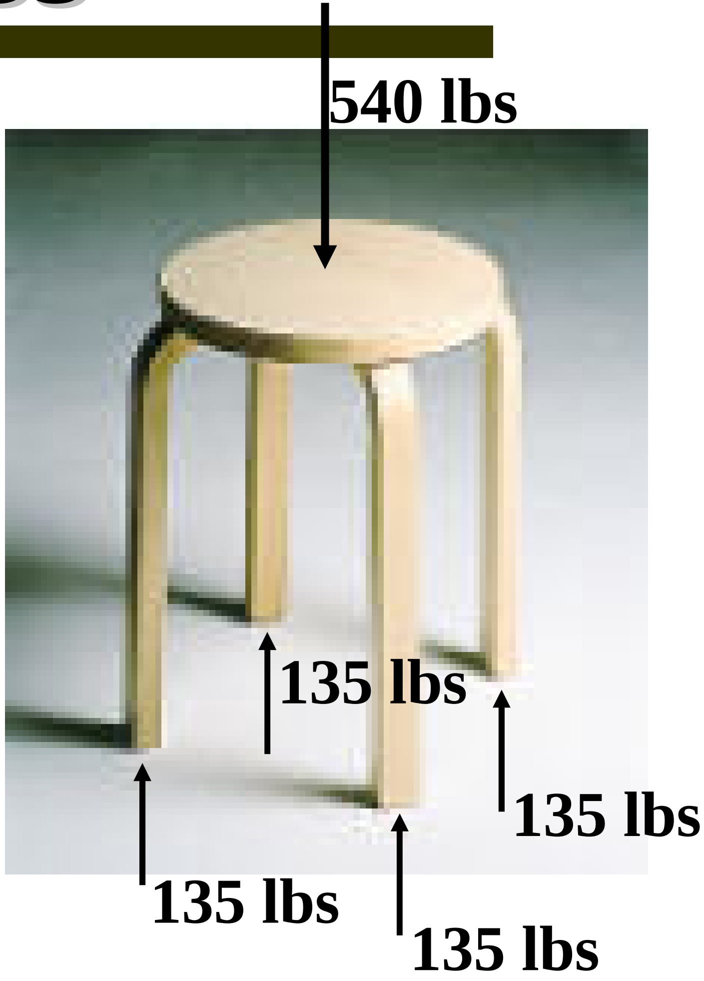
As they “squash,” the other two legs will start to carry load also



Collapse of a 4-Legged Stool

At final collapse state, all four legs carry 135 pounds and the stool carries 540 pounds.

This occurs only if the structure is ductile (ie, if the legs can "squash")



Ductile Collapse

So small imperfections do not matter, as long as the structural elements are ductile

The forces in a hyperstatic structure cannot be known exactly, and the solutions depend on the assumptions for the supports

Internal forces are *unknowable*

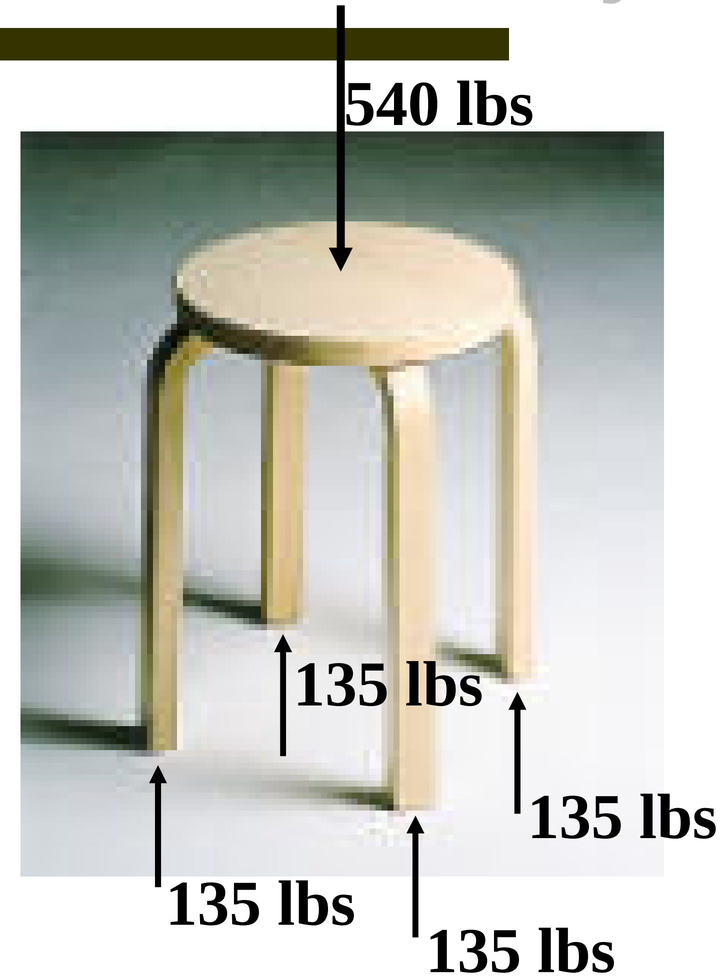
(only the structure knows)

Lower Bound Theorem of Plasticity

If you can find one possible set of forces, then the structure can find a possible set of forces

It does not have to be correct, as long as the structure has capacity for displacements (ductility)

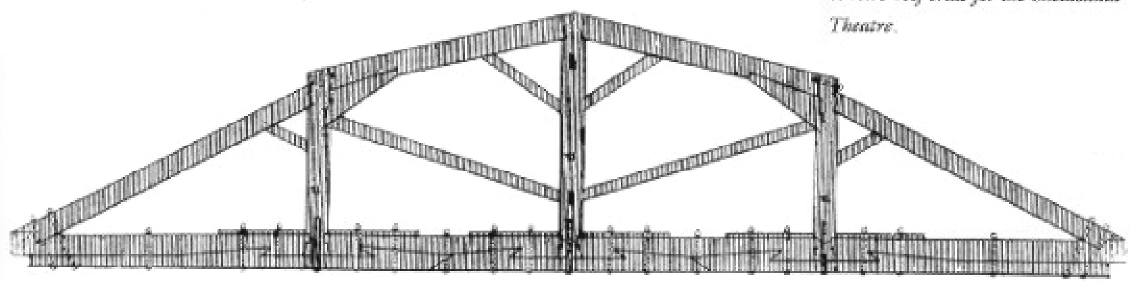
For indeterminate structures, we cannot be certain of the internal state of the forces



Examples of Statically Determinate Structures

- **Unstressed by support movements or temperature changes**
  - **Three-legged stool**
  - **Simply supported beam**
  - **Cantilever beam**
  - **Three-hinged arch**
  - **Triangulated truss**

Determinate or indeterminate?



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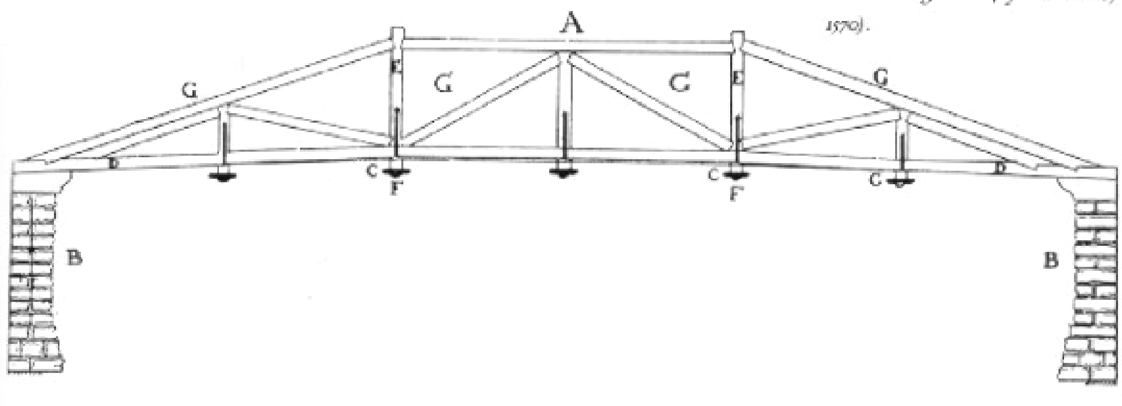

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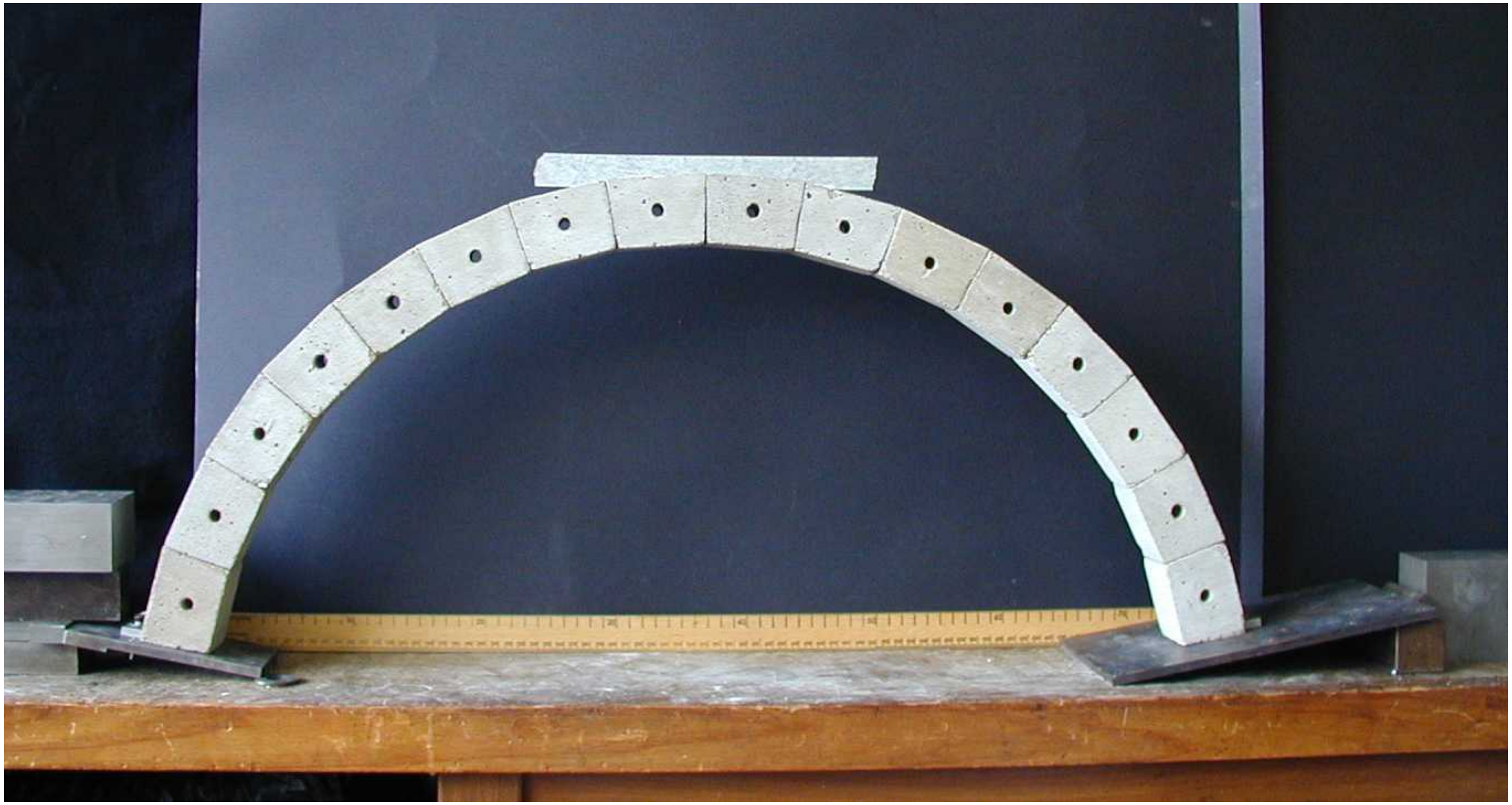
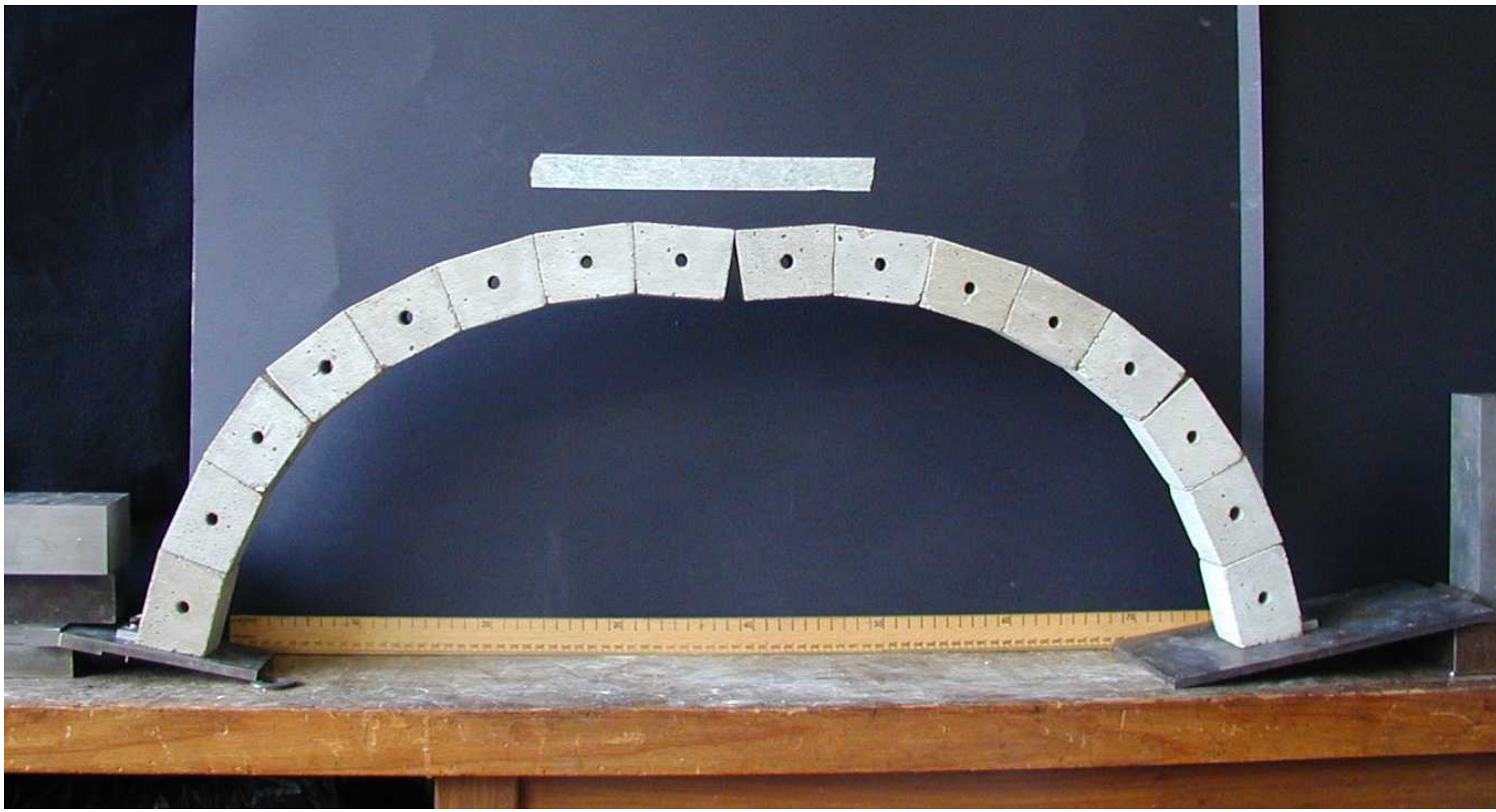
Timber bridge truss (after Palladio, 1570).

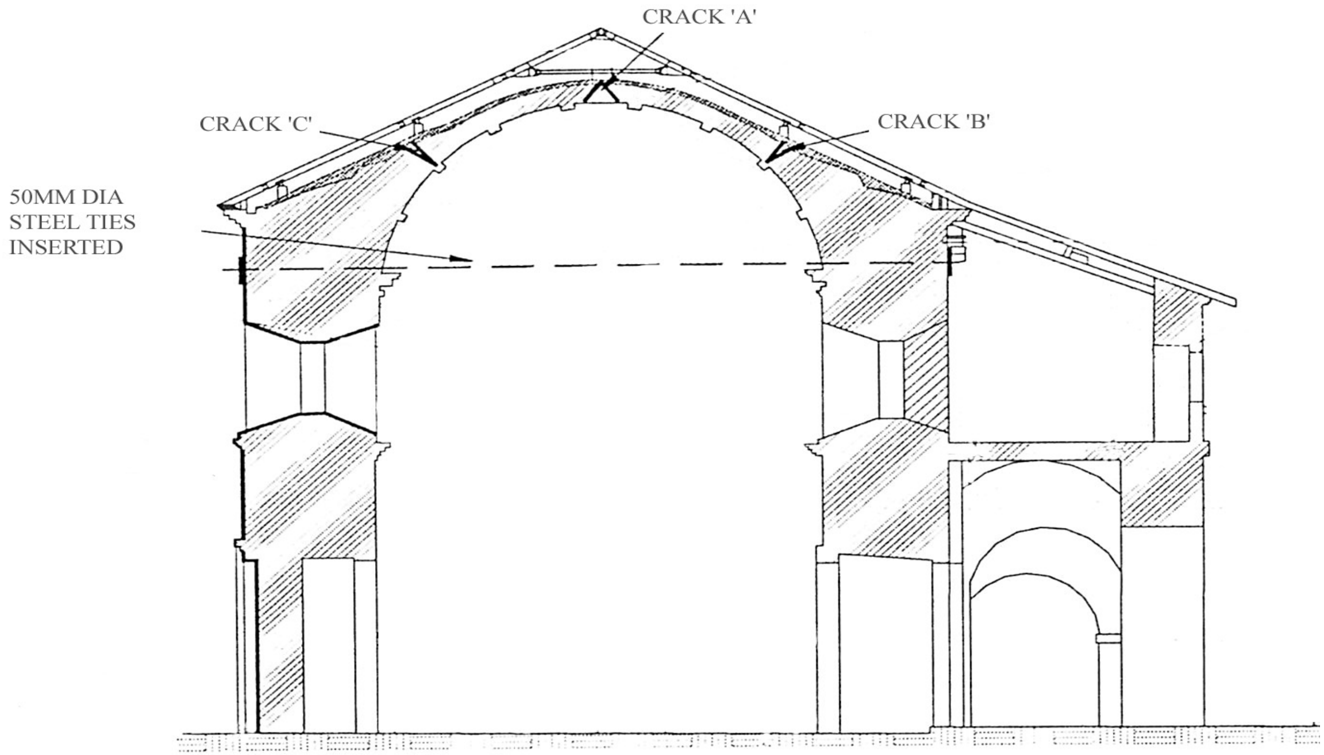
Model Arch Experiment



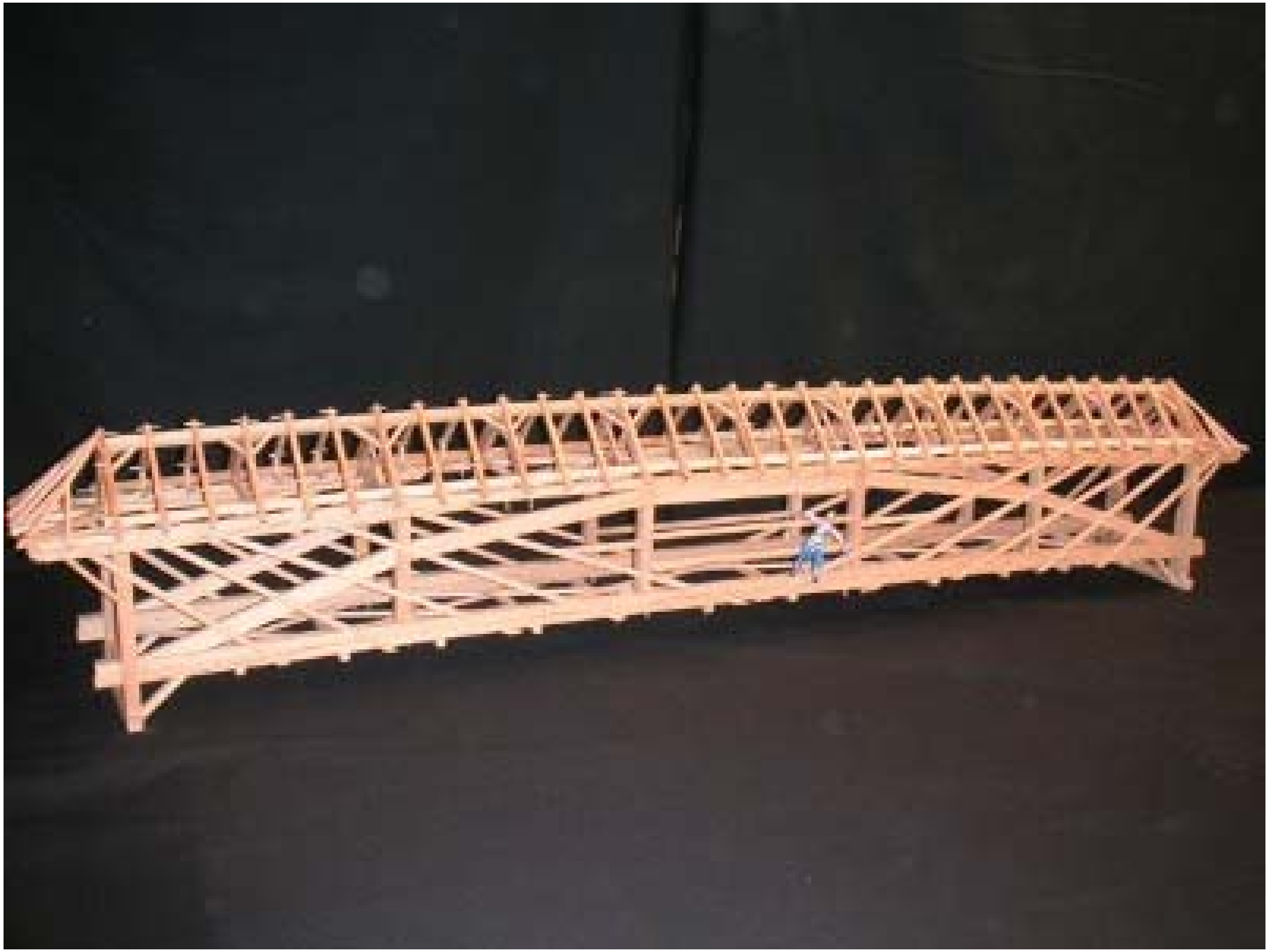
Model Arch Experiment



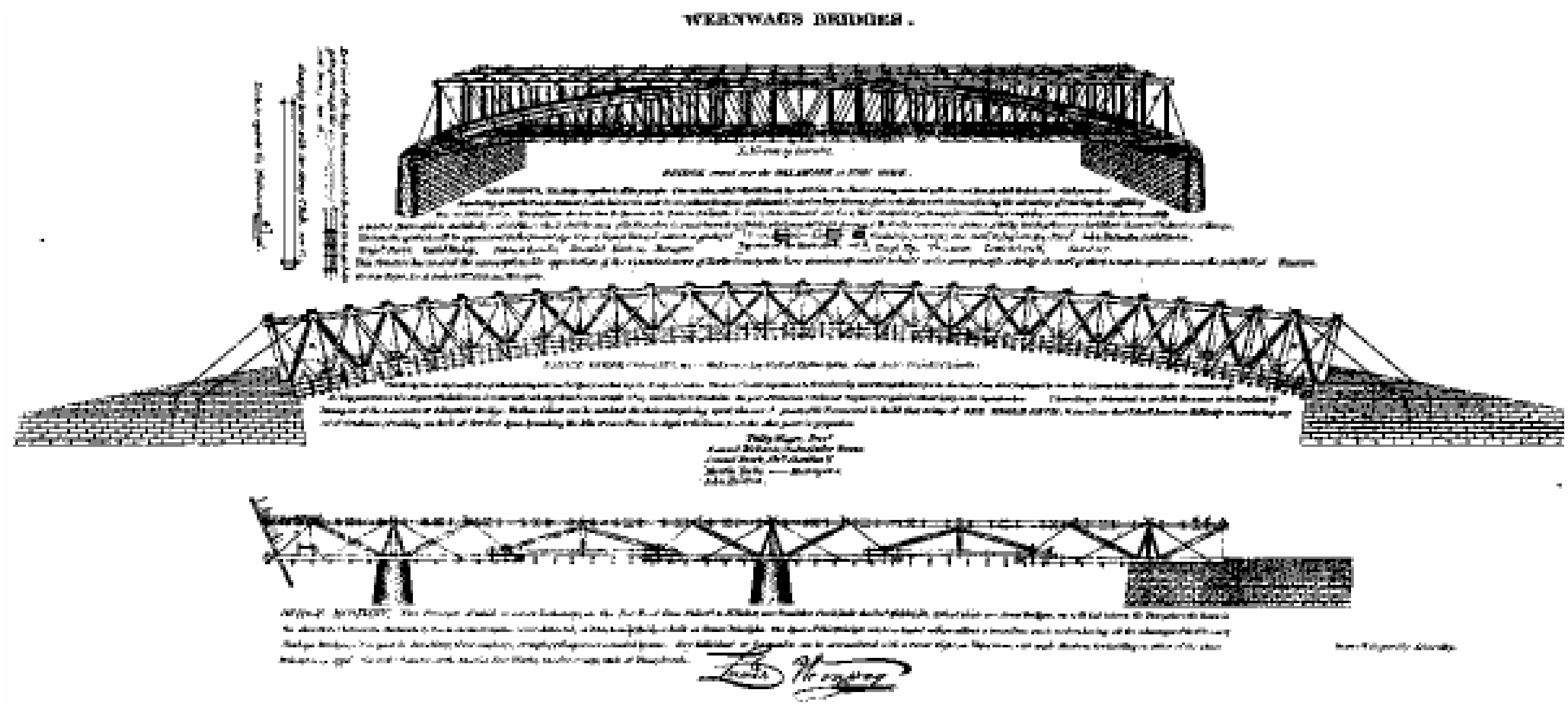
Case Study:
16th C. Church in Goa, India



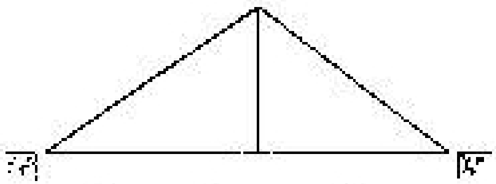
Determinate or indeterminate?



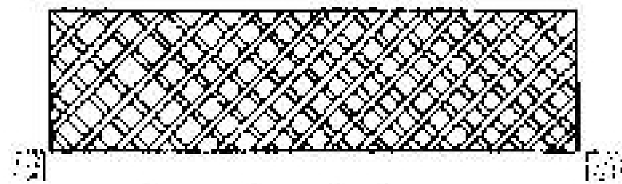
"Colossus" over Schuylkill River in Philadelphia



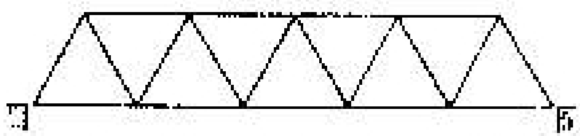
Determinate or indeterminate?



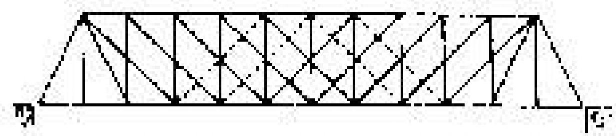
King Post Truss



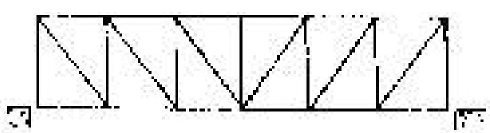
Lattice Truss



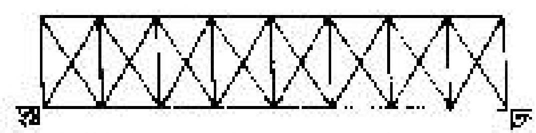
James Warren Truss



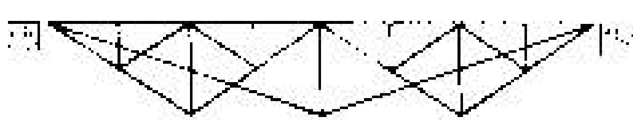
Squire Whipple Truss



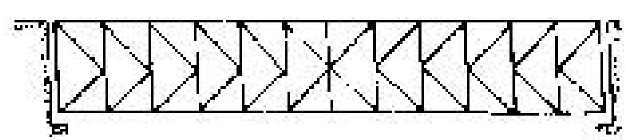
Pratt Truss



William Howe Truss

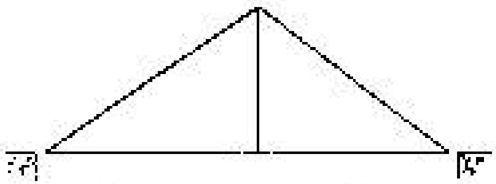


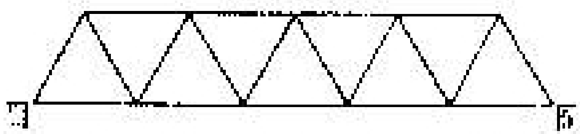
Albert Fink Truss

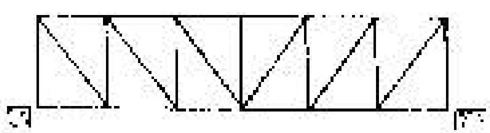


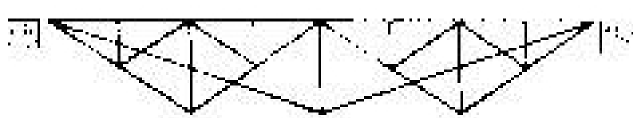
K-Truss

Statically determinate

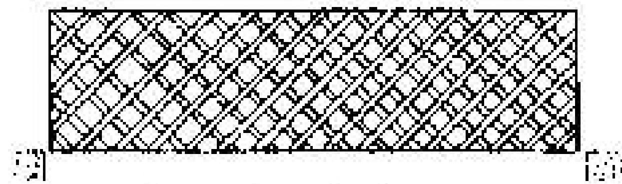

King Post Truss

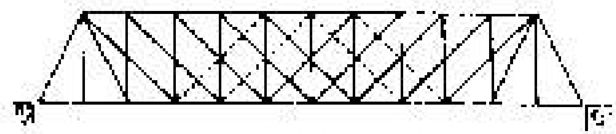

James Warren Truss

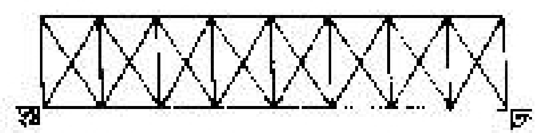

Pratt Truss

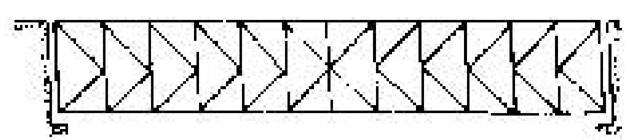

Albert Fink Truss

Statically indeterminate


Lattice Truss


Squire Whipple Truss


William Howe Truss


K-Truss

Hammer-Beam Roof systems

- Typical in England
  - Case study of Westminster Hall
  - Used to help span longer distances
- Limit to span for a single beam
  - Diameter of trees
  - Length of elements
  - Consistency of materials

Conclusions

- **Like traditional masonry structures…**
  - **Timber has low stresses**
  - **Most are statically indeterminate. There is not one answer for internal forces; depends on supports and assumptions.**
- **For indeterminate structures, you must explore various possibilities (support conditions are most important)**
- **Equilibrium is the bedrock of our analysis**

Analysis of Timber Structures

- Static equilibrium is the guiding principle (stresses are low)
- Assumptions greatly influence the results (joints and supports)
- Statically determinate or indeterminate structures behave in fundamentally different ways. Be clear about which type of structure you are dealing with.

Conclusions

- The distanced spanned by wood is limited by the size of trees
- Trusses allow for longer spans
- Many subjects of historic timber construction have not been studied
- Apply simple truss analysis in most cases