5.73

Quiz 25 ANSWERS

1.

$$\mathbf{L}\cdot\mathbf{S} = \tfrac{1}{2}\left[\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2\right]$$
$$\mathbf{L}\cdot\mathbf{S} = \mathbf{L}_z\mathbf{S}_z + \tfrac{1}{2}\left[\mathbf{L}_+\mathbf{S}_- + \mathbf{L}_-\mathbf{S}_+\right]$$
$$\mathbf{H}^{\mathrm{SO}} = \zeta\mathbf{L}\bullet\mathbf{S}$$
$$\mathbf{H}^{\mathrm{Zeeman}} = -\gamma B_z\left(\mathbf{L}_z + 2\mathbf{S}_z\right)$$

A. For a $^2F_{5/2}$ state:

(i) What is **S**? **S** = 1/2

(ii) What is **L**? **L** ≡ 3

(iii) What is **J**? **J** = 5/2

(iv) Evaluate $\left\langle {}^2F_{5/2}, M_J \middle| \mathbf{H}^{\mathrm{SO}} \middle| {}^2F_{5/2}, M_J \right\rangle$.

$$= \frac{1}{2}\zeta\left[\frac{5}{2}\frac{7}{2} - 3\cdot 4 - \frac{1}{2}\frac{3}{2}\right]h^2$$
$$= \frac{1}{2}\zeta\left[8-12\right] = -2\zeta h^2$$

(v) Evaluate $\left\langle {}^2F_{7/2}, M_J = 7/2 \middle| \mathbf{H}^{\mathrm{Zeeman}} \middle| {}^2F_{7/2}, M_J = 7/2 \right\rangle$.

$= -\gamma B_z[3 + 1] = = -4\gamma B_z$
because $|{}^2F_{7/2}, M_J = 7/2\rangle$ is an extreme state
$= |{}^2F_{7/2}, M_L = 3, M_S = 1/2\rangle$

B.

Apply $\mathbf{J}^- = \mathbf{L}^- + \mathbf{S}^-$ to both sides of $\left|{}^2F_{7/2}, M_J = 7/2\right\rangle = \left|{}^2F, M_L = 3, M_S = 1/2\right\rangle$ where the two basis states are the "extreme" states, respectively in the coupled and uncoupled basis sets:

$$\mathbf{J}^-\left|{}^2F_{7/2}, M_J = 7/2\right\rangle = \left(\mathbf{L}^- + \mathbf{S}^-\right)\left|{}^2F, M_L = 3, M_S = 1/2\right\rangle.$$

Find the normalized combination of $\left|{}^2F, M_L = 2, M_S = 1/2\right\rangle$ and $\left|{}^2F, M_L = 3, M_S = -1/2\right\rangle$ that corresponds to $\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$.

$$\mathbf{J}^-\left|{}^2F_{7/2}, M_J = 7/2\right\rangle = \hbar\left[7/2\cdot 9/2 - 7/2\cdot 5/2\right]^{1/2}\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$$
$$= \hbar[7]\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$$
$$\left(\mathbf{L}^- + \mathbf{S}^-\right)\left|{}^2F_{7/2}, M_L = 3, M_S = 1/2\right\rangle = \hbar\left[3\cdot 4 - 3\cdot 2\right]^{1/2}$$
$$\times\left|{}^2F_{7/2}, M_L = 2, M_S = -1/2\right\rangle + \hbar\left[1/2\cdot 3/2 - 1/2(-1/2)\right]^{1/2}$$
$$\times\left|{}^2F_{7/2}, M_L = 3, M_S = -1/2\right\rangle$$
$$\left|{}^2F_{7/2}, M_J = 5/2\right\rangle = \left(\frac{6}{7}\right)^{1/2}\left|{}^2F_{7/2}, M_L = 2, M_S = 1/2\right\rangle$$
$$+\left(\frac{1}{7}\right)^{1/2}\left|{}^2F_{7/2}, M_L = 3, M_S = -1/2\right\rangle$$

C. Verify that $|{}^2F_{5/2}, M_J=5/2\rangle = -7^{-1/2}|{}^2F, M_L = 2, M_S = 1/2\rangle + (6/7)^{1/2}|{}^2F, M_L = 3, M_S = -1/2\rangle$ is orthogonal to $\left|{}^2F_{7/2}, M_J = 5/2\right\rangle$ which you obtained in part B.

$$\left\langle {}^2F_{7/2}, 5/2\middle|{}^2F_{5/2}, 5/2\right\rangle = -\left(\frac{6}{7}\right)^{1/2}\left(\frac{1}{7}\right)^{1/2} + \left(\frac{1}{7}\right)^{1/2}\left(\frac{6}{7}\right)^{1/2} = 0$$

D. Evaluate $\left\langle {}^2F_{5/2}, M_J = 5/2\middle|\mathbf{H}^{\text{Zeeman}}\middle|{}^2F_{5/2}, M_J = 5/2\right\rangle$.

$$-\gamma B_z\left\langle {}^2F_{5/2}, M_J = 5/2\middle|\mathbf{L}_z + 2\mathbf{S}_z\middle|{}^2F_{5/2}, M_J = 5/2\right\rangle$$
$$= -\gamma B_z\left[(2+1)\frac{1}{7} + (3-1)\frac{6}{7}\right] = -\hbar\gamma B_z\left(\frac{15}{7}\right)$$

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