# 5.73

## Quiz 27 ANSWERS

$$\mathbf{T}^{(1)}_{\pm 1} = \mp 2^{-1/2}(\mathbf{x} \pm i\mathbf{y}), \quad \mathbf{T}^{(1)}_{0} = \mathbf{z}$$

$$\left[\mathbf{J}_i, \mathbf{q}_j\right] = i\hbar \sum_k \varepsilon_{ijk} \mathbf{q}_k$$

A. Show that $\left[\mathbf{J}_z, \mathbf{T}^{(1)}_{-1}\right] = \hbar \mathbf{T}^{(1)}_{-1}$

$$\mathbf{T}^{(1)}_{-1} = +2^{-1/2}(\mathbf{x} - i\mathbf{y})$$
$$\left[\mathbf{J}_z, \mathbf{T}^{(1)}_{-1}\right] = 2^{-1/2}\left(\left[\mathbf{J}_z, \mathbf{x}\right] - \left[\mathbf{J}_z, i\mathbf{y}\right]\right)$$
$$= 2^{-1/2} i\hbar(\mathbf{y} + i\mathbf{x}) = 2^{-1/2}\hbar(-\mathbf{x} + i\mathbf{y})$$
$$= -2^{-1/2}\hbar \mathbf{T}^{(1)}_{-1}$$

B. Show that $\left[\mathbf{J}_-, \mathbf{T}^{(1)}_{-1}\right] = 0$

$$\left[\mathbf{J}_-, T^{(1)}_{-1}\right] = \left[\mathbf{J}_x - i\mathbf{J}_y, 2^{-1/2}(\mathbf{x} - i\mathbf{y})\right]$$
$$= 2^{-1/2}\left(\left[\mathbf{J}_x, x\right] - i\left[\mathbf{J}_x, y\right] - i\left[\mathbf{J}_y, x\right] - \left[\mathbf{J}_y, y\right]\right)$$
$$= 2^{-1/2}\left(0 - i(i\hbar)z - i(i\hbar)(-z) + 0\right)$$
$$= 2^{-1/2}\left[\hbar z - \hbar z\right] = 0$$

C. If $\mathbf{T}^{(\omega)}_{\mu}$ satisifes the $\left[\mathbf{J}_{\pm}, \mathbf{T}^{(\omega)}_{\mu}\right] = \hbar\left[\omega(\omega+1) - \mu(\mu \pm 1)\right]^{1/2} \mathbf{T}^{(\omega)}_{\mu \pm 1}$ and $\left[\mathbf{J}_z, \mathbf{T}^{(\omega)}_{\mu}\right] = \hbar\mu \mathbf{T}^{(\omega)}_{\mu}$ definitions, then we are supposed to know all selection rules for matrix elements of $\mathbf{T}^{(\omega)}_{\mu}$ in the $|JM_J\rangle$ basis set. What are the ΔJ and $\Delta M_J$ selection rules for $\mathbf{T}^{(3)}_{+2}$?

ΔJ = ±3, ±2, ±1, 0
ΔM = +2

ΔJ = $\qquad\qquad$ $\Delta M_J$ =

D. Show that the operator $(\mathbf{L}_+)^2$ satisfies at least one part of the commutation rule definition for $\mathbf{T}^{(2)}_{+2}$: $\left[\mathbf{J}_z, \mathbf{T}^{(2)}_{2}\right] = \hbar 2 \mathbf{T}^{(2)}_{2}$.

$$\left[\mathbf{J}_z, \mathbf{L}^2_+\right] = \mathbf{L}_+\left[\mathbf{J}_z, \mathbf{L}_+\right] + \left[\mathbf{J}_z, \mathbf{L}_+\right]\mathbf{L}_+$$