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Quiz 27 ANSWERS

$$\mathbf{T}_{\pm 1}^{(1)} = \mp 2^{-1/2} (\mathbf{x} \pm i\mathbf{y}), \quad \mathbf{T}_0^{(1)} = \mathbf{z}$$

$$[\mathbf{J}_i, \mathbf{q}_j] = i\hbar \sum_k \varepsilon_{ijk} \mathbf{q}_k$$

A. Show that $[\mathbf{J}_z, \mathbf{T}_{-1}^{(1)}] = \hbar \mathbf{T}_{-1}^{(1)}$

$$\begin{aligned} \mathbf{T}_{-1}^{(1)} &= +2^{-1/2} (\mathbf{x} - i\mathbf{y}) \\ [\mathbf{J}_z, \mathbf{T}_{-1}^{(1)}] &= 2^{-1/2} ([\mathbf{J}_z, \mathbf{x}] - [\mathbf{J}_z, i\mathbf{y}]) \\ &= 2^{-1/2} i\hbar (\mathbf{y} + i\mathbf{x}) = 2^{-1/2} \hbar (-\mathbf{x} + i\mathbf{y}) \\ &= -2^{-1/2} \hbar \mathbf{T}_{-1}^{(1)} \end{aligned}$$

B. Show that $[\mathbf{J}_-, \mathbf{T}_{-1}^{(1)}] = 0$

$$\begin{aligned} [\mathbf{J}_-, \mathbf{T}_{-1}^{(1)}] &= [\mathbf{J}_x - i\mathbf{J}_y, 2^{-1/2} (\mathbf{x} - i\mathbf{y})] \\ &= 2^{-1/2} ([\mathbf{J}_x, x] - i[\mathbf{J}_x, y] - i[\mathbf{J}_y, x] - [\mathbf{J}_y, y]) \\ &= 2^{-1/2} (0 - i(i\hbar)z - i(i\hbar)(-z) + 0) \\ &= 2^{-1/2} [\hbar z - \hbar z] = 0 \end{aligned}$$

C. If $\mathbf{T}_\mu^{(\omega)}$ satisfies the $[\mathbf{J}_\pm, \mathbf{T}_\mu^{(\omega)}] = \hbar[\omega(\omega+1) - \mu(\mu\pm 1)]^{1/2} \mathbf{T}_{\mu\pm 1}^{(\omega)}$ and $[\mathbf{J}_z, \mathbf{T}_\mu^{(\omega)}] = \hbar\mu \mathbf{T}_\mu^{(\omega)}$ definitions, then we are supposed to know all selection rules for matrix elements of $\mathbf{T}_\mu^{(\omega)}$ in the $|JM_J\rangle$ basis set. What are the ΔJ and ΔM_J selection rules for $\mathbf{T}_{+2}^{(3)}$?

$$\begin{aligned} \Delta J &= \pm 3, \pm 2, \pm 1, 0 \\ \Delta M &= +2 \end{aligned}$$

D. Show that the operator $(\mathbf{L}_+)^2$ satisfies at least one part of the commutation rule definition for $\mathbf{T}_{+2}^{(2)}$: $[\mathbf{J}_z, \mathbf{T}_{+2}^{(2)}] = \hbar 2 \mathbf{T}_{+2}^{(2)}$.

$$[\mathbf{J}_z, \mathbf{L}_+^2] = \mathbf{L}_+ [\mathbf{J}_z, \mathbf{L}_+] + [\mathbf{J}_z, \mathbf{L}_+] \mathbf{L}_+$$

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