### 5.73

## Quiz 35 ANSWERS

For $\mathrm{p}^{3}$ configuration:
The $M_{L}=0, M_{S}=1 / 2$ block contains $\|1 \alpha 0 \alpha-1 \beta\|,\|\mid 1 \alpha 0 \beta-1 \alpha\|$, and $\|1 \beta 0 \alpha-1 \alpha\|$. For $M_{L}=0, \mathbf{L}^{2}$ may be replaced by $\mathbf{L}_{+} \mathbf{L}_{-}$.
$\mathbf{L}^{2}| | 1 \alpha 0 \alpha-1 \beta \|=\hbar^{2}[2| | 1 \alpha 0 \alpha-1 \beta| |-2| | 1 \alpha 0 \beta-1 \alpha| |]$
$\mathbf{L}^{2}| | 1 \alpha 0 \beta-1 \alpha| |=\hbar^{2}[4| | 1 \alpha 0 \beta-1 \alpha| |-2| | 1 \beta 0 \alpha-1 \alpha| |-2| | 1 \alpha 0 \alpha-1 \beta| |]$
$\mathbf{L}^{2} \mid 11 \beta 0 \alpha-1 \alpha \|=\hbar^{2}[2| | 1 \beta 0 \alpha-1 \alpha \|-2| | 1 \alpha 0 \beta-1 \alpha| |]$
A. Set up the $\mathbf{L}^{2}$ matrix for the $\mathrm{M}_{\mathrm{L}}=0, \mathrm{M}_{\mathrm{S}}=1 / 2$ block.

$$
\begin{aligned}
& \text { Row Label } \\
& \|1 \alpha 0 \alpha-1 \beta\| \\
& \|1 \alpha 0 \beta-1 \alpha\| \\
& \|\mid \beta 0 \alpha-1 \alpha\|
\end{aligned} \quad \hbar^{2}\left(\begin{array}{ccc}
\mathbf{L}^{2} \text { matrix } \\
2 & -2 & 0 \\
-2 & 4 & -2 \\
0 & -2 & 2
\end{array}\right)
$$

B. Find the normalized eigenvector of $\mathbf{L}^{2}$ that corresponds to
$\left|{ }^{2} D M_{L}=0, M_{S}=1 / 2\right\rangle$
$\left(\mathbf{L}^{2}\right)\left(\begin{array}{l}a \\ b \\ c\end{array}\right)=\hbar^{2} 6\left(\begin{array}{l}a \\ b \\ c\end{array}\right) \quad 1=\left[|a|^{2}+|b|^{2}+|c|^{2}\right]^{1 / 2}$
$\mathbf{L}^{2}$
$\left.\hbar^{2}\left(\begin{array}{ccc}2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2\end{array}\right)\left(\begin{array}{c}\downarrow \\ a \\ b \\ c\end{array}\right)=\hbar^{2} \stackrel{\downarrow}{\uparrow} \begin{array}{c}\text { eigenvector } \\ \end{array}\right)$

$$
\begin{array}{lll} 
& L=2 & \mathbf{L}^{2}=\hbar^{2} 2 \cdot 3=\hbar^{2} 6 \\
2 a-2 b=6 a & -2 b=4 a & b=-2 a \\
-2 a+4 b-2 c=6 b & \text { not needed } \\
-2 b+2 c=6 c & -2 b=4 c & b=-2 c \\
& \mathrm{a}=\mathrm{c} & \\
{\left[a^{2}+b^{2}+c^{2}\right]^{1 / 2}=1} & & \\
{\left[a^{2}+4 a^{2}+a^{2}\right]^{1 / 2}=1} & {\left[6 a^{2}\right]^{1 / 2}=1} & a=6^{-1 / 2}
\end{array}
$$

Thus: $a=6^{-1 / 2}, b=-2 \cdot 6^{-1 / 2}, c=6^{-1 / 2}$

Verify:

$$
\begin{aligned}
\hbar^{2}\left(\begin{array}{ccc}
2 & -2 & 0 \\
-2 & 4 & -2 \\
0 & -2 & 2
\end{array}\right) 6^{-1 / 2}\left(\begin{array}{c}
1 \\
-2 \\
1
\end{array}\right) & =\hbar^{2} 6^{-1 / 2}\left(\begin{array}{c}
6 \\
-12 \\
6
\end{array}\right) \\
& =6 \hbar^{2} 6^{-1 / 2}\left(\begin{array}{c}
1 \\
-2 \\
1
\end{array}\right)
\end{aligned}
$$

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### 5.73 Quantum Mechanics I

Fall 2018

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