

5.73

Quiz 5 ANSWERS

$$\delta(a(x-b)) = \frac{1}{|a|} \delta(x-b).$$

$$\delta(g(x)) = \sum_i \left| \frac{dg(x_i)}{dx} \right|^{-1} \delta(x-x_i); x_i \text{ are zeroes of } g(x).$$

$\delta'(x-d)$ means derivative of δ -function evaluated at $x=d$.

Infinite box of width L : $E_n = \frac{n^2 h^2}{8mL^2}, k_n = \pm \frac{n\pi}{L}, n = 1, 2, \dots$

1. $f(x) = (x-3)(4x+8)$

A. Evaluate $\int \delta(x-d)f(x)dx$. $= f(d) = (d-3)(4d+8)$

B. Evaluate $\int \delta'(x-d)f(x)dx$. $= f'(d) = 8d-4$

C. Express $\delta(f(x))$ in terms of $\delta(x-x_i)$.

zeroes of $f(x)$ are $x = 3$ and $x = -2$

$$\left. \frac{df}{dx} \right|_{x=3} = 20, \quad \left. \frac{df}{dx} \right|_{x=-2} = -20$$

$$\delta(f(x)) = \frac{1}{20} [\delta(x-3) + \delta(x+2)]$$

2. Density of States.

A. Compute $\frac{dn}{dk}$.

$$= 1 / \frac{dk}{dn} = 1 / (\pm \pi / L) = \pm L / \pi$$

B. Compute $\frac{dn}{dE}$.

$$n = \left(\frac{8mL^2 E_n}{h^2} \right)^{1/2} \quad \frac{dn}{dE} = \frac{1}{2} \left(\frac{8mh^2}{h^2} \right) E_n^{-1/2}$$

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