

5.73

Quiz 26

$$\mathbf{H}^{SO} = \frac{\zeta}{\hbar} \mathbf{L} \cdot \mathbf{S} = \frac{1}{2} \frac{\zeta}{\hbar} [\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2]$$

$$\mathbf{H}^{\text{Zeeman}} = -\gamma B_z (\mathbf{L}_z + 2\mathbf{S}_z)$$

Using ladders plus orthogonality, you should have obtained

$$|^2F_{7/2}, M_J = 5/2\rangle = \left(\frac{6}{7}\right)^{1/2} |^2F, M_L = 2, M_S = 1/2\rangle + \left(\frac{1}{7}\right)^{1/2} |^2F, M_L = 3, M_S = -1/2\rangle$$

and

$$|^2F_{5/2}, M_J = 5/2\rangle = -\left(\frac{1}{7}\right)^{1/2} |^2F, M_L = 2, M_S = 1/2\rangle + \left(\frac{6}{7}\right)^{1/2} |^2F, M_L = 3, M_S = -1/2\rangle.$$

A. Compute $E^{SO} (^2F_{7/2}) = \langle ^2F_{7/2}, M_J = 5/2 | \mathbf{H}^{SO} | ^2F_{7/2}, M_J = 5/2 \rangle$.

B. Compute $E^{SO} (^2F_{5/2}) = \langle ^2F_{5/2}, M_J = 5/2 | \mathbf{H}^{SO} | ^2F_{5/2}, M_J = 5/2 \rangle$.

C. Compute $E^Z (^2F_{5/2}, 5/2) = \langle ^2F_{5/2}, M_J = 5/2 | \mathbf{H}^{\text{Zeeman}} | ^2F_{5/2}, M_J = 5/2 \rangle$.

D. Compute the off-diagonal matrix element:

$$\mathbf{H}_{5/2, 5/2; 7/2, 5/2}^Z = \langle ^2F_{5/2}, M_J = 5/2 | \mathbf{H}^{\text{Zeeman}} | ^2F_{7/2}, M_J = 5/2 \rangle.$$

- E. Use second order perturbation theory to derive the Zeeman tuning rate for the nominal ${}^2F_{5/2}$, $M_J = 5/2$ state. The Zeeman tuning rate is $\frac{dE}{dB_Z}$.

$$E^{(0)}({}^2F_{5/2}, M_J = 5/2) = E^{SO}({}^2F_{5/2}) + E^Z({}^2F_{5/2}, 5/2)$$

$$E^{(2)}({}^2F_{5/2}, M_J = 5/2) = \frac{|\mathbf{H}_{5/2, 5/2; 7/2, 5/2}^Z|^2}{E^{SO}({}^2F_{5/2}) - E^{SO}({}^2F_{7/2})}$$

MIT OpenCourseWare
<https://ocw.mit.edu/>

5.73 Quantum Mechanics I
Fall 2018

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.