5.73

Quiz 26

$$\mathbf{H}^{SO} = \frac{\zeta}{\hbar}\mathbf{L}\bullet\mathbf{S} = \frac{1}{2}\frac{\zeta}{\hbar}\left[\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2\right]$$
$$\mathbf{H}^{\text{Zeeman}} = -\gamma B_z(\mathbf{L}_z + 2\mathbf{S}_z)$$

Using ladders plus orthogonality, you should have obtained

$$\left|{}^2F_{7/2}, M_J = 5/2\right\rangle = \left(\frac{6}{7}\right)^{1/2}\left|{}^2F, M_L = 2, M_S = 1/2\right\rangle + \left(\frac{1}{7}\right)^{1/2}\left|{}^2F, M_L = 3, M_S = -1/2\right\rangle$$

and

$$\left|{}^2F_{5/2}, M_J = 5/2\right\rangle = -\left(\frac{1}{7}\right)^{1/2}\left|{}^2F, M_L = 2, M_S = 1/2\right\rangle + \left(\frac{6}{7}\right)^{1/2}\left|{}^2F, M_L = 3, M_S = -1/2\right\rangle.$$

A. Compute $E^{SO}\left({}^2F_{7/2}\right) = \left\langle {}^2F_{7/2}, M_J = 5/2 \middle| \mathbf{H}^{SO} \middle| {}^2F_{7/2}, M_J = 5/2 \right\rangle$.

B. Compute $E^{SO}\left({}^2F_{5/2}\right) = \left\langle {}^2F_{5/2}, M_J = 5/2 \middle| \mathbf{H}^{SO} \middle| {}^2F_{5/2}, M_J = 5/2 \right\rangle$.

C. Compute $E^{Z}\left({}^2F_{5/2}, 5/2\right) = \left\langle {}^2F_{5/2}, M_J = 5/2 \middle| \mathbf{H}^{Zeeman} \middle| {}^2F_{5/2}, M_J = 5/2 \right\rangle$.

D. Compute the off-diagonal matrix element:
$\mathbf{H}^{Z}_{5/2\,5/2;7/2\,5/2} = \left\langle {}^2F_{5/2}, M_J = 5/2 \middle| \mathbf{H}^{Zeeman} \middle| {}^2F_{7/2}, M_J = 5/2 \right\rangle$.

E. Use second order perturbation theory to derive the Zeeman tuning rate for the nominal $^2F_{5/2}$, $M_J = 5/2$ state. The Zeeman tuning rate is $\dfrac{dE}{dB_Z}$.

$$E^{(0)}\left({}^2F_{5/2}, M_J = 5/2\right) = E^{SO}\left({}^2F_{5/2}\right) + E^Z\left({}^2F_{5/2}, 5/2\right)$$

$$E^{(2)}\left({}^2F_{5/2}, M_J = 5/2\right) = \frac{\left|\mathbf{H}^Z_{5/2\,5/2;7/2\,5/2}\right|^2}{E^{SO}\left({}^2F_{5/2}\right) - E^{SO}\left({}^2F_{7/2}\right)}.$$