# 5.73

## Quiz 17

1.

$$\psi^{(0)}_{v_1 v_2} = \phi_{v_1}(x_1)\phi_{v_2}(x_2)$$

$$E^{(0)}_{v_1 v_2} = \hbar\left[\omega_1(v_1 + 1/2) + \omega_2(v_2 + 1/2)\right]$$

$$\mathbf{H}^{(1)} = k_{122} x_1 x_2^2$$

$$\mathbf{H}^{(1)}_{n_1 k_1 n_2 k_2} = k_{122}\left\langle n_1 \middle| x_1 \middle| k_1 \right\rangle \left\langle n_2 \middle| x_2^2 \middle| k_2 \right\rangle$$

$$\mathbf{x}_1 = \left(\frac{m_1\omega_1}{2\hbar}\right)^{1/2}\left(\mathbf{a}_1 + \mathbf{a}_1^{\dagger}\right)$$

$$\mathbf{x}_2 = \left(\frac{m_2\omega_2}{2\hbar}\right)^{1/2}\left(\mathbf{a}_2 + \mathbf{a}_2^{\dagger}\right)$$

A. Matrix elements of $\mathbf{H}^{(1)}$ have four indices, $n_1$ and $k_1$ specify the final and initial state quantum numbers for oscillator #1, $n_2$ and $k_2$ the final and initial state quantum numbers for oscillator #2. What are the selection rules for nonzero matrix elements of $\mathbf{H}^{(1)}$?

$n_1 - k_1 =$

$n_2 - k_2 =$

B. What are the zero-order energy differences that correspond to each of the nonzero matrix elements of $\mathbf{H}^{(1)}_{n_1 k_1 n_2 k_2}$?

$$E^{(0)}_{n_1 n_2} - E^{(0)}_{k_1 k_2} = \hbar\left[\omega_1(\qquad) + \omega_2(\qquad)\right]$$

C. Evaluate at least two of the six nonzero values of the off-diagonal elements of $\mathbf{H}^{(1)}$.

(i) e.g. $\mathbf{H}_{n_1 n_1+1 n_2 n_2+2} = \gamma\left[(n_1+1)(n_2+2)(n_2+1)\right]^{1/2}$

(ii) $\mathbf{H}_{n_1 n_1+1 n_2 n_2} = \gamma[ \qquad\qquad\qquad ]$

DO THIS ONE! (iii) $\mathbf{H}_{n_1 n_1+1 n_2 n_2-2} = \gamma[ \qquad\qquad\qquad ]$

(iv) $\mathbf{H}_{n_1 n_1-1 n_2 n_2+2} = \gamma[ \qquad\qquad\qquad ]$

(v) $\mathbf{H}_{n_1 n_1-1 n_2 n_2} = \gamma[ \qquad\qquad\qquad ]$

(vi) $\mathbf{H}_{n_1 n_1-1 n_2 n_2-2} = \gamma[ \qquad\qquad\qquad ]$

where $\gamma \equiv \left(\frac{m_1\omega_1}{2\hbar}\right)^{1/2}\left(\frac{m_2\omega_2}{2\hbar}\right)^{1}$.

D. The term $\frac{\left|\mathbf{H}^{(1)}_{n_1 n_1+1 n_2 n_2-2}\right|^2}{E^{(0)}_{n_1 n_2} - E^{(0)}_{n_1+1 n_2-2}}$

appears in the second-order perturbation summation for $E^{(2)}_{n_1 n_2}$.
Evaluate this term (based on your answers to parts B and C(iii)).

E. What happens to the term in part D if $\omega_1 = 2\omega_2$?