Tunneling Applications

Outline

- Barrier Reflection and Penetration
- Electron Conduction
- Scanning Tunneling Microscopy
- Flash Memory

Reflection of EM Waves and QM Waves

$$P = \hbar\omega \times \frac{photons}{s\, cm^2}$$

$$J = q \times \frac{electrons}{s\, cm^2}$$

$$P = \frac{|E|^2}{\eta}$$

$$J = \rho v = q|\psi|^2(\hbar k/m)$$

$$R = \frac{P_{reflected}}{P_{incident}} = |\frac{E_o^r}{E_o^i}|^2$$

$$R = \frac{J_{reflected}}{J_{incident}} = \frac{|\psi_B|^2}{|\psi_A|^2}$$

Then for optical material when $\mu = \mu_0$:

$$R = |\frac{B}{A}|^2 = |\frac{k_1 - k_2}{k_1 + k_2}|^2$$

$$= |\frac{n_1 - n_2}{n_1 + n_2}|^2$$

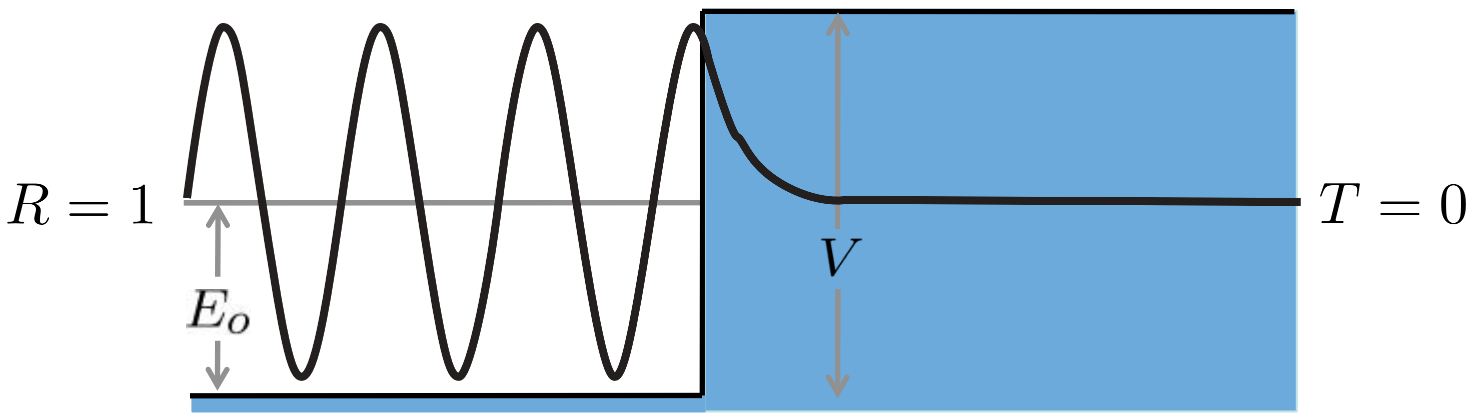
= probability of a particular photon being reflected

$$R = |\frac{B}{A}|^2 = |\frac{k_1 - k_2}{k_1 + k_2}|^2$$

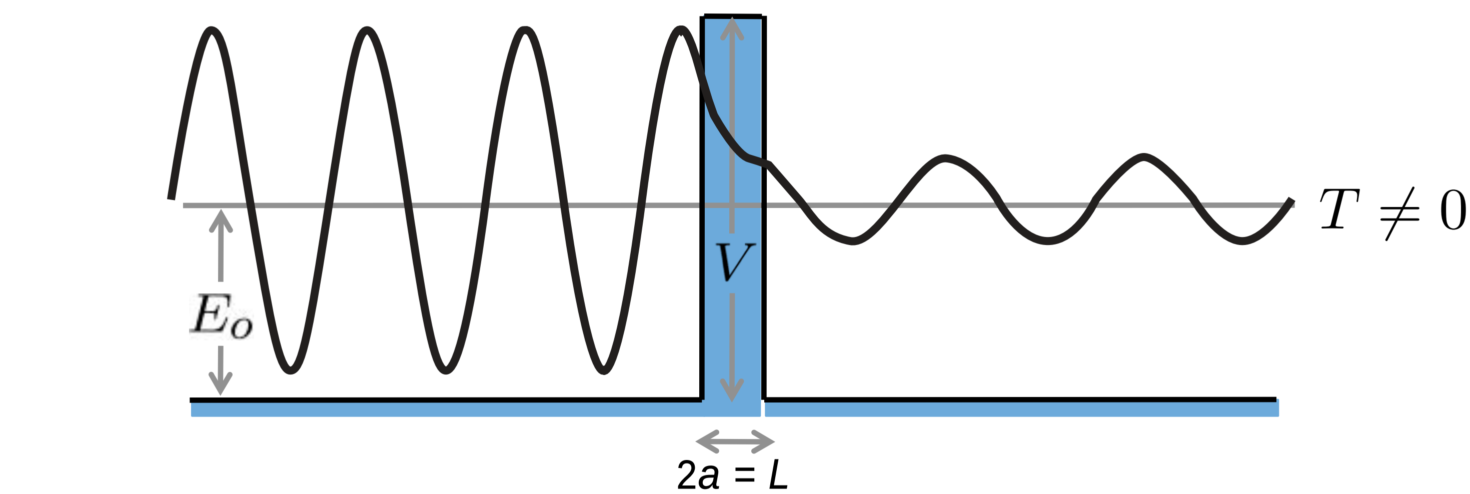
= probability of a particular electron being reflected

Quantum Tunneling Through a Thin Potential Barrier

Total Reflection at Boundary

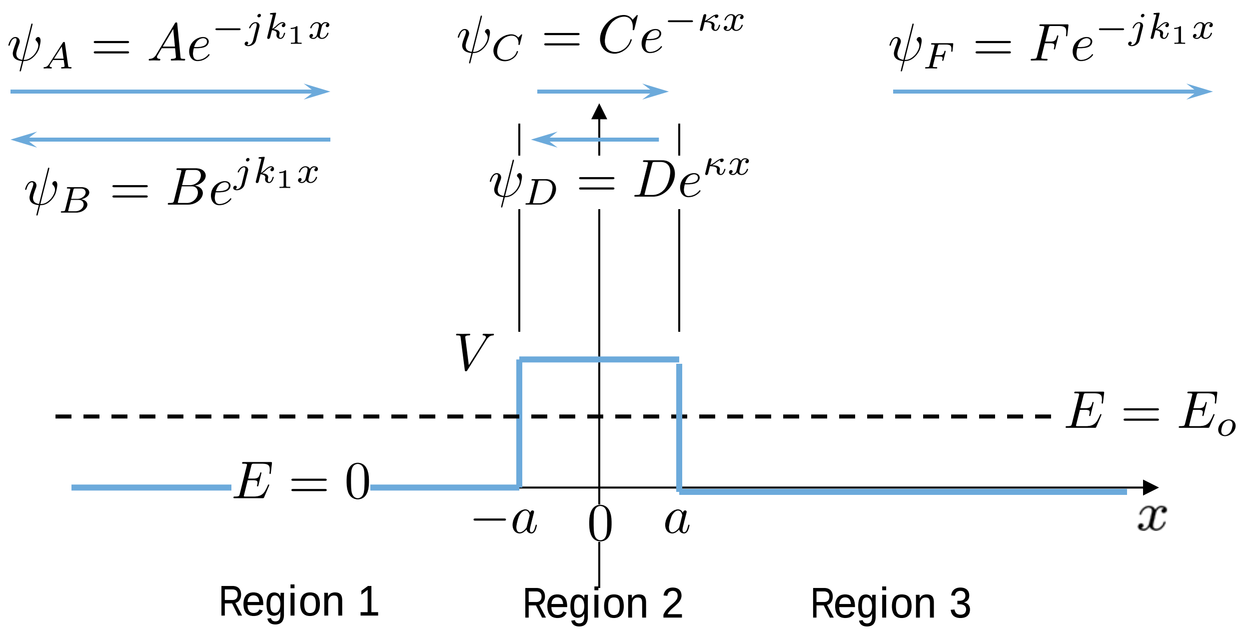


Frustrated Total Reflection (Tunneling)



A Rectangular Potential Step

CASE II : $E_o < V$



In Regions 1 and 3: $$E_o\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad k_1^2 = \frac{2mE_o}{\hbar^2}$$

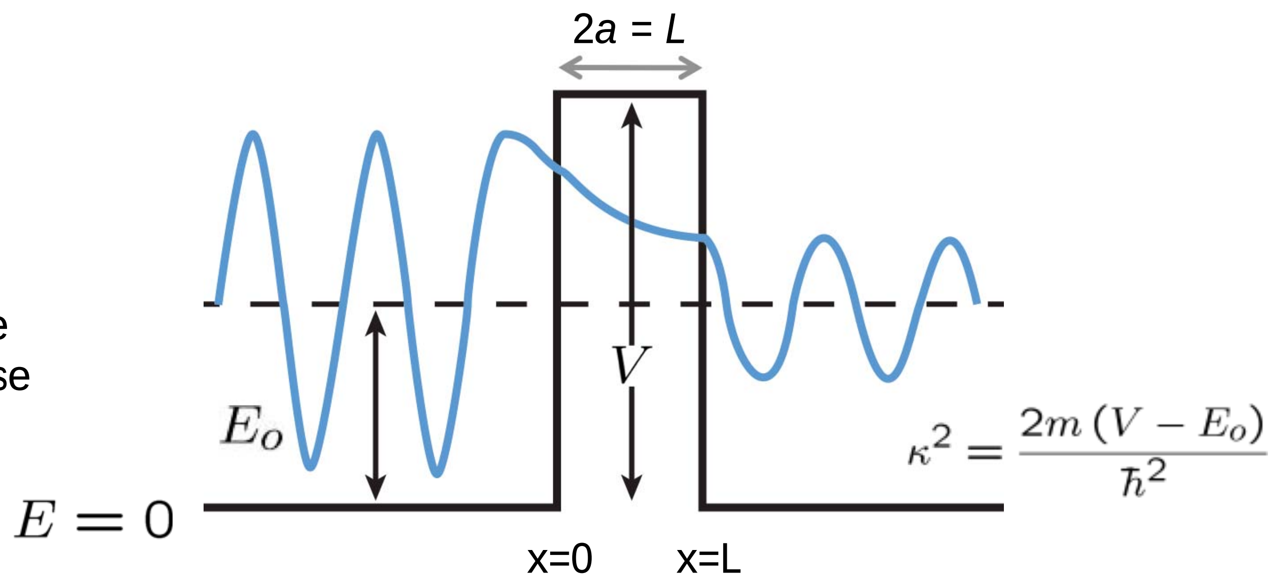
In Region 2: $$(E_o - V)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \quad \Rightarrow \quad \kappa^2 = \frac{2m(V - E_o)}{\hbar^2}$$

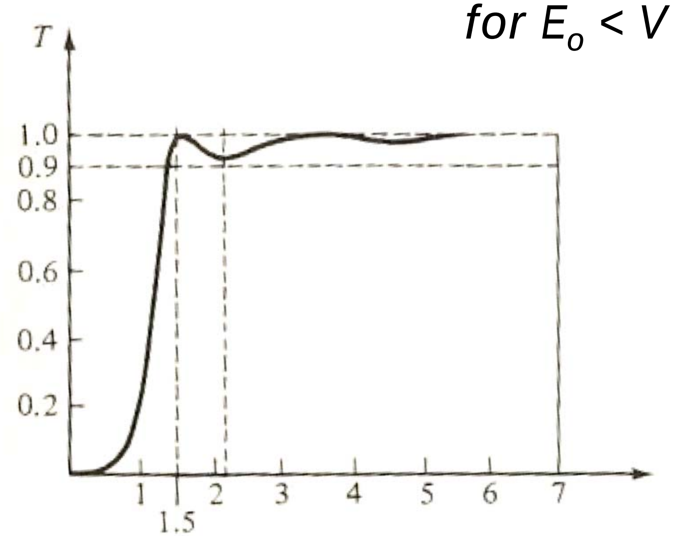
for $E_o < V$:

$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}\sinh^2(2\kappa a)}$$

A Rectangular Potential Step

Real part of Ψ for $E_o < V$, shows hyperbolic (exponential) decay in the barrier domain and decrease in amplitude of the transmitted wave.





Transmission Coefficient versus E_o/V for barrier with $2m(2a)^2V/\hbar = 16$

for $E_o < V$:

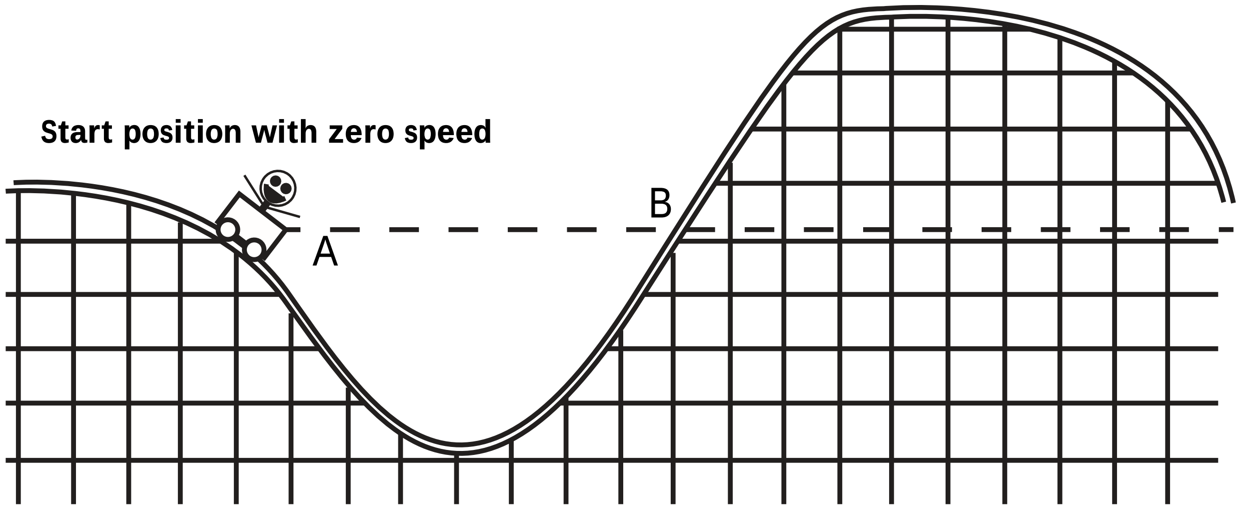
$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}\sinh^2(2\kappa a)}$$

$$\sinh^2(2\kappa a) = \left[e^{2\kappa a} - e^{-2\kappa a}\right]^2 \approx e^{-4\kappa a}$$

$$T = \left|\frac{F}{A}\right|^2 \approx \frac{1}{1 + \frac{1}{4}\frac{V^2}{E_o(V-E_o)}} e^{-4\kappa a}$$

Tunneling Applet: http://www.colorado.edu/physics/phet/dev/quantum-tunneling/1.07.00/

Imagine the Roller Coaster …



- Normally, the car can only get as far as B, before it falls back again

- But a fluctuation in energy could get it over the barrier to E!

- A particle ‘borrows’ an energy ΔE to get over a barrier

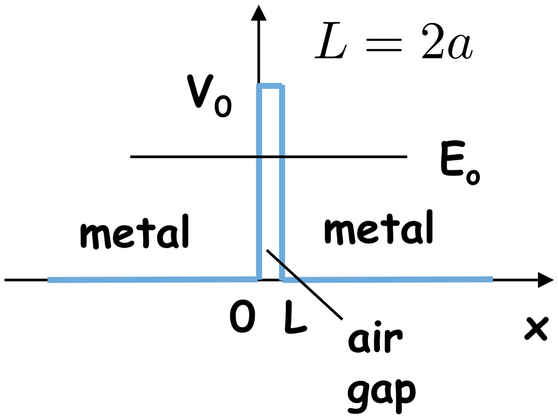
- Does not violate the uncertainty principle, provided this energy is repaid within a certain time Δt

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

Example: Barrier Tunneling

- Let's consider a tunneling problem:

An electron with a total energy of E_o= 6 eV approaches a potential barrier with a height of V_0 = 12 eV. If the width of the barrier is L = 0.18 nm, what is the probability that the electron will tunnel through the barrier?



$$T = \left|\frac{F}{A}\right|^2 \approx \frac{16E_o(V - E_o)}{V^2} e^{-2\kappa L}$$

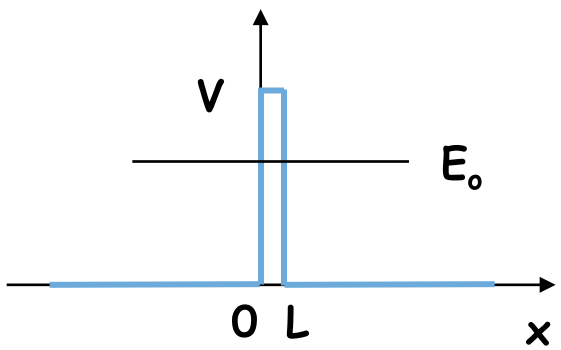
$$\kappa = \sqrt{\frac{2m_e}{\hbar^2}(V - E_o)} = 2\pi\sqrt{\frac{2m_e}{h^2}(V - E_o)} = 2\pi\sqrt{\frac{6\text{eV}}{1.505\text{eV-nm}^2}} \approx 12.6 \text{ nm}^{-1}$$

$$T = 4e^{-2(12.6 \text{ nm}^{-1})(0.18 \text{ nm})} = 4(0.011) = \boxed{4.4\%}$$

Question: What will T be if we double the width of the gap?

Multiple Choice Questions

Consider a particle tunneling through a barrier:



1. Which of the following will increase the likelihood of tunneling?

a. decrease the height of the barrier
b. decrease the width of the barrier
c. decrease the mass of the particle

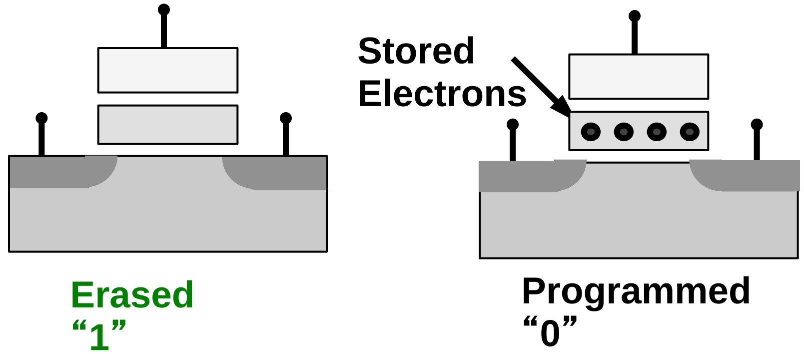
2. What is the energy of the particles that have successfully "escaped"?

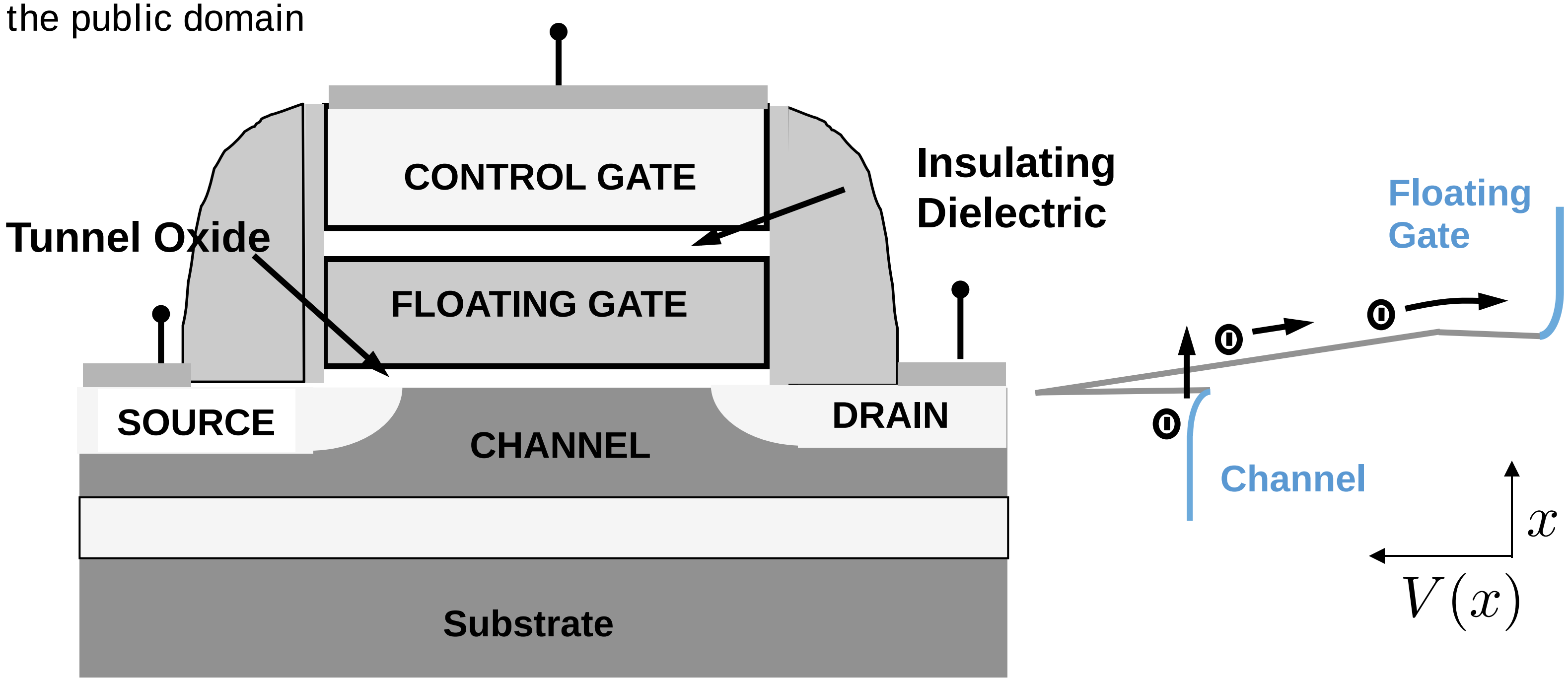
a. < initial energy
b. = initial energy
c. > initial energy

Although the *amplitude* of the wave is smaller after the barrier, no energy is lost in the tunneling process

Flash Memory

Image is in the public domain





Electrons tunnel preferentially when a voltage is applied

MOSFET: Transistor in a Nutshell

Conduction electron flow

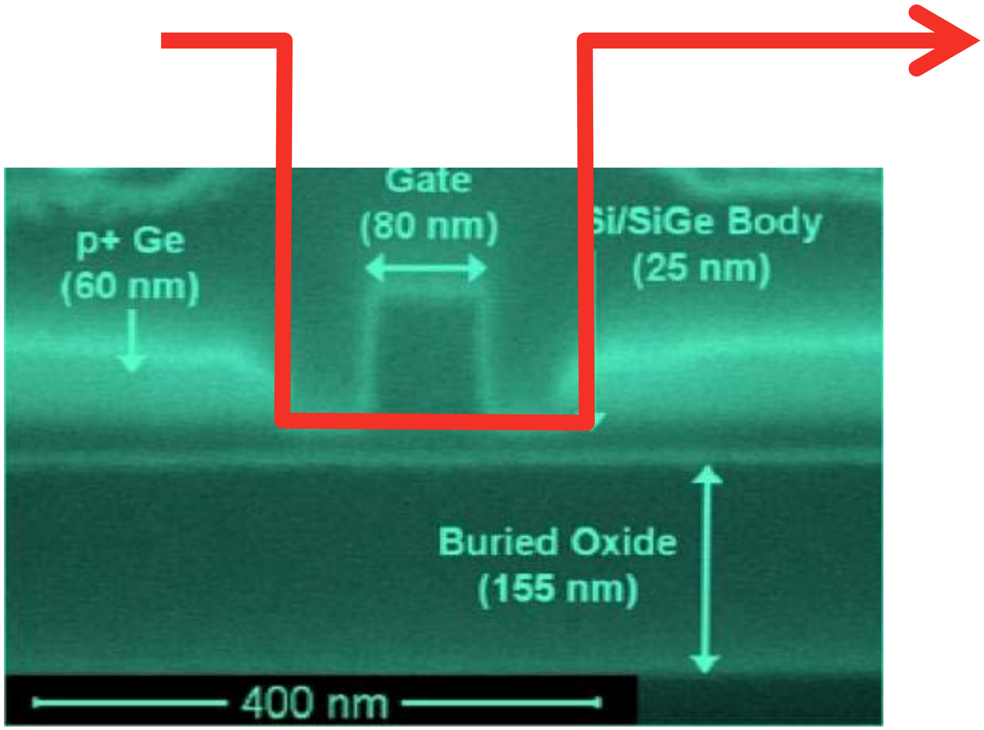


Image courtesy of J. Hoyt Group, EECS, MIT. Photo by L. Gomez

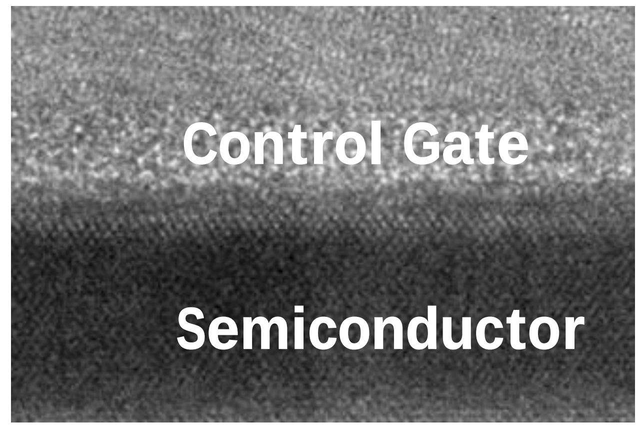


Image courtesy of J. Hoyt Group, EECS, MIT.
Photo by L. Gomez

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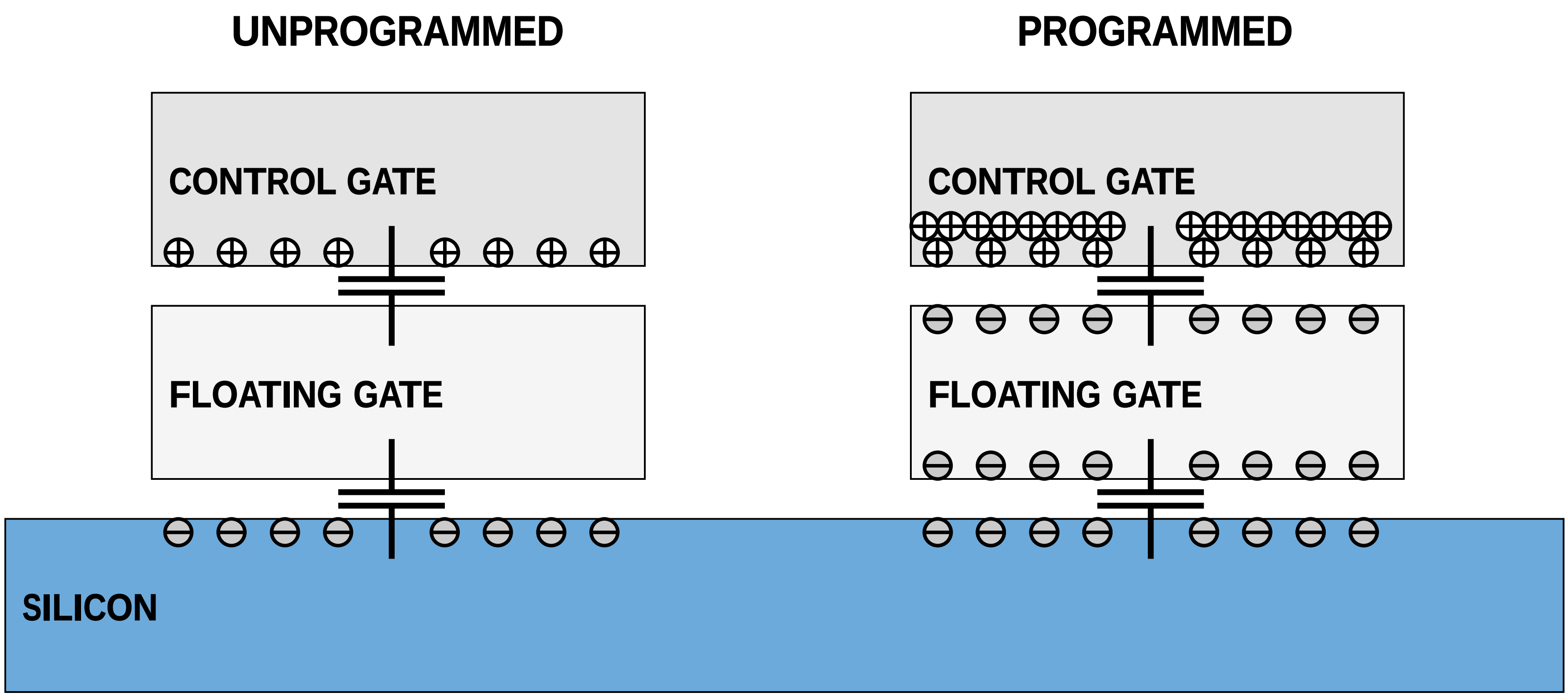
Tunneling causes thin insulating layers
to become leaky !

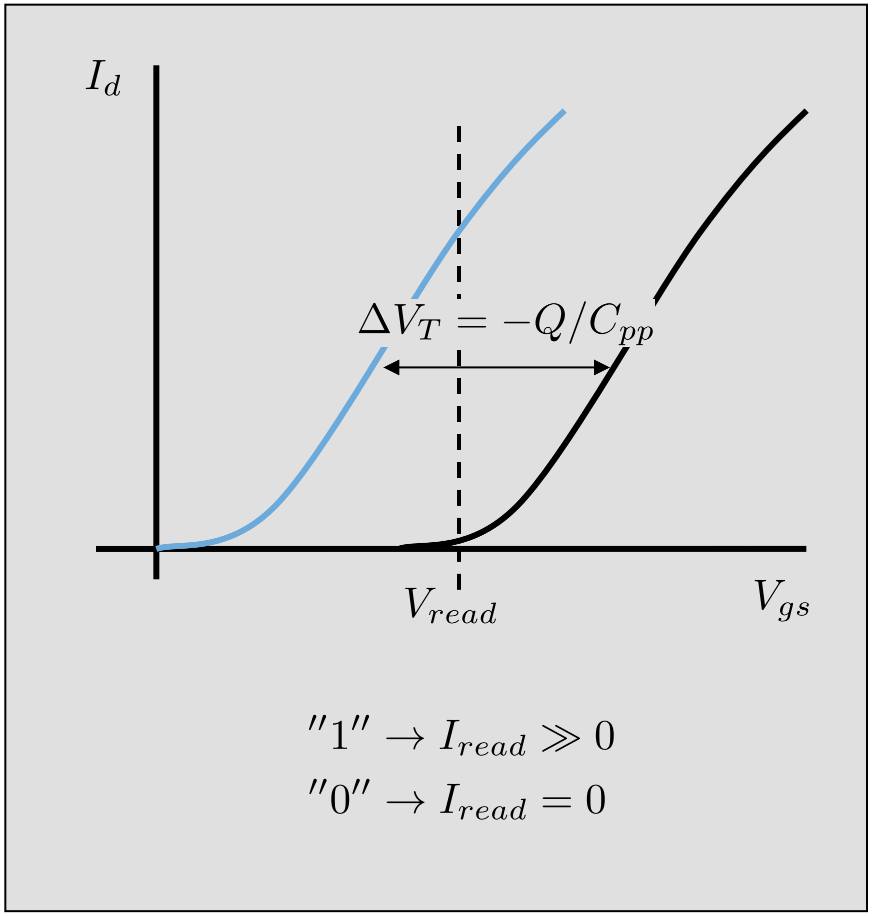
Reading Flash Memory



To obtain the same channel charge, the programmed gate needs a higher control-gate voltage than the unprogrammed gate

How do we WRITE Flash Memory ?

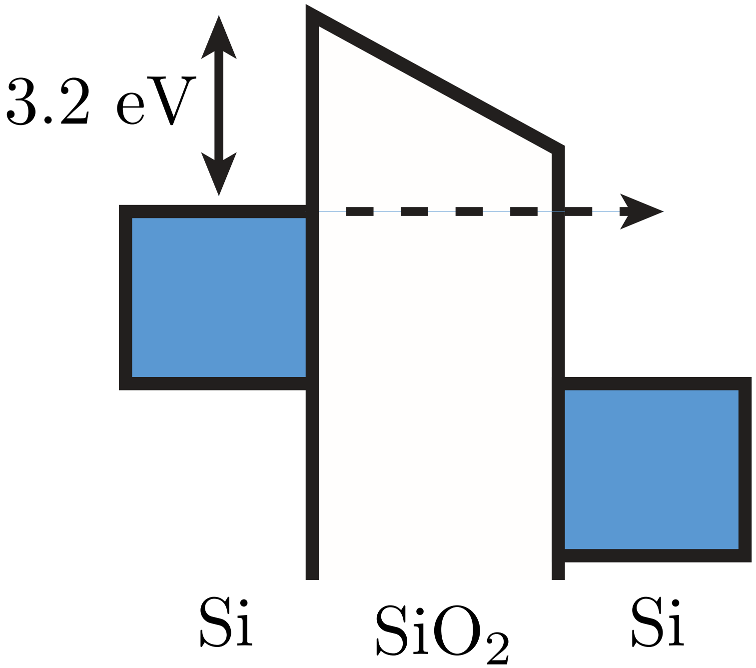
Reading Flash Memory



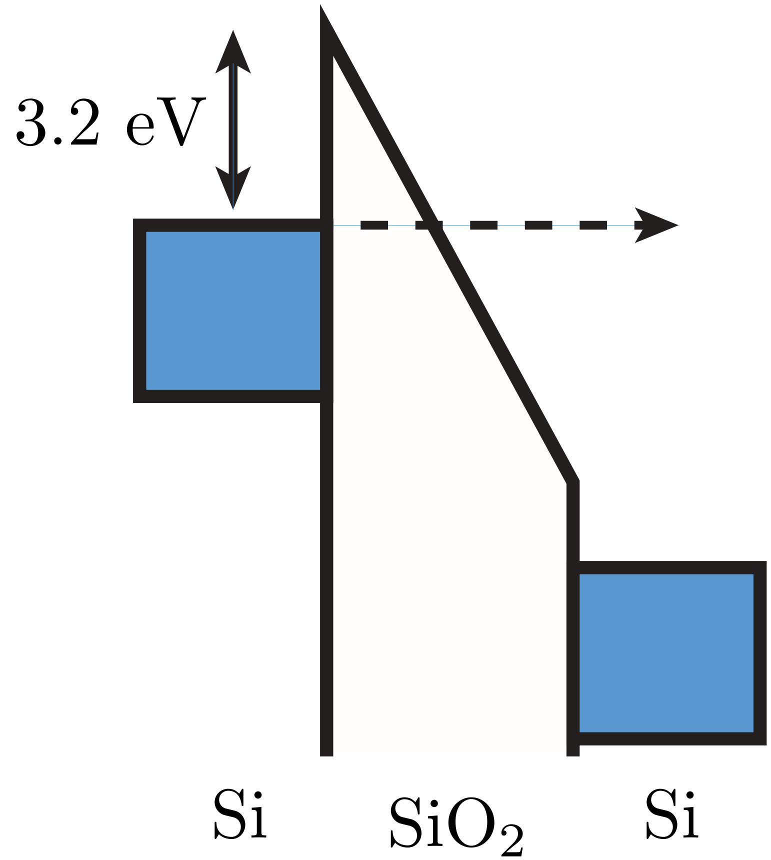
Reading a *bit* means:

1. Apply V_{read} on the control gate
2. Measure drain current I_d of the floating-gate transistors

Erasing Flash Memory



Fowler-Nordheim tunneling

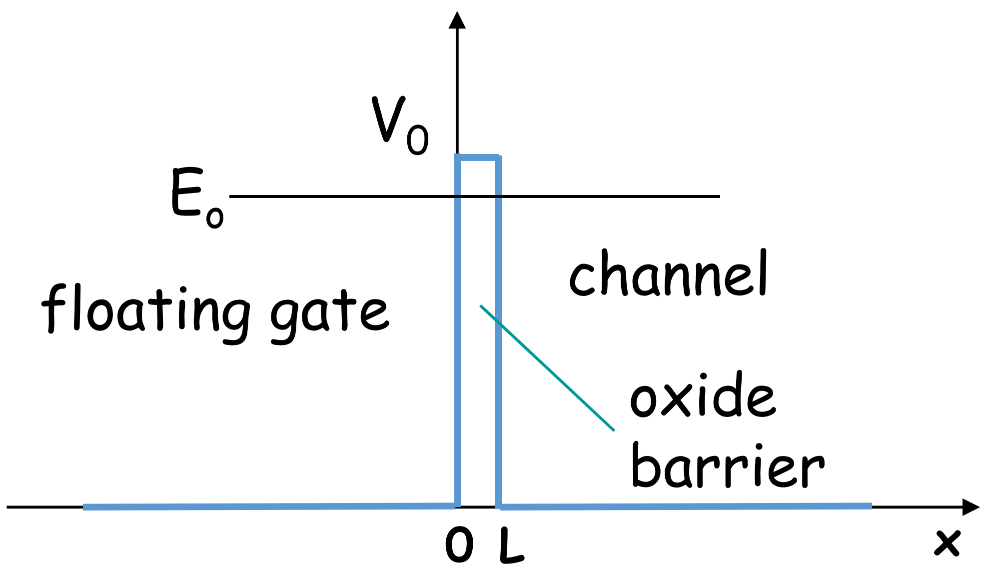


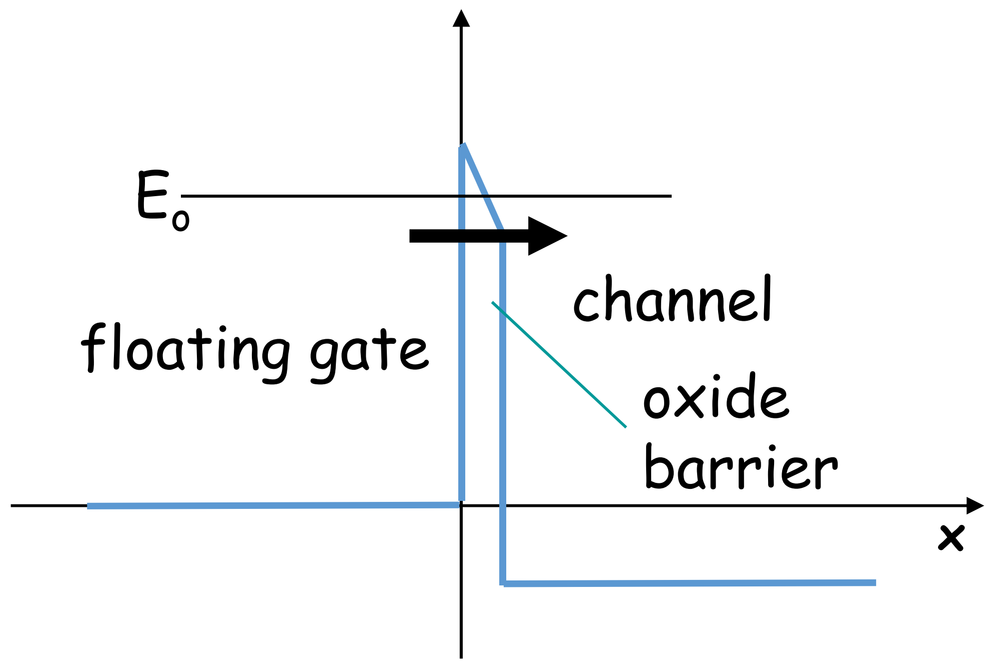
Direct tunneling

Effective thickness decreases with voltage...

$$I = AK_1V^2e^{-K_2/V}$$

Flash Memory





$$T = \left|\frac{F}{A}\right|^2 \approx \frac{16E_o\left(V - E_o\right)}{V^2} e^{-2\kappa L}$$

Retention = the ability to hold on to the charge

Tunnel oxide thickness	Time for 20% charge loss
4.5 nm	4.4 minutes
5 nm	1 day
6 nm	½ - 6 years

Effective thickness of the tunneling barrier decreases, as the applied voltage bends the potential energy levels

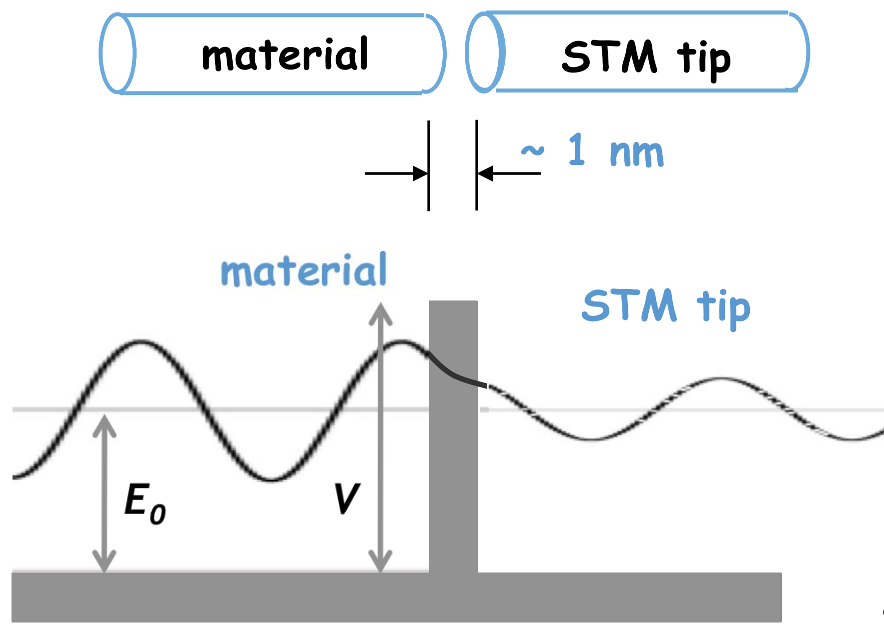
7-8 nm oxide thickness is the bare minimum, so that the flash memory chip can retain charge in the floating gates for at least 20 years

Application of Tunneling:

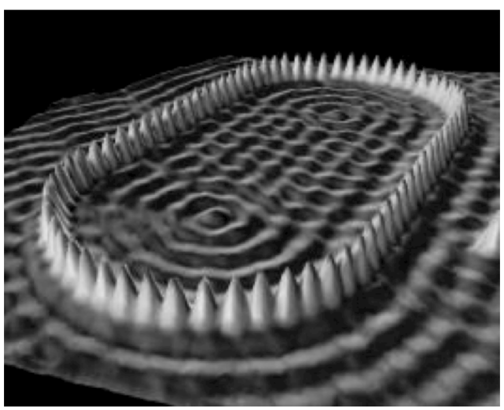
Scanning Tunneling Microscopy (STM)

Due to the quantum effect of "barrier penetration," the electron density of a material extends beyond its surface:

One can exploit this to measure the electron density on a material's surface:



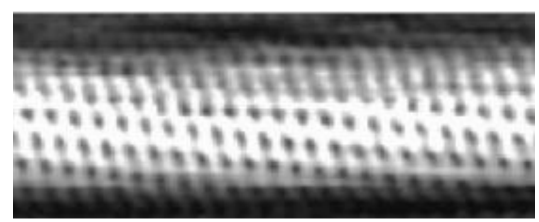
Sodium atoms on metal:



← **STM images** →

Image originally created by IBM Corporation

Single walled carbon nanotube:



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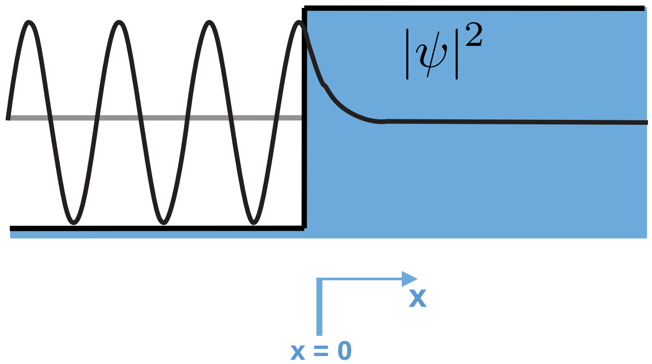
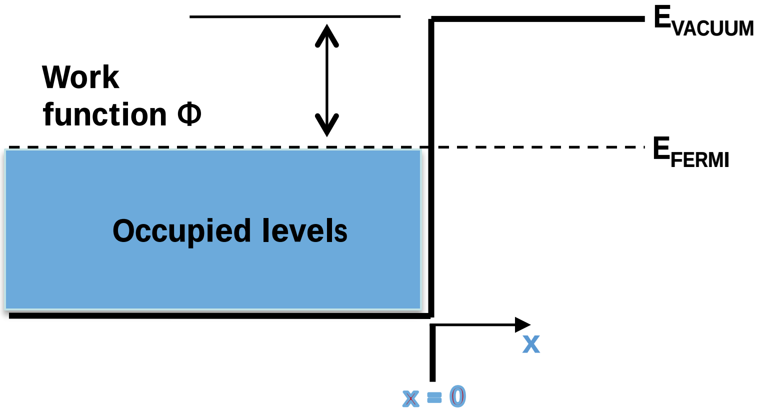
Leaky Particles

Due to "barrier penetration", the electron density of a metal actually extends outside the surface of the metal !





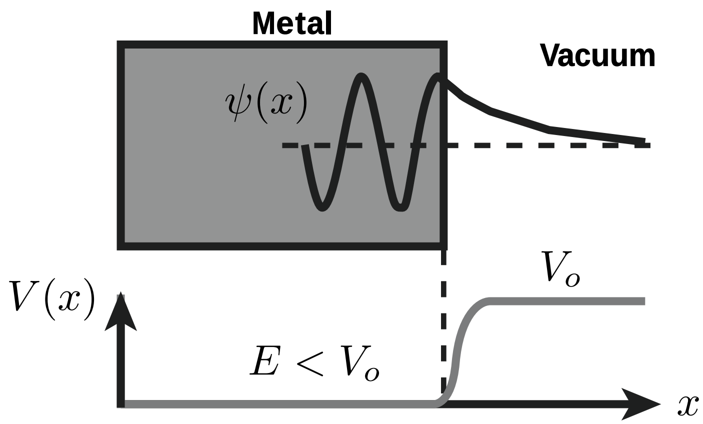
Assume that the work function (i.e., the energy difference between the most energetic conduction electrons and the potential barrier at the surface) of a certain metal is $\Phi = 5$ eV. Estimate the distance x outside the surface of the metal at which the electron probability density drops to 1/1000 of that just inside the metal.

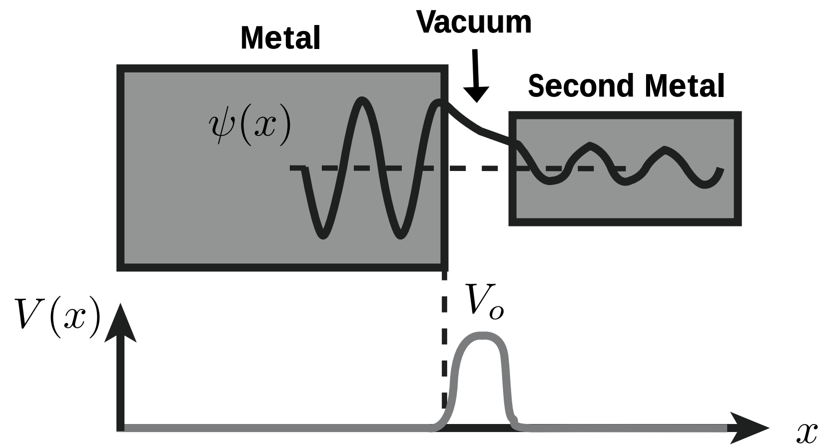
(Note: in previous slides the thickness of the potential barrier was defined as $x = 2a$)

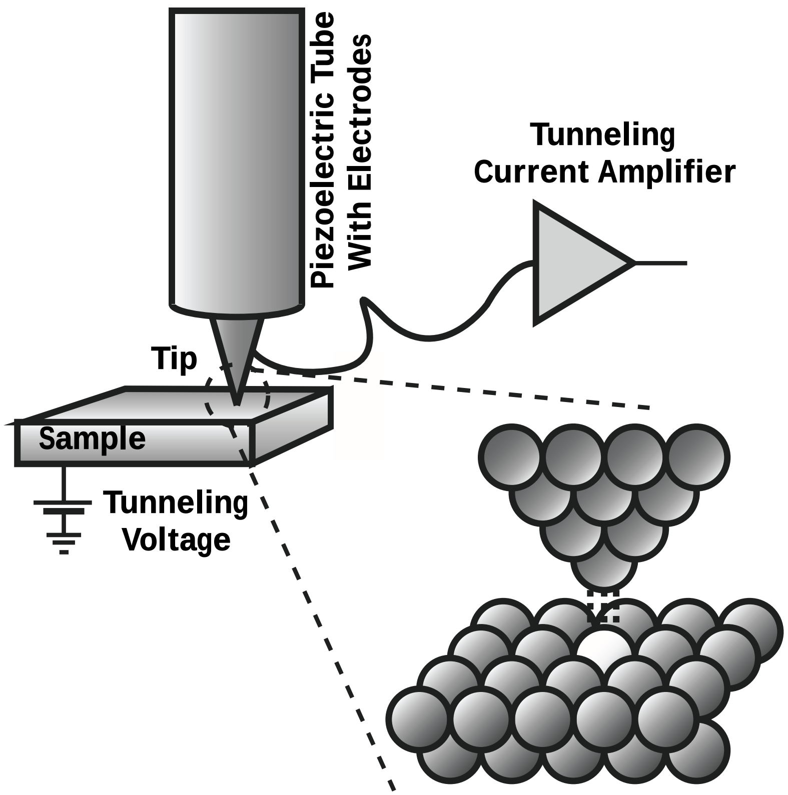
$$\frac{|\psi(x)|^2}{|\psi(0)|^2} = e^{-2\kappa x} \approx \frac{1}{1000} \quad \Rightarrow \quad x = -\frac{1}{2\kappa} \ln\left(\frac{1}{1000}\right) \approx 0.3 \text{ nm}$$

using $$\kappa = \sqrt{\frac{2m_e}{\hbar^2}(V_o - E)} = 2\pi\sqrt{\frac{2m_e}{h^2}\Phi} = 2\pi\sqrt{\frac{5 \text{ eV}}{1.505 \text{ eV}\cdot\text{nm}^2}} = 11.5 \text{ nm}^{-1}$$

Application: Scanning Tunneling Microscopy







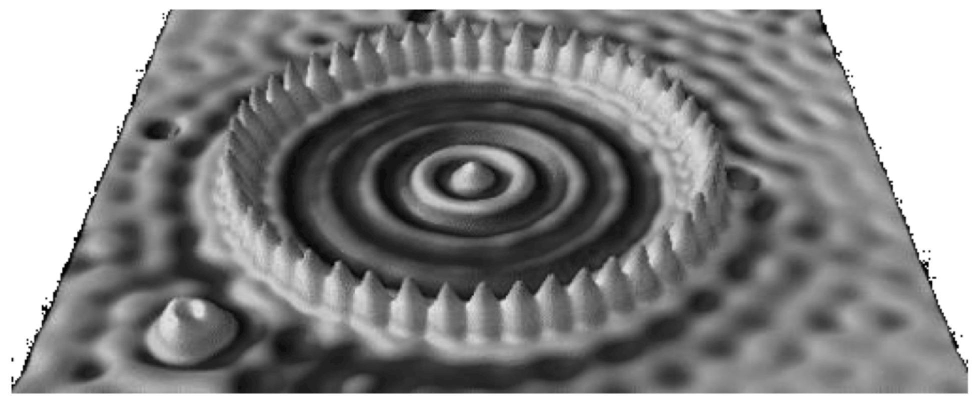


Image originally created by IBM Corporation.

Application of Tunneling:

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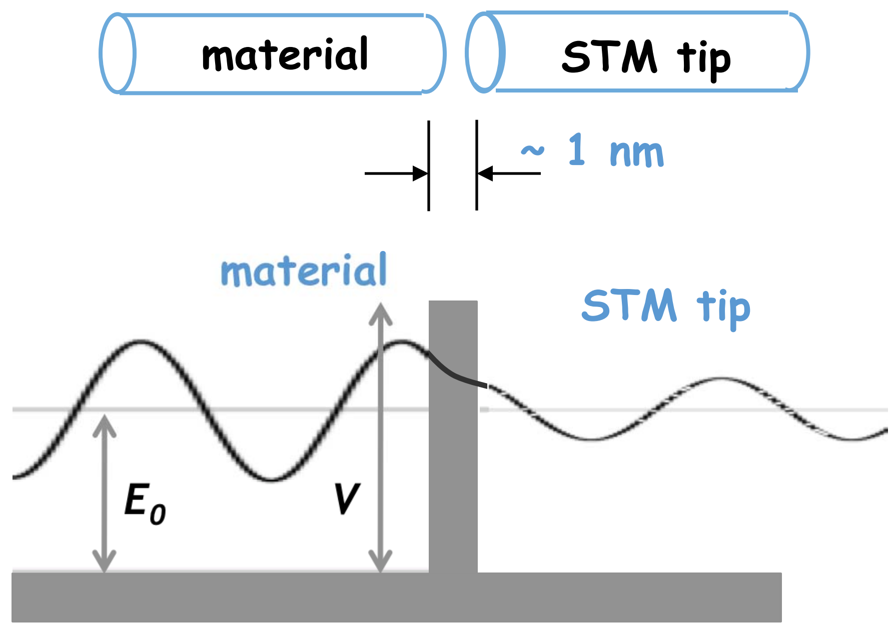

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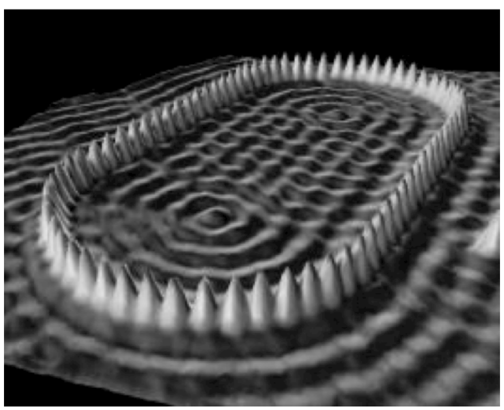


Image originally created by IBM Corporation.

← **STM images** →

DNA Double Helix:

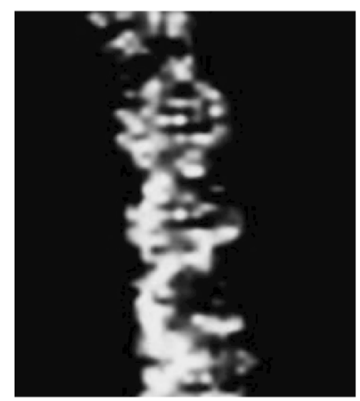
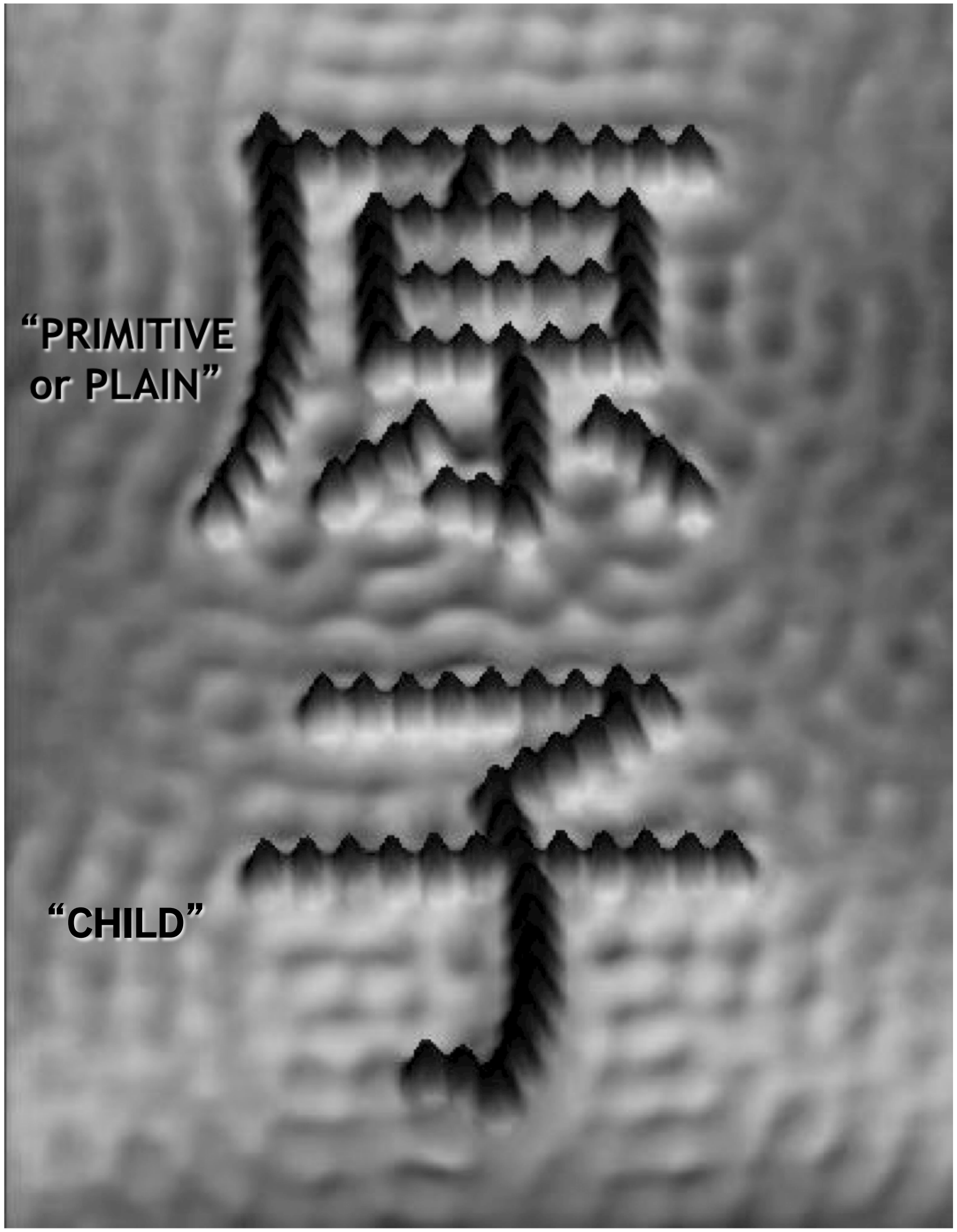
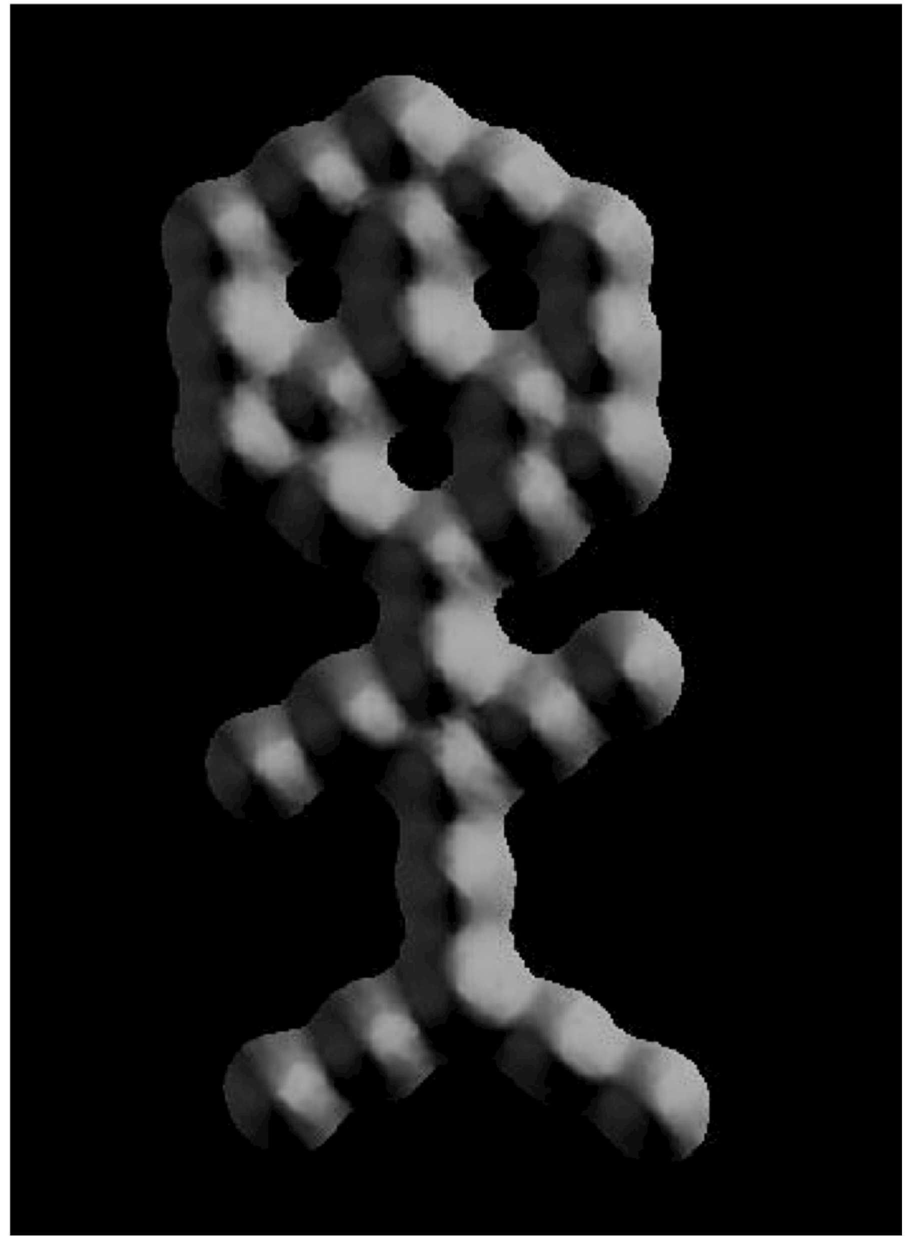


Image by Wolfgang Schonert of GSI Biophysics Research Group

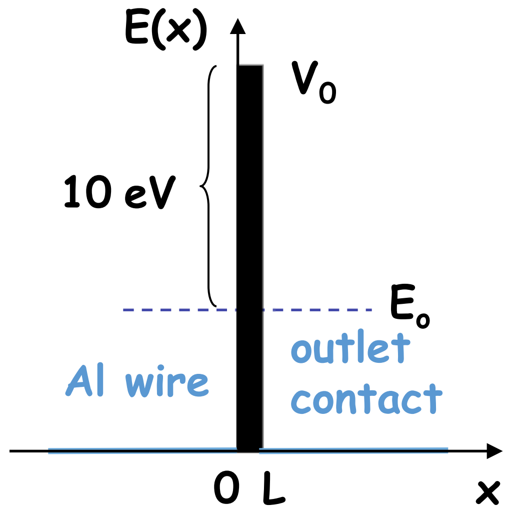


= “ATOM”



Images originally created by IBM Corporation.

Example: Al wire contacts



"Everyday" problem:

You're putting the electrical wiring in your new house, and you're considering using Aluminum wiring, which is cheap and a good conductor. However, you also know that aluminum tends to form an oxide surface layer (Al_2O_3) which can be as much as several nanometers thick.

This oxide layer could cause a problem in making electrical contacts with outlets, for example, since it presents a barrier of roughly 10 eV to the flow of electrons in and out of the Al wire.

Your requirement is that your transmission coefficient across any contact must be $T > 10^{-10}$, or else the resistance will be too high for the high currents you're using, causing a fire risk.

Should you use aluminum wiring or not?

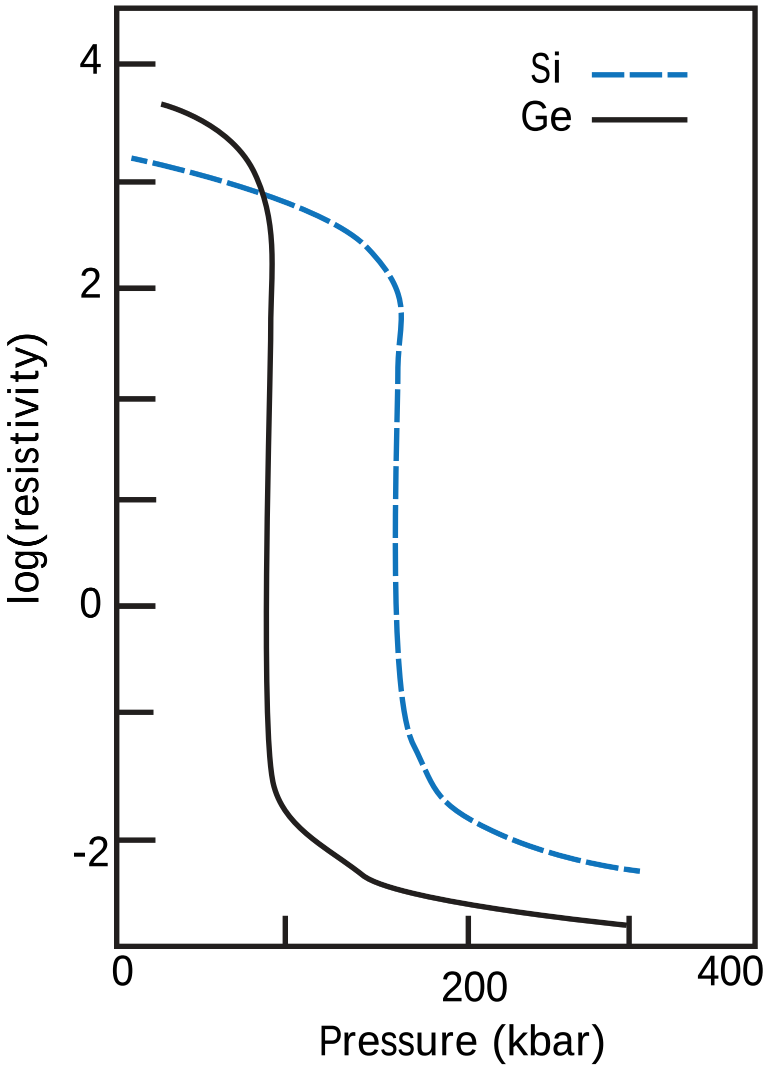
Compute L:

$$T \approx e^{-2\kappa L} \approx 10^{-10} \Rightarrow L \approx -\frac{1}{2\kappa}\ln(10^{-10}) \approx 0.72 \text{ nm}$$

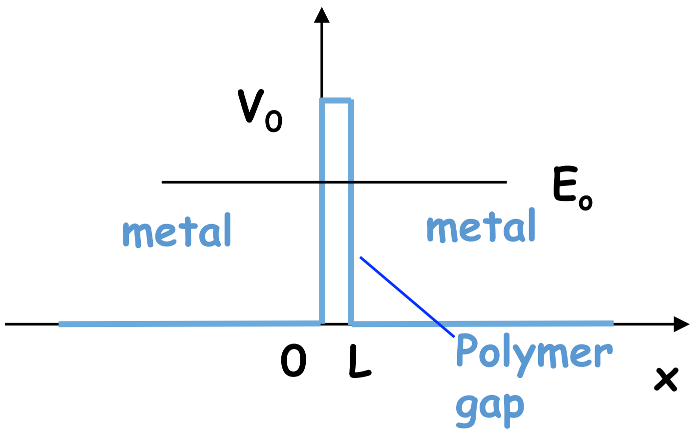
$$\kappa = \sqrt{\frac{2m_e}{\hbar^2}(V_o - E)} = 2\pi\sqrt{\frac{2m_e}{h^2}(V_o - E)} = 2\pi\sqrt{\frac{10 \text{ eV}}{1.505 \text{ eV}\cdot\text{nm}^2}} = 16 \text{ nm}^{-1}$$

Oxide is thicker than this, so go with Cu wiring! (Al wiring in houses is illegal for this reason)

Tunneling and Electrical Conduction



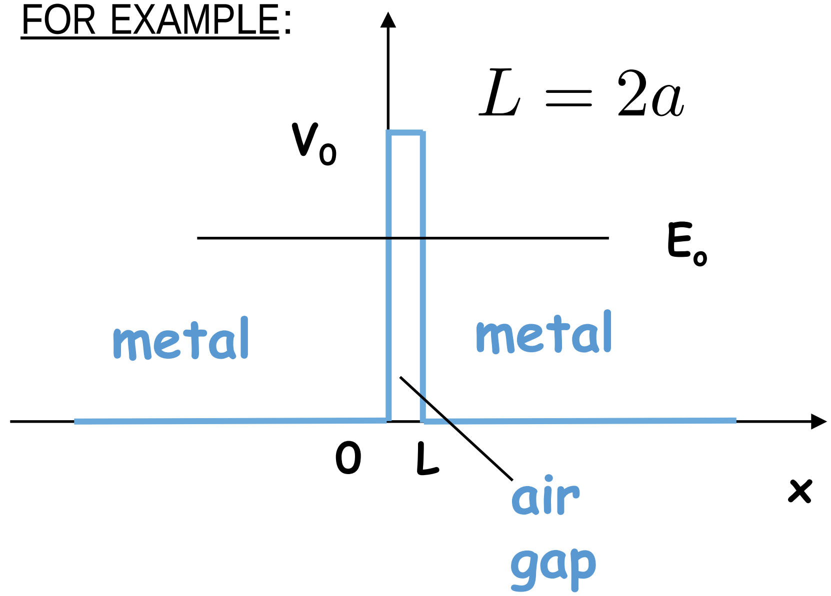
Squeezing a material can reduce the width of the tunneling barrier and turn an 'insulator' into a 'metal'



Key Takeaways

$$T = \left| \frac{F}{A} \right|^2 \approx \frac{16E_o(V - E_o)}{V^2} e^{-2\kappa L}$$

FOR EXAMPLE:



6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011