Energy in Electrical Systems

<u>Outline</u>

Review of Last time

Electric Fields and Work

Conservation Laws
- Kirchhoff’s Voltage Law
- Kirchhoff’s Current Law

Energy in Capacitors, Batteries and Molecules

TRUE or FALSE?

1. In steady state operation of a motor, all of the energy that goes in is lost to heat due to friction. _____

2. In a motor, the back EMF is proportional to the frequency of rotation. $V_{\mathrm{Bemf}} = K\omega$ _____

3. Gauss' Law states that $\int_S \mathbf{E} \cdot \mathbf{dA} = \int_{\mathbf{V}} \rho \mathbf{dV}$
$= Q_{enclosed}$ _____

Today
we will consider
different methods of

Energy Storage in Hybrid Vehicles

to solve the "mystery" of
why gasoline is so efficient
in storing energy

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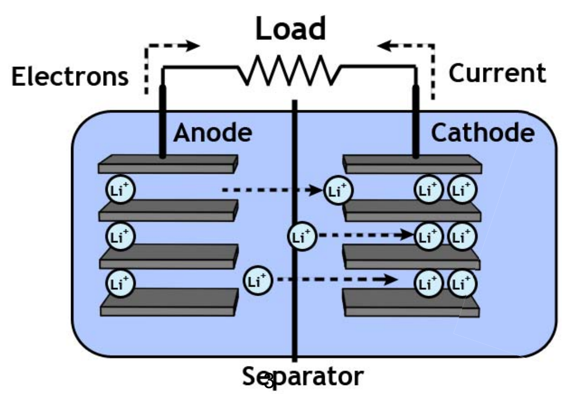
Capacitor

Image is in the public domain

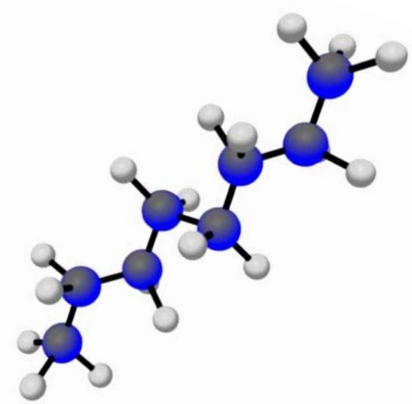
Battery

Prius NiMH: 1.8 kW-h
: 52 kg



Gasoline

1 gallon : 40 kW-h
: 2.75 kg



A REVIEW OF YESTERDAY'S LECTURE

When motor turns it generates "Back EMF" $V_{\mathrm{Bemf}} = K\omega$

- The 1st Law requires that any contraption that can be used as an electromechanical actuator can use its actuation to generate EM fields:
  - a motor can also be used as a generator.
  - a loudspeaker can also be used as a microphone

- Torque of a motor = motor constant × motor current $\tau_m = KI$

- When frictional torque in a motor equals the morque from the Lorentz force inside the motor, the net torque is zero and a steady operation is achieved.

Frictional Torque = constant × angular velocity $\tau_{\mathrm{f}} = \beta\omega$

- Operated at CONSTANT CURRENT – motors have steady torque
- Operated at CONSTANT VOLTAGE – motors have steady angular velocity

- Energy stored in 1 gallons of gasoline is 35 kW-hr (or rounding-up 40 kW-hr)
- Electric vehicles are more economical per mile traveled, however, they can travel fewer miles since the batteries store less energy per kg than fuel does

Hypothetical Gas-Powered Go-Cart:

Let's make a comparison to a similar go-cart powered by a reasonably sized gasoline engine and gas tank. Let's replace the 36 pounds of batteries by a gas tank that holds 36 pounds of gasoline.

- Gasoline weighs 6 pounds per gallon,
- Gasoline stores 40,000 Watt-hours of heat energy per gallon. That is, if you burned a gallon of gasoline, you would get 40,000 Watt-hours worth of heat.

How many joules are stored in a gallon of gasoline?	144,000,000 J
How many gallons of gas are stored in the hypothetical go-cart gas tank?	6 gallons
How many joules of heat energy are stored in the go-cart gas tank?	864 MJ

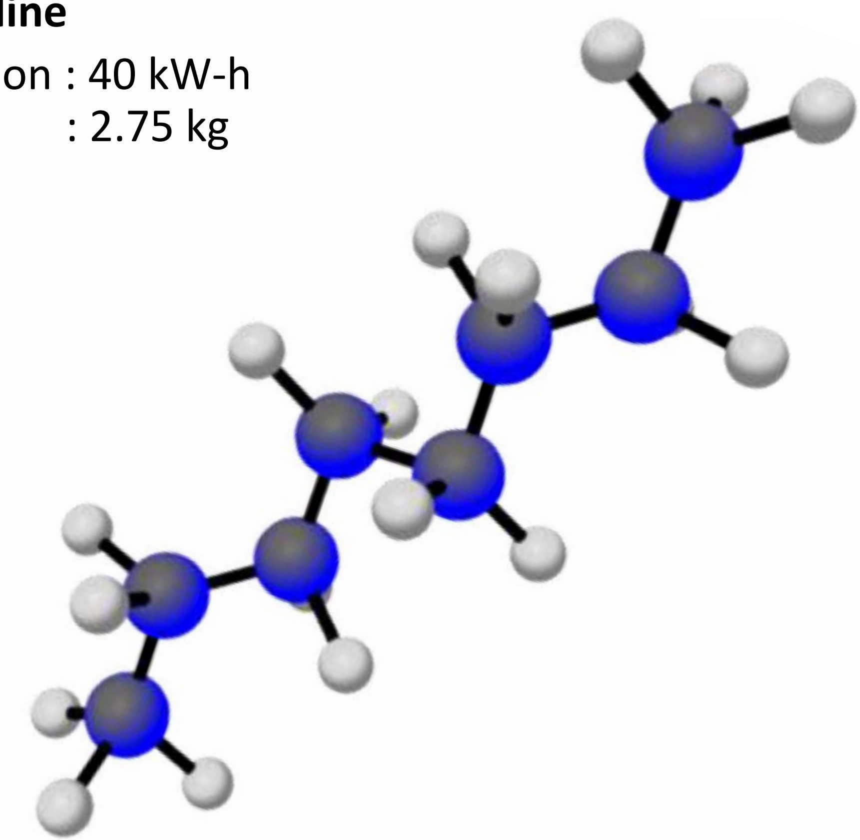
In comparison, the battery stores 2.2 ___________ MJ

Main Question for Today

Why does gasoline have such a large energy density

Gasoline

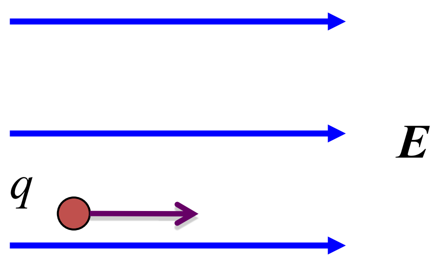
1 gallon : 40 kW-h
: 2.75 kg



How does E&M help me?

Energy and Electric Fields

$$\overline{f} = q\overline{E}$$



$$\int_a^b \overline{f} \cdot dx = -W_{ab}$$

$$= q \int_a^b \overline{E} \cdot d\bar{l}$$

Can define electrostatic potential (potential energy of a positive test charge) …

$$\overline{E} = -\frac{d\phi}{dl}$$

$$W_{ab} = q \int_a^b -\overline{E} \cdot d\bar{l} = q \int_a^b \frac{d\phi}{dl} dl$$

$$= q \int_a^b d\phi = q\left[\phi(b) - \phi(a)\right]$$

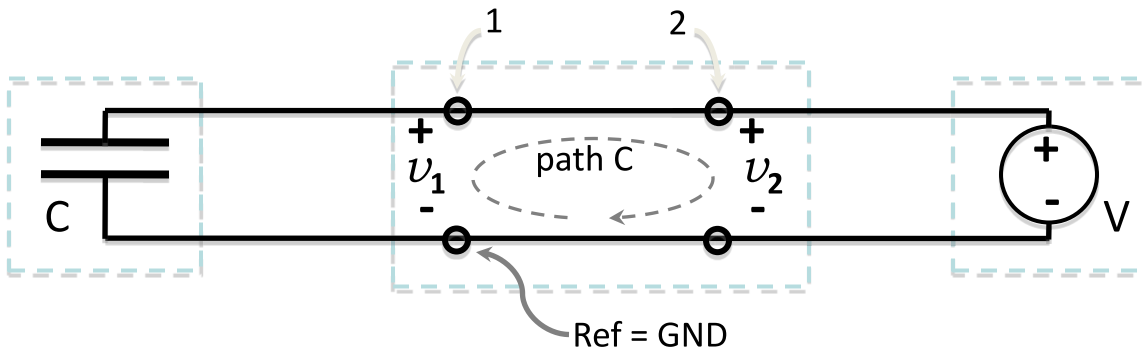
Potential changes by
a certain amount (measured in Volts) over
certain distance (measured in meters)

independent of path

For "conservative" fields like $\overline{E}$ in the electrostatic system,
the work W_{ab} done by $\overline{f}$ does not depend on the path taken !

Energy Conservation and Conversion

Consider the following circuit:



For a closed loop (C), a=b so the energy expended in moving charge is zero…

$$W_{ab} = q \int_C -\overline{E} \cdot d\overline{l} = 0$$

Voltage drops around a closed loop (circuit) must sum to zero..

$$0 = \int_C \overline{E} \cdot d\overline{l} = \int_{ref}^{1} \overline{E} \cdot d\overline{l} + \int_{1}^{2} \overline{E} \cdot d\overline{l} + \int_{2}^{ref} \overline{E} \cdot d\overline{l} + \int_{ref}^{ref} \overline{E} \cdot d\overline{l}$$

$$= v_1 + 0 + (-v_2) + 0$$

Kirchhoff's Voltage Law (KVL)

Regenerative Brakes

Hybrid cars:

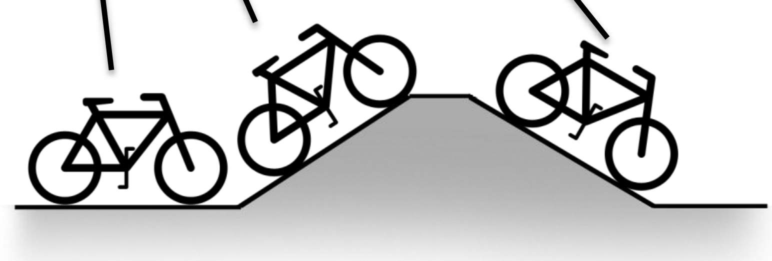
Energy from regenerative braking is stored in banks of capacitors or the battery.

Image is in the public domain

1. Bicycling along with kinetic energy

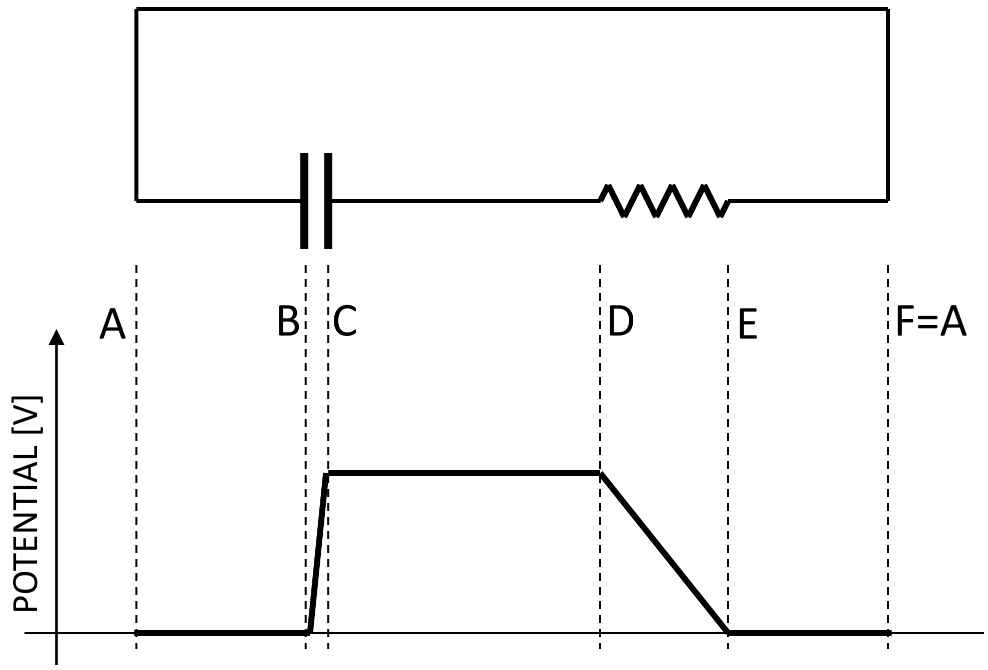
2. Brake going up ramp. Kinetic energy stored as potential energy.

3. Roll down ramp to convert stored potential energy back to kinetic energy!



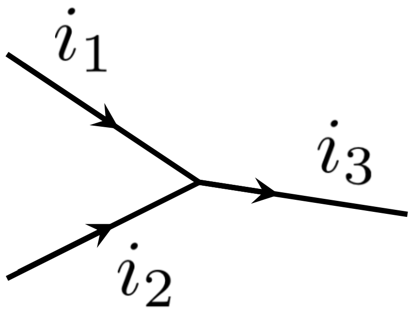
Example of KVL: Capacitor in Series with a Resistor

The stored energy in regenerative brakes of a hybrid-car (represented as capacitor) can power a motor load (represented as resistor)

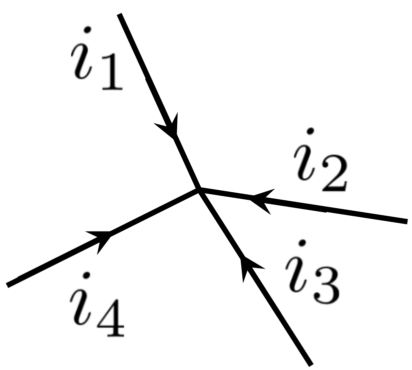


KVL is a statement of **Conservation of Energy**!

Kirchhoff's Current Law (KCL)



$$i_1 + i_2 = i_3$$



$$\sum_k i_k = 0$$

Summed over all currents entering a node

KCL is a statement of **Conservation of Mass** !

Energy Stored in Capacitors

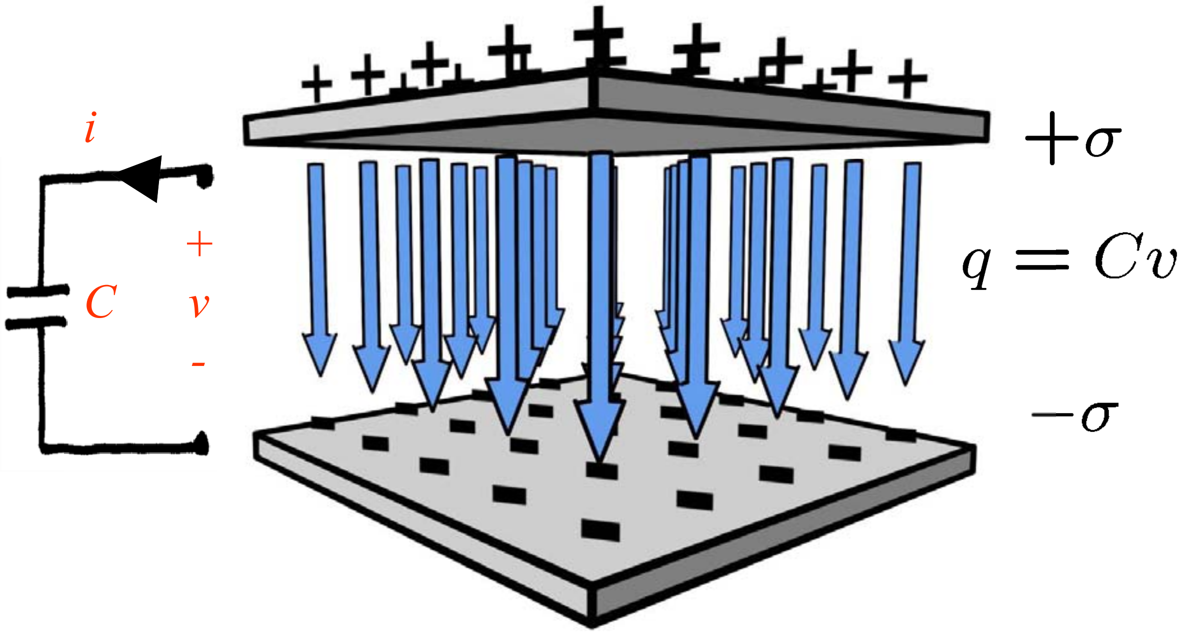
(we will assume that the structure is mechanically rigid)

$$P = \frac{dW_s}{dt} = i \cdot v$$

$$= \frac{dq}{dt} \int_a^b \overline{E} \cdot d\bar{l}$$

$$= C\frac{dv}{dt}v$$

$$= \frac{d}{dt}\left(\frac{1}{2}Cv^2\right)$$



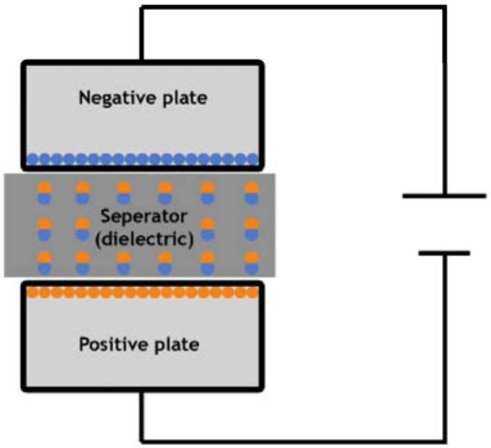
$$W_s = \frac{1}{2}Cv^2$$

How do we increase the energy storage?

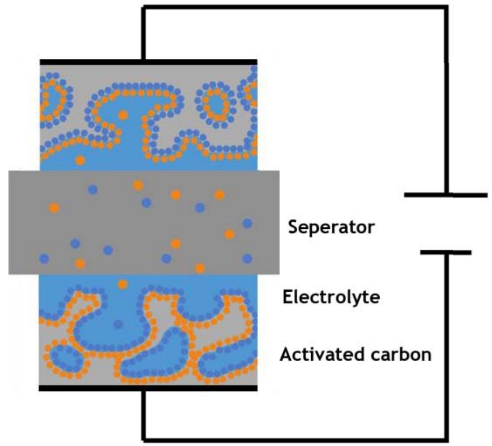
$$C = \frac{\varepsilon A}{d}$$

Ultracapacitors

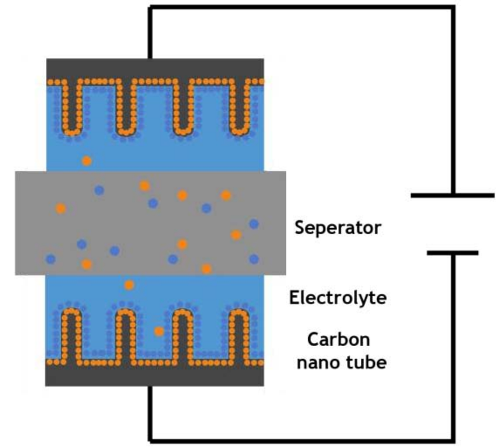
Traditional capacitor

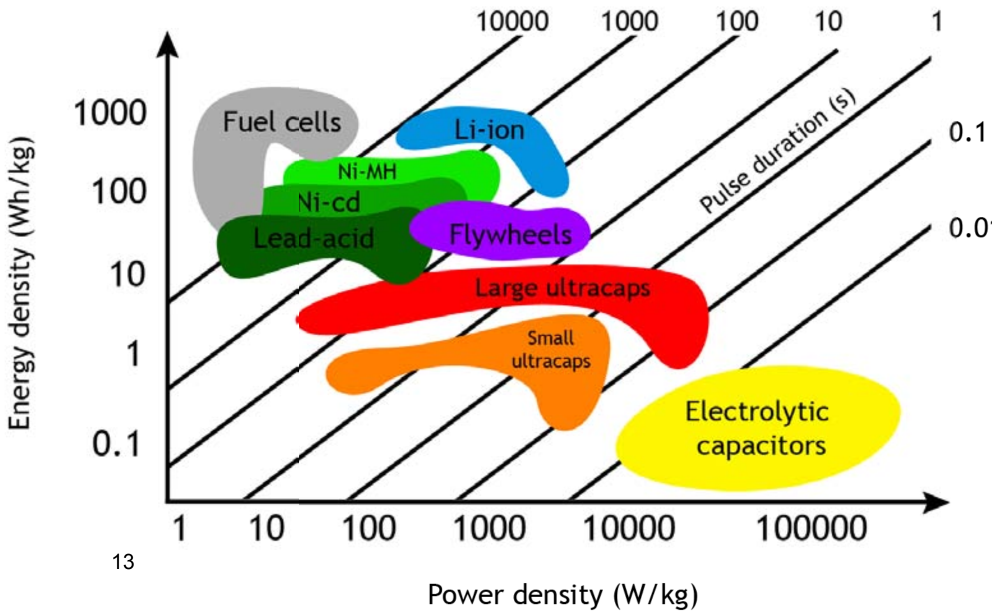


Ultracapacitor



Ultracapacitor with carbon nanotubes





Today's Culture Moment

Georg Stahl

Phlogiston

The "element" that was heat itself. This idea was perpetuated by Georg Stahl and was widespread in use by scientists in the 1700s.

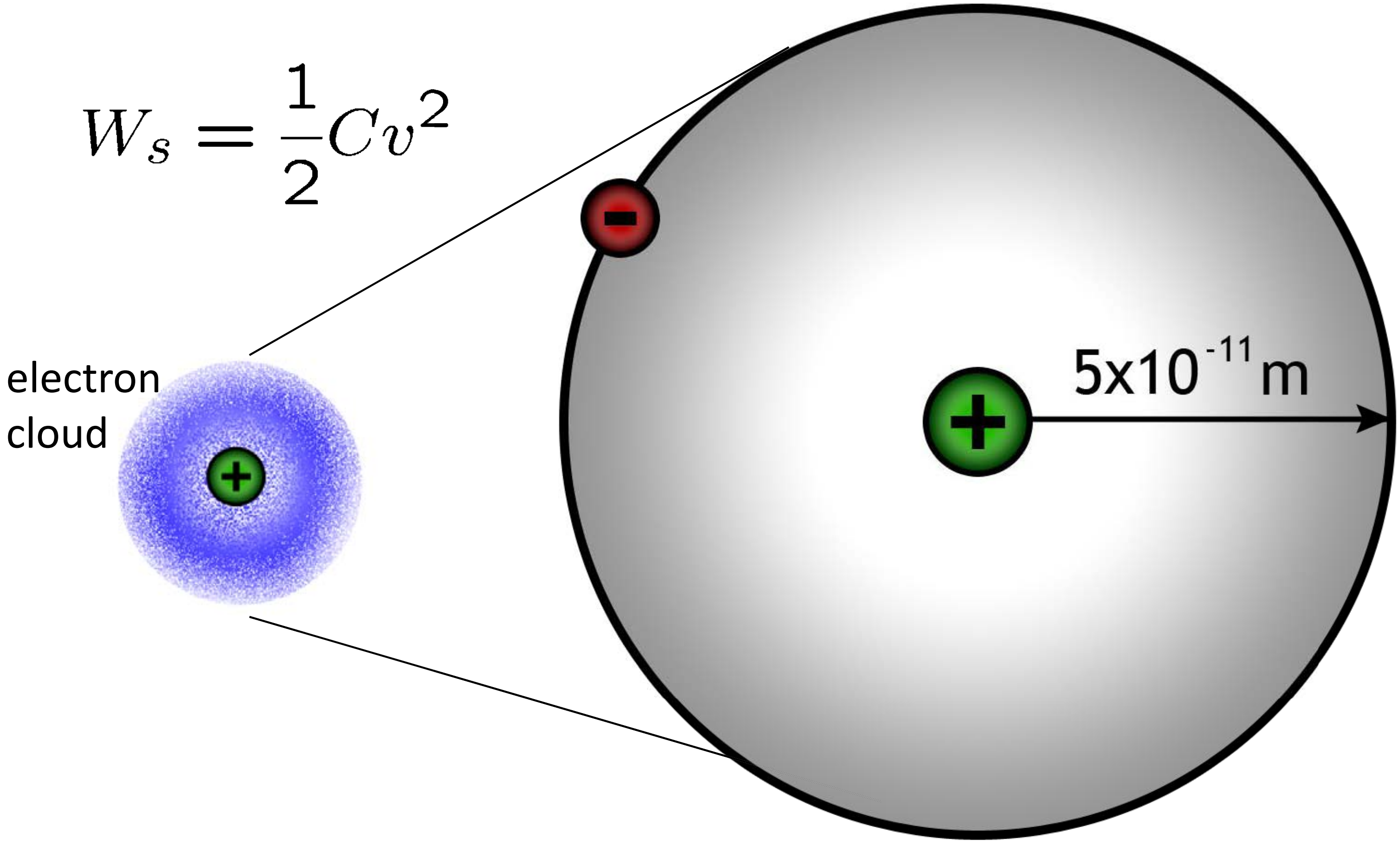
Phlogiston

"The remarkable thing about this amazing theory was that it seemed to have worked, and it had been used by eminent and respected early scientists for an entire century before it was finally proven wrong….Phlogiston theory was perhaps the most persistent, widespread, and totally wrong mistake made by scientists all through the age when science, as we know it today, was developed."

Fawcett, *100 Mistakes that Changed History*

Atomic Capacitors

$$W_s = \frac{1}{2}Cv^2$$



Hydrogen Atom

Lets approximate the electron cloud as a spherical shell of charge (-q) surrounding a positive nucleus (+q) …

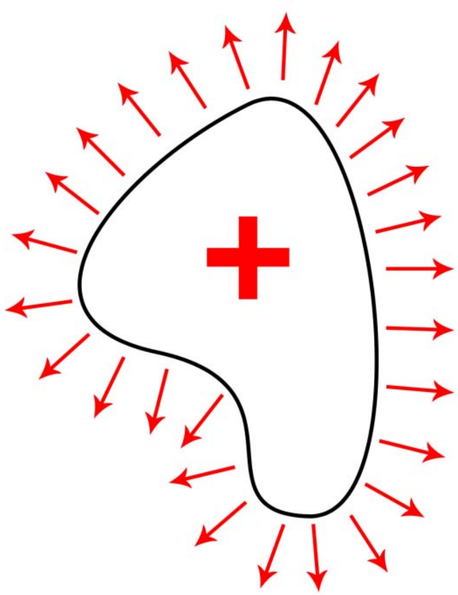
… and try to estimate the capacitance, voltage, and energy …

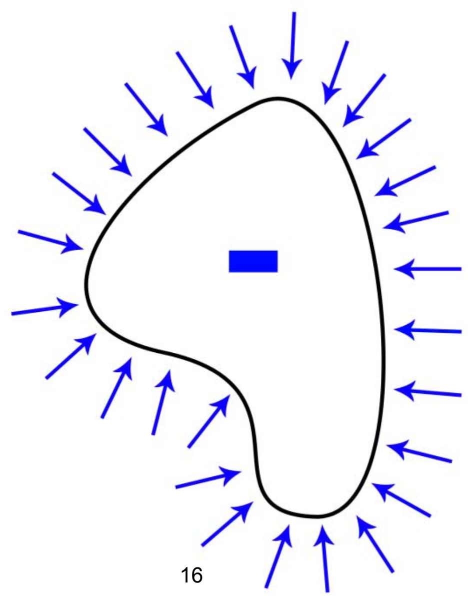
Gauss's Law

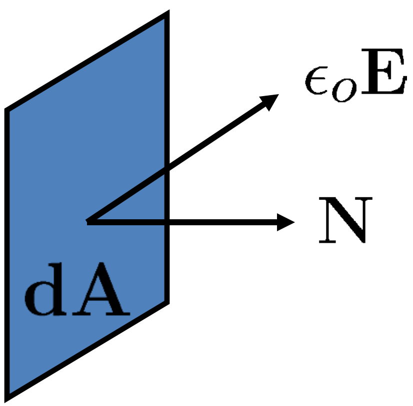
Flux of $\epsilon_o \mathbf{E}$ through closed surface ***S*** = net charge inside V

$$\int_S \epsilon_o \mathbf{E} \cdot \mathbf{dA} = \int_{\mathbf{V}} \rho \mathbf{dV}$$

$$= Q_{enclosed}$$

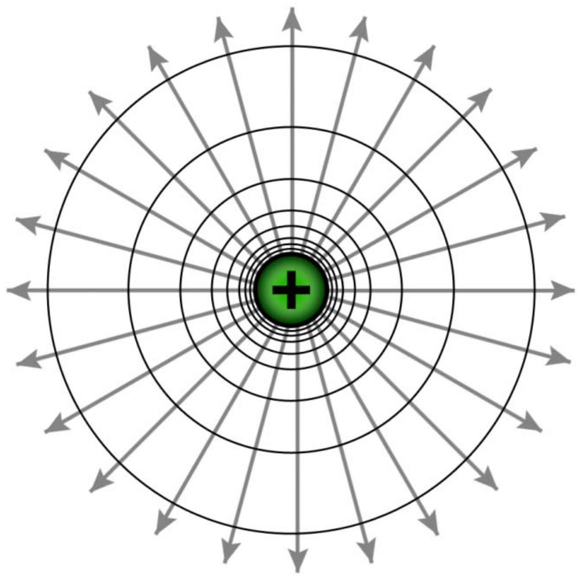






Atomic Capacitor

Field from a point charge



Area integral gives a measure of the net charge enclosed; Divergence of the electric field gives the density of the sources.

$$\overline{E}_{pointcharge} =$$

$$V_{Hydrogen} =$$

$$W_{S_{Hydrogen}} =$$

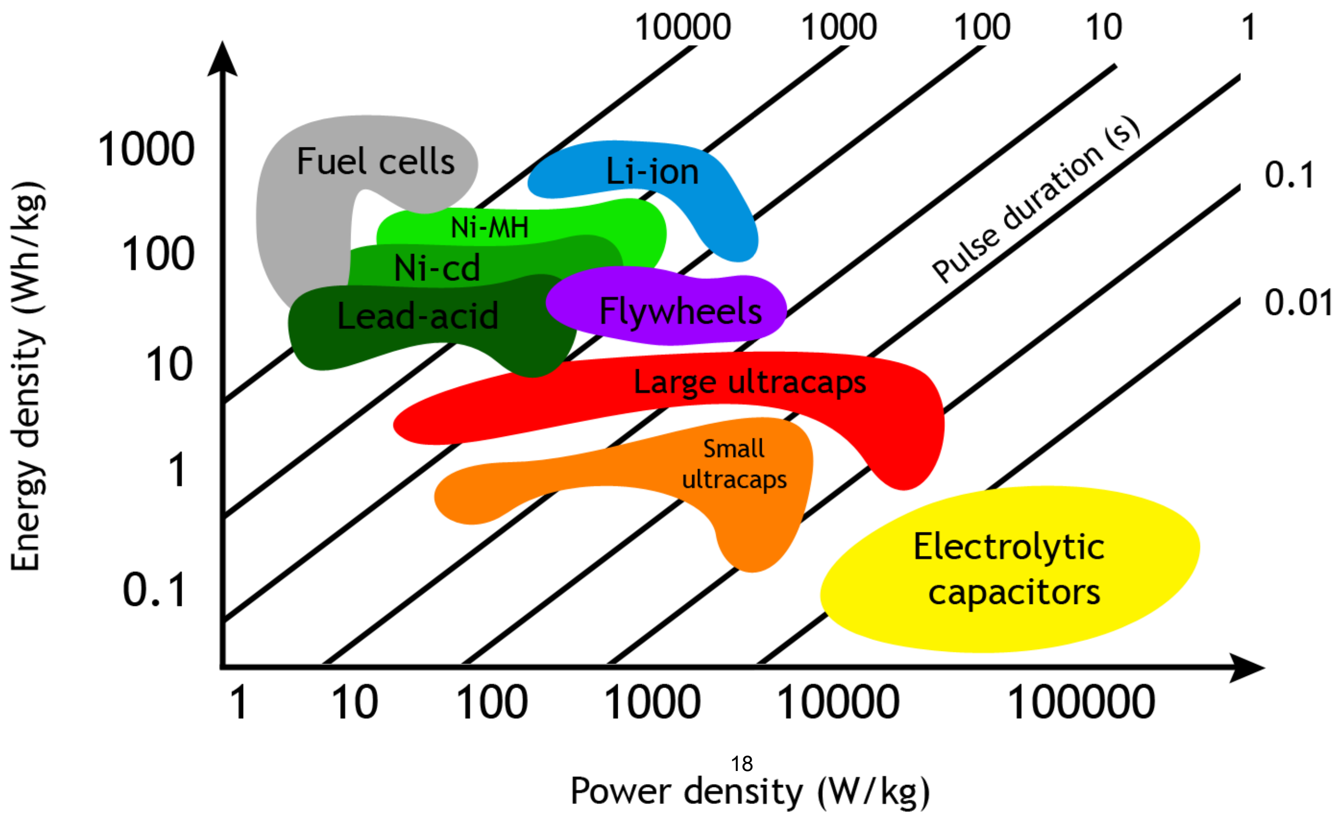
(~ 1000 kW-hr per kg)

Atomic Capacitor

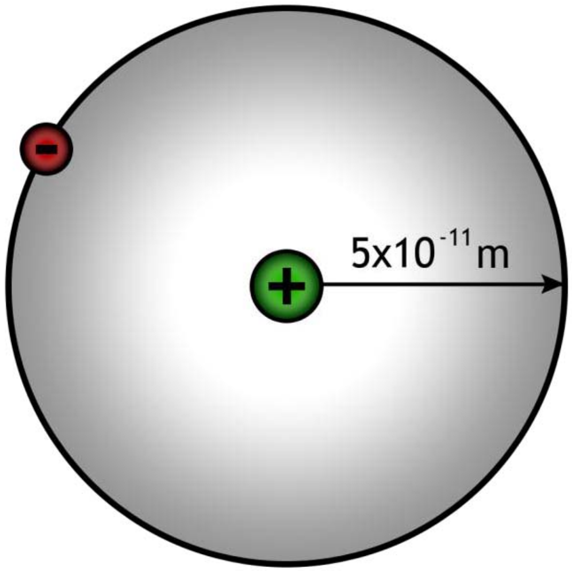
Stored energy for atomic hydrogen…

$$W \approx \tfrac{1}{2} q v \approx 14.4 \text{ eV} \sim 1000 \text{ kW-hr/kg}$$

Remember this unit of energy:
$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$



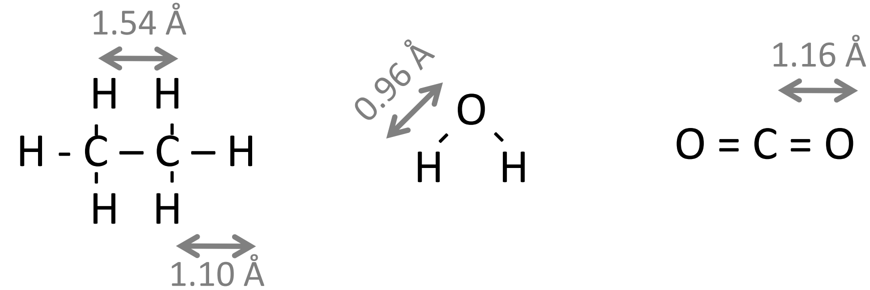
$$\phi(r) = \frac{q}{4\pi\epsilon_o r}$$

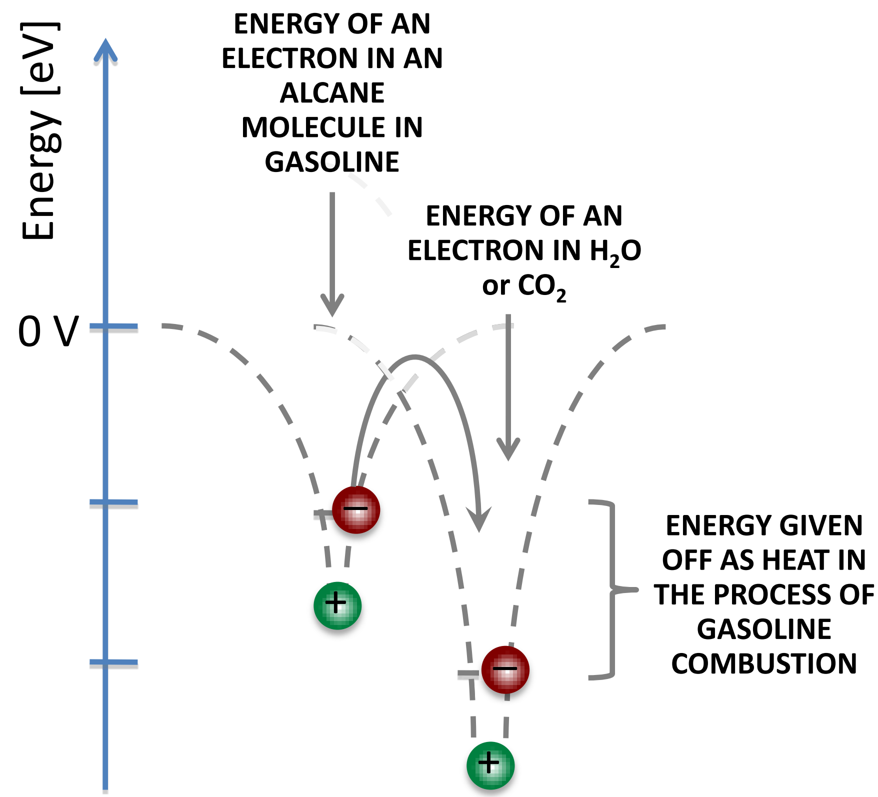


Hydrogen Atom

Hydrogen ground state energy is -13.6 eV

If the hydrogen radius was twice as long, what would be the ground state energy ?





Remember this unit of energy:

1 eV = 1.6×10^{-19} J

<u>*Batteries*</u>

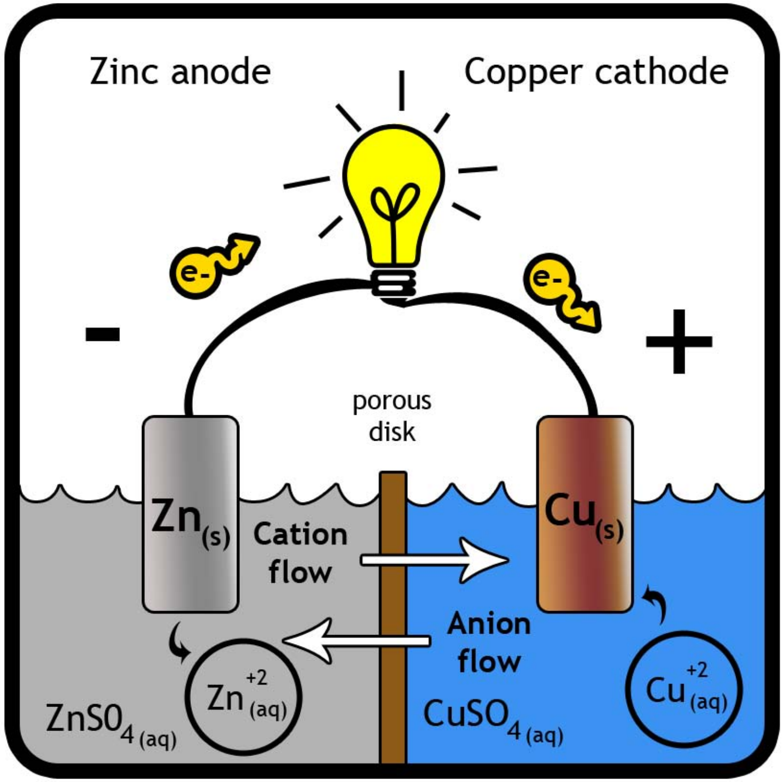
In 1780, Luigi Galvani discovered that when two different metals (copper and zinc for example) were connected together and then both touched to different parts of a nerve of a frog leg at the same time, they made the leg contract. He called this "animal electricity". The Voltaic pile invented by Alessandro Volta in the 1800s is similar to the galvanic cell. These discoveries paved the way for electrical batteries.

From Wikipedia article on the Galvanic cell.

Capacity of Batteries:

AAA – 1250 mAh
AA – 2850 mAh
C – 8350 mAh
D – 20500 mAh

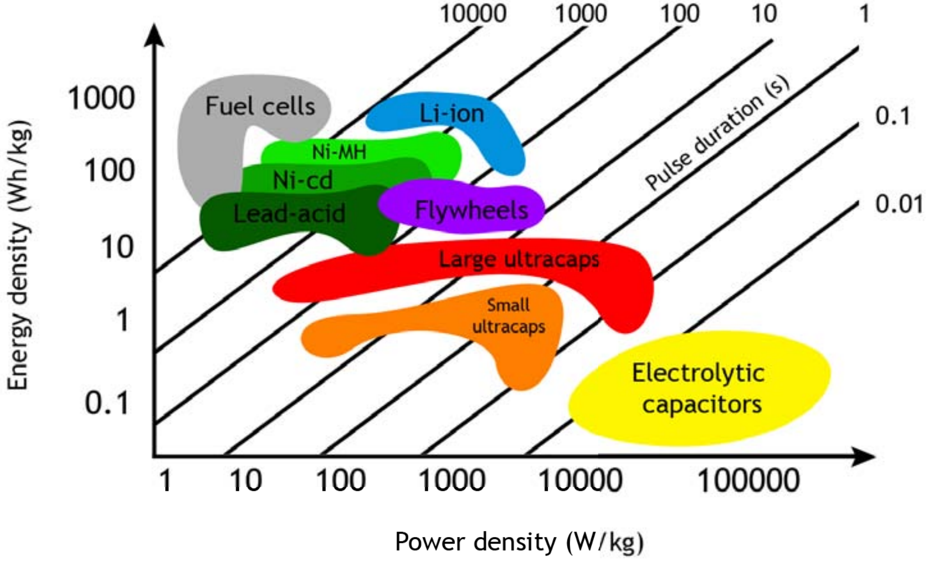
iPhone – 1400 mAh @3.7V



Galvanic Cell

The more electrolyte and electrode material there is in the cell, the greater the capacity of the cell. Thus a small cell has less capacity than a larger cell, given the same chemistry (e.g. alkaline cells), though they develop the same open-circuit voltage.

Energy-to-Weight Ratio



FUELS

Hydrogen	79,000 W-hr/kg
Gasoline	13,000 W-hr/kg

ELECTROCHEMICAL CELLS

Nickel-Metal Hydride	30~80 W-hr/kg
Nickel-Iron	50 W-hr/kg
Lead-Acid	30 W-hr/kg

ATOMIC WEIGHTS:

H_2 – 2 g/mol
O_2 – 32 g/mol
S – 32 g/mol
C – 12 g/mol
Cu – 64 g/mol
Zn – 65 g/mol
Pb – 207 g/mol

DIFFERENCES ARE DUE TO THE RELATIVE MOLECULAR WEIGHTS OF CONSTITUENT MATERIALS AND A NEED FOR MECHANICAL STRUCTURE THAT SUPPORTS THE ELECTROCHEMICAL CELLS

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to solve the "mystery" of why gasoline is so efficient in storing energy

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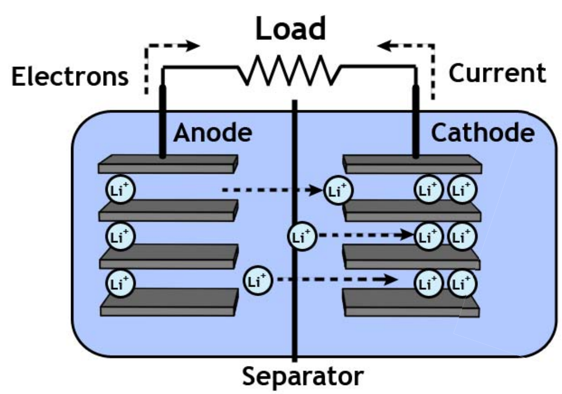

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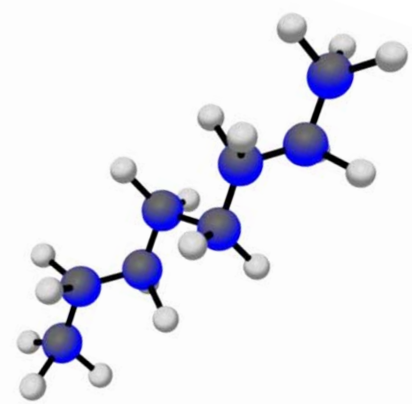

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3. Gauss' Law states that $\int_S \mathbf{E} \cdot \mathbf{dA} = \int_{\mathbf{V}} \rho \mathbf{dV}$
$= Q_{enclosed}$ F

Summary

$$W_{ab} = q \int_a^b \overline{E} \cdot d\bar{l}$$

$$W_s = \frac{1}{2} C v^2$$

Conservation Laws

Kirchhoff's Voltage Law

Kirchhoff's Current Law

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http://www.flickr.com/photos/hisgett/5126927680/
on flickr

Energy in capacitors, batteries, and gasoline:

DIFFERENCES ARE DUE TO THE <u>RELATIVE MOLECULAR WEIGHTS</u> OF CONSTITUENT MATERIALS AND A NEED FOR <u>MECHANICAL STRUCTURE</u> THAT SUPPORTS THE ELECTROCHEMICAL CELLS

Extra Example: Power from Stored Electrostatic Energy

Electrical power expended in moving charge from *a to b*…

$$P = \frac{dW_{ab}}{dt} = \frac{d}{dt}\left[q\int_a^b \overline{E}\cdot d\bar{l}\right]$$

$$= \left[\frac{dq}{dt}\int_a^b \overline{E}\cdot d\bar{l} + q\int_a^b \frac{d\overline{E}}{dt}\cdot d\bar{l}\right]$$

For electrical quasistatic systems, EQS, (slowly varying E-fields)..

$$P = \frac{dq}{dt}\int_a^b \overline{E}\cdot d\bar{l}$$

$$= i\left[\phi(b) - \phi(a)\right] = iv$$

Exactly what you were expecting !