Reflection & Transmission of EM Waves

Reading – Shen and Kong – Ch. 4

Outline

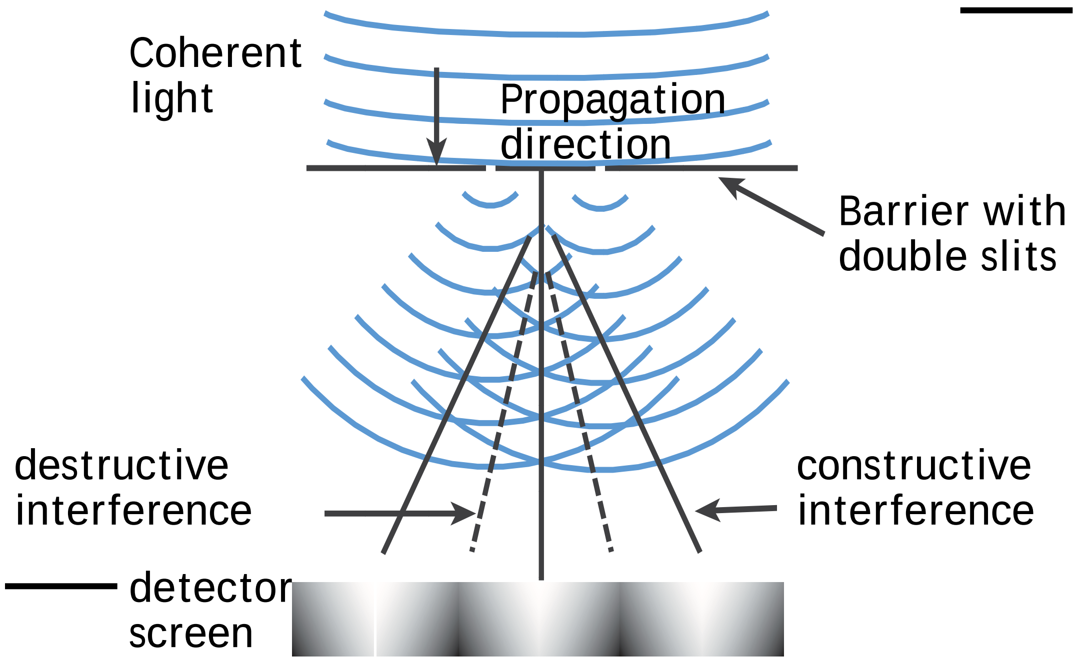
- Everyday Reflection
- Reflection & Transmission (Normal Incidence)
- Reflected & Transmitted Power
- Optical Materials, Perfect Conductors, Metals

TRUE or FALSE

1. Destructive interference occurs when two waves are offset by a phase of ½πm, or half a wavelength. ____

2. The intensity of a plane wave oscillates in time. This means it is always constructively and destructively interfering with itself. ____

3. In a double-slit experiment, as you decrease the space between the slits, the interference peaks decrease proportionally. ____

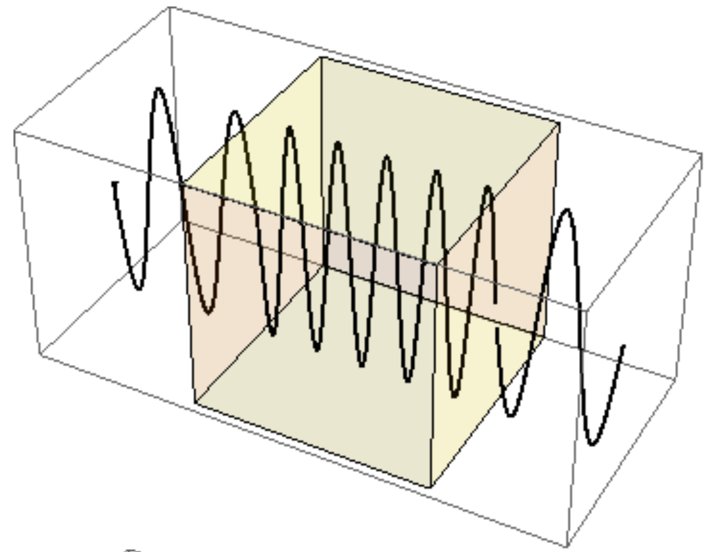


Waves in Materials

$$\tilde{k} = \frac{\omega}{v_p} = \frac{\tilde{n}\omega}{c} \qquad \tilde{k} = (n - j\kappa)\frac{\omega}{c}$$

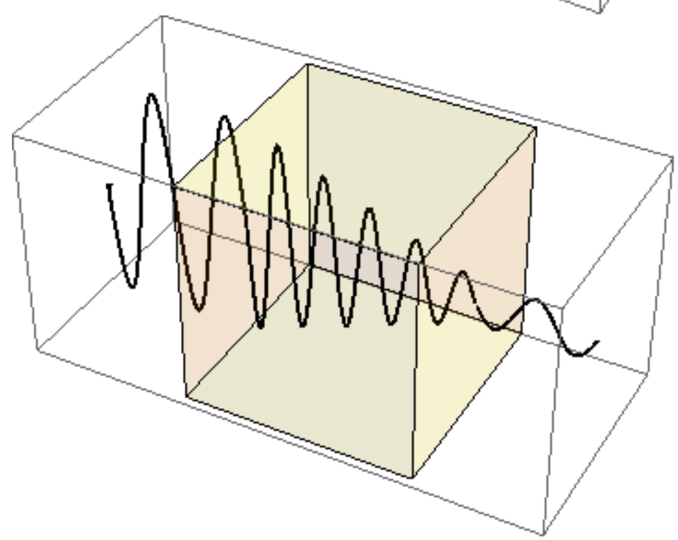
Index of refraction

$$n \equiv \frac{c}{v_p}$$



Absorption coefficient

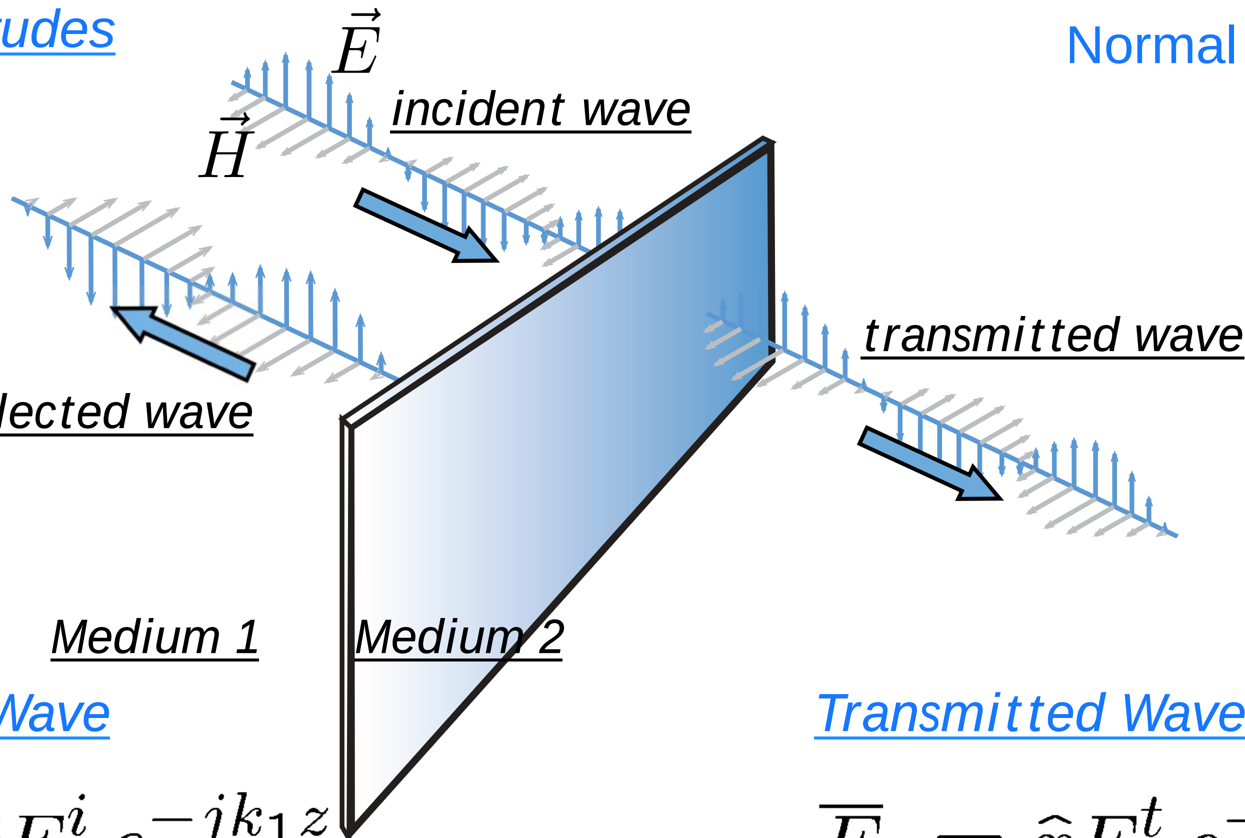
$$\alpha = \frac{2\kappa\omega}{c} = \frac{4\pi\kappa}{\lambda}$$



Incident and Transmitted Waves

same amplitudes

Normal Incidence



Incident Wave

$$\overline{E}_i = \hat{x} E_o^i e^{-jk_1 z}$$

$$k_1 = \omega\sqrt{\epsilon_1 \mu_1}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

Transmitted Wave

$$\overline{E}_t = \hat{x} E_o^t e^{-jk_2 z}$$

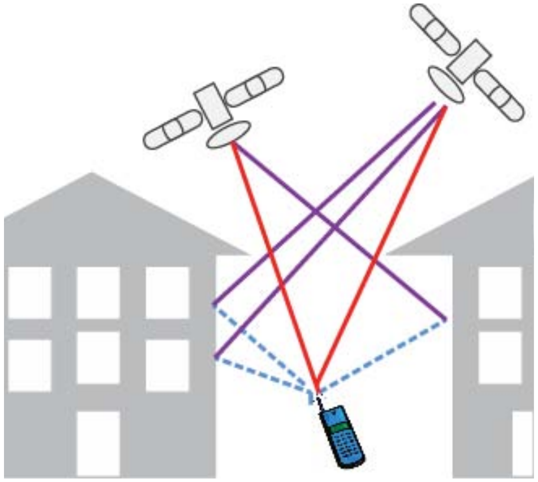
$$k_2 = \omega\sqrt{\epsilon_2 \mu_2}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

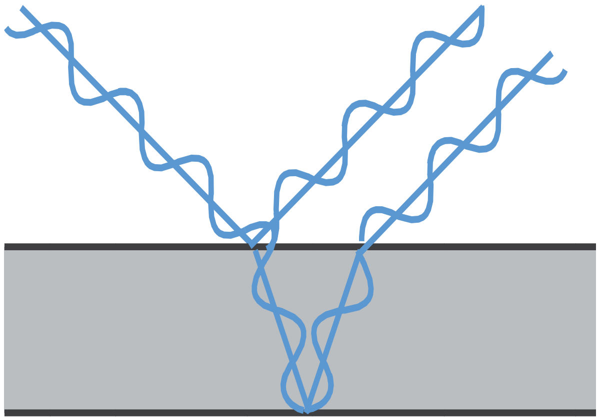
EM Wave Reflection

Image in the Public Domain

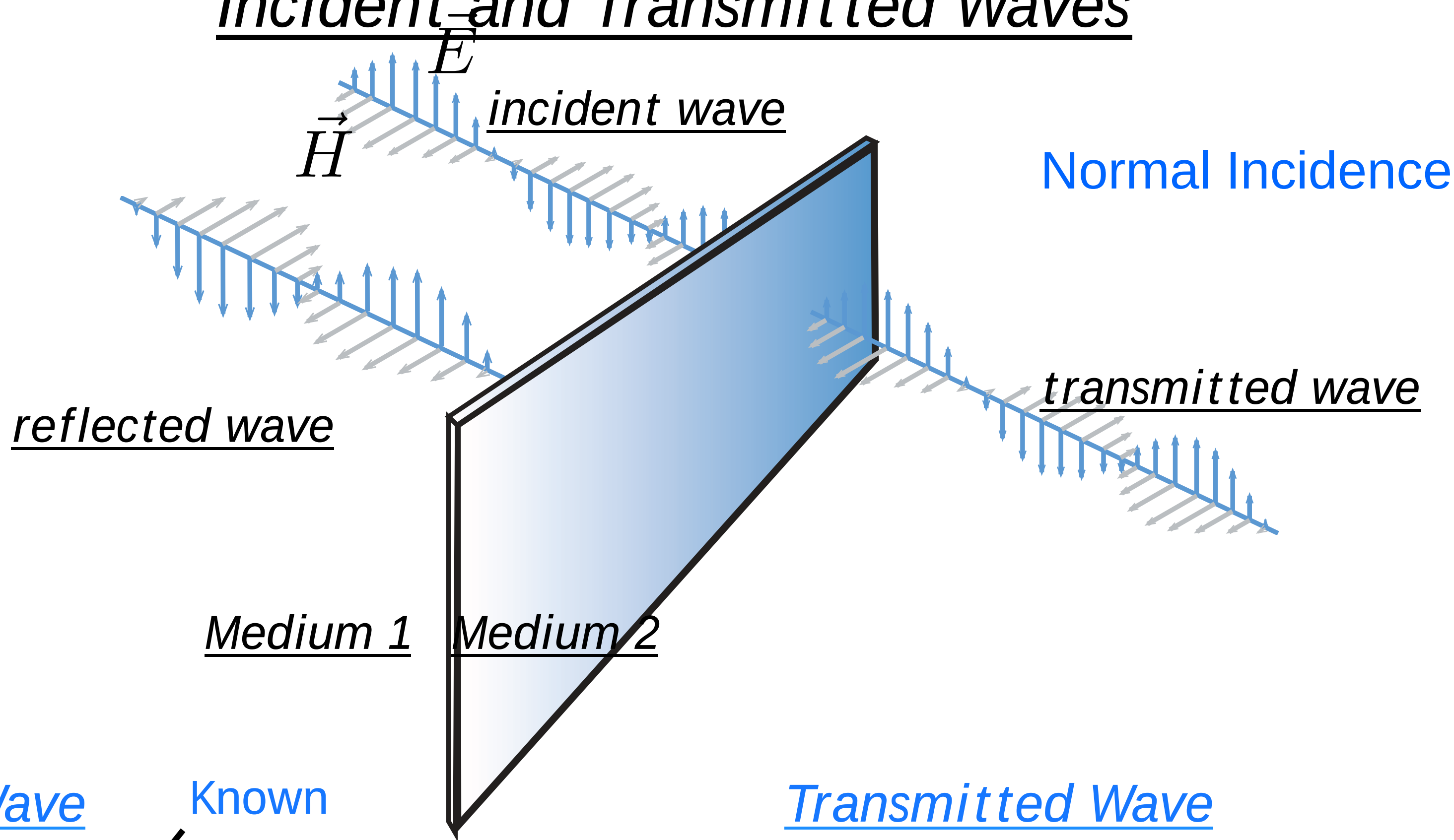
Image by Ali Smiles :) http://www.flickr.com/photos/77682540@N00/2789338547/ on flickr



Cell Phone Reflection



Incident and Transmitted Waves



Incident Wave

Known

$$\overline{E}_i = \hat{x} E_o^i e^{-jk_1 z}$$

Reflected Wave

$$\overline{E}_r = \hat{x} E_o^r e^{+jk_1 z}$$

Transmitted Wave

$$\overline{E}_t = \hat{x} E_o^t e^{-jk_2 z}$$

Define reflection coefficient as $r = \dfrac{E_o^r}{E_o^i}$

Define transmission coefficient as $t = \dfrac{E_o^t}{E_o^i}$

Key Takeaways

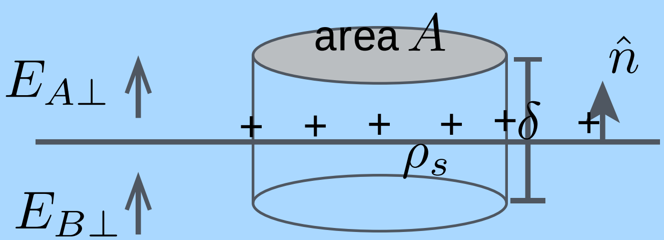
- Define the *reflection coefficient* as

$$r = \frac{E_o^r}{E_o^i} = \frac{n_1 - n_2}{n_1 + n_2}$$

- Define the *transmission coefficient* as

$$t = \frac{E_o^t}{E_o^i} = \frac{2\,n_1}{n_1 + n_2}$$

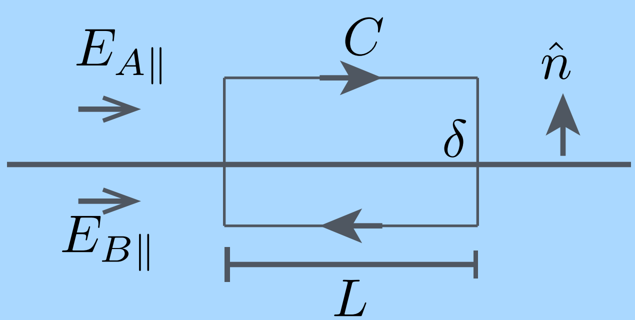
E-Field Boundary Conditions



$$\lim_{\delta\to 0} \ Gauss \Rightarrow (\epsilon_0 E_{A\perp} - \epsilon_0 E_{B\perp})A = \rho_s A$$

$$\hat{n}\cdot(\epsilon_0 E_A - \epsilon_0 E_B) = \rho_s$$

Normal $\vec{E}$ is discontinuous at a surface charge.



$$\lim_{\delta\to 0} \ Faraday \Rightarrow (E_{A\|} - E_{B\|})L = 0$$

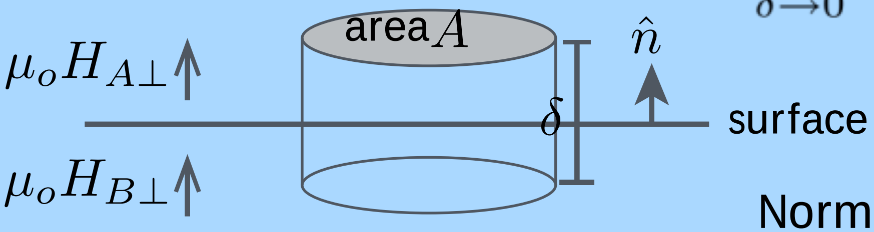
$$\hat{n}\times(E_A - E_B) = 0$$

Tangential $\vec{E}$ is continuous at a surface.

Known

$$E_o^i + E_o^r = E_o^t$$

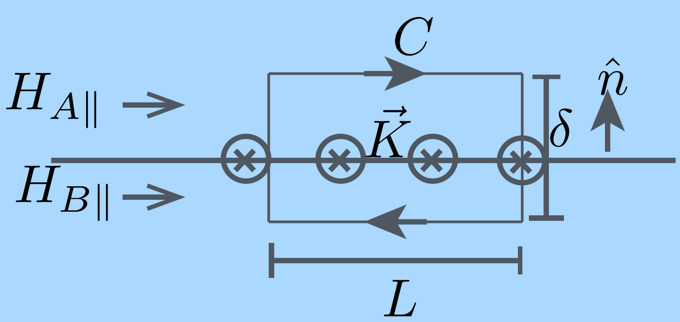
H-Field Boundary Conditions



$$\lim_{\delta \to 0} \; Gauss \Rightarrow (\mu_0 H_{A\perp} - \mu_0 H_{B\perp})A = 0$$

$$\hat{n} \cdot (\mu_0 H_A - \mu_0 H_B) = 0$$

Normal $\mu_o \vec{H}$ is continuous at a surface.

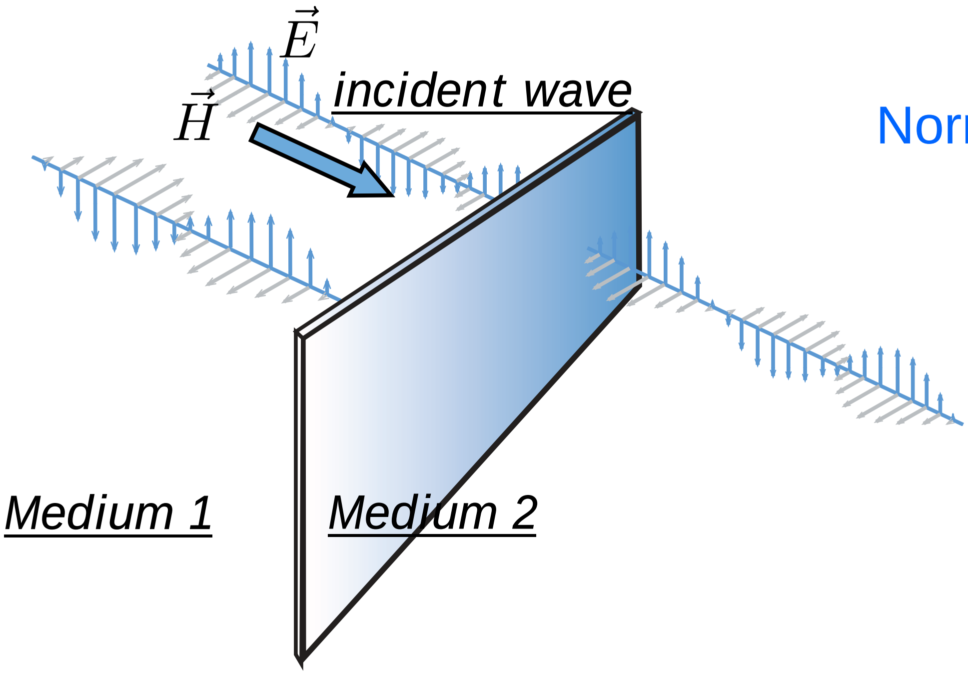


$$\lim_{\delta \to 0} \; Ampere \Rightarrow (H_{A\|} - H_{B\|})L = K\,L$$

$$\hat{n} \times (H_A - H_B) = \vec{K}$$

Tangential $\vec{H}$ is discontinuous at a surface current $\vec{K}$.

Incident EM Waves at Boundaries



Normal Incidence

Incident Wave

Known

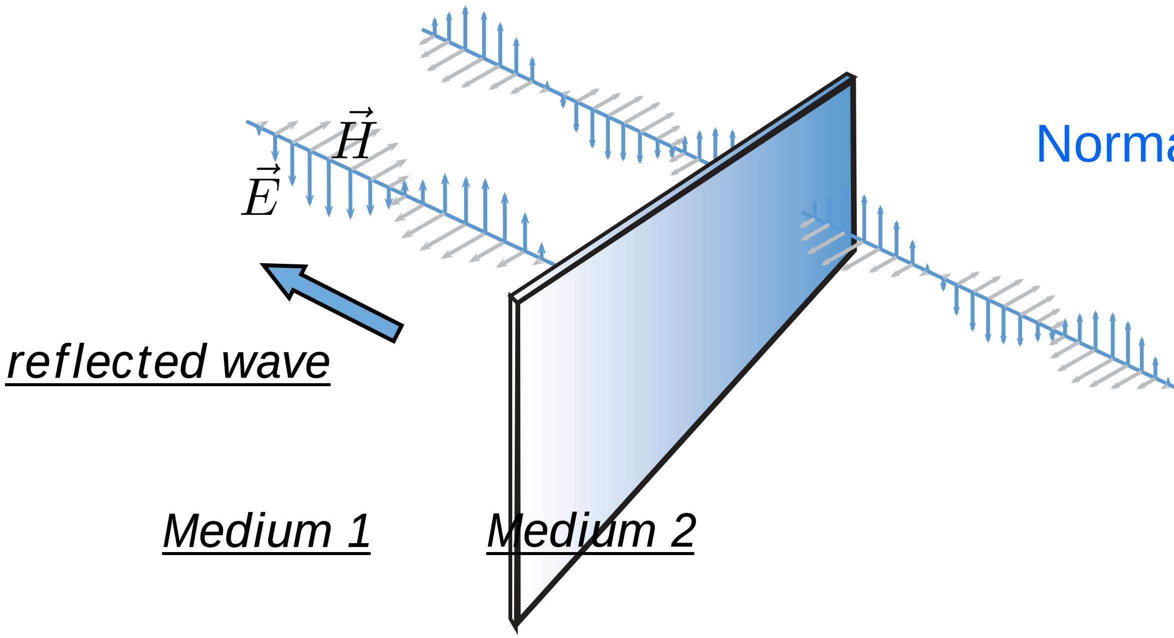
$$\vec{E}_i = \hat{x} E_o^i e^{-jk_1 z}$$

$$\vec{H}_i = \frac{1}{\eta_1} \hat{z} \times \vec{E}_i = \hat{y} \frac{1}{\eta_1} E_o^i e^{-jk_1 z}$$

$$k_1 = \omega \sqrt{\epsilon_1 \mu_1}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

Reflected EM Waves at Boundaries



Normal Incidence

Reflected Wave

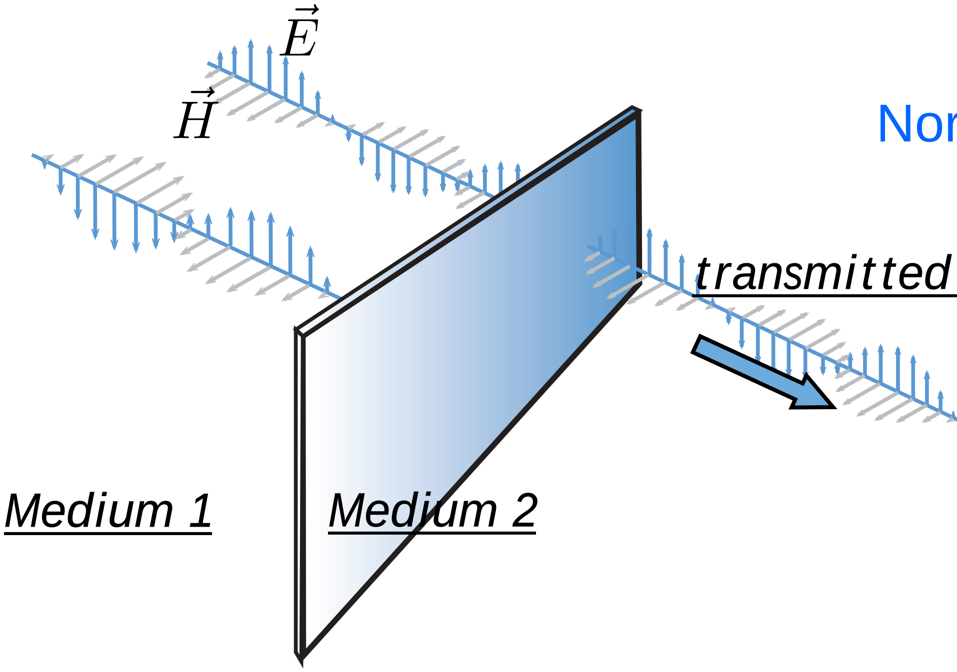
Unknown

$$\vec{E}_r = \hat{x} E_o^r e^{+jk_1 z}$$

$$\vec{H}_r = \frac{1}{\eta_1}(-\hat{z}) \times \vec{E}_r = -\hat{y}\frac{E_o^r}{\eta_1} e^{+jk_1 z}$$

DEFINE REFLECTION COEFFICIENT AS $r = \frac{E_o^r}{E_o^i}$

Transmitted EM Waves at Boundaries



Normal Incidence

Transmitted Wave

Unknown

$$\vec{E}_r = \hat{x} E_o^t e^{-jk_2 z}$$

$$\vec{H}_t = \frac{1}{\eta_2} \hat{z} \times \vec{E}_t = \hat{y} \frac{E_o^t}{\eta_2} e^{-jk_2 z}$$

DEFINE TRANSMISSION COEFFICIENT AS $t = \frac{E_o^t}{E_o^i}$

$$k_2 = \omega \sqrt{\epsilon_2 \mu_2}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

Reflection & Transmission of EM Waves at Boundaries

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r$$
$$= \hat{x}\left(E_o^i e^{-jk_1 z} + E_o^r e^{+jk_1 z}\right)$$

$$\vec{E}_2 = \vec{E}_t$$
$$= \hat{x} E_o^t e^{-jk_2 z}$$

Medium1 Medium 2

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r$$
$$= \hat{y}\left(\frac{E_o^i}{\eta_1} e^{-jk_1 z} - \frac{E_o^r}{\eta_1} e^{+jk_1 z}\right)$$

$$\vec{H}_2 = \vec{H}_r$$
$$= \hat{y}\frac{E_o^t}{\eta_2} e^{-jk_2 z}$$

$$\overline{E}_{1(z=0)} = \overline{E}_{2(z=0)}$$
$$\overline{H}_{1(z=0)} = \overline{H}_{2(z=0)}$$

Reflection of EM Waves at Boundaries

$$\vec{E}_{1(z=0)} = \vec{E}_{2(z=0)}$$

$$\Rightarrow \quad E_o^i + E_o^r = E_o^t$$

$$\vec{H}_{1(z=0)} = \vec{H}_{2(z=0)}$$

$$\Rightarrow \quad \frac{E_o^i}{\eta_1} - \frac{E_o^r}{\eta_1} = \frac{E_o^t}{\eta_2} \qquad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$r = \frac{E_o^r}{E_o^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$t = \frac{E_o^t}{E_o^i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Reflectivity & Transmissivity of Waves

- Define the *reflection coefficient* as

$$r = \frac{E_o^r}{E_o^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

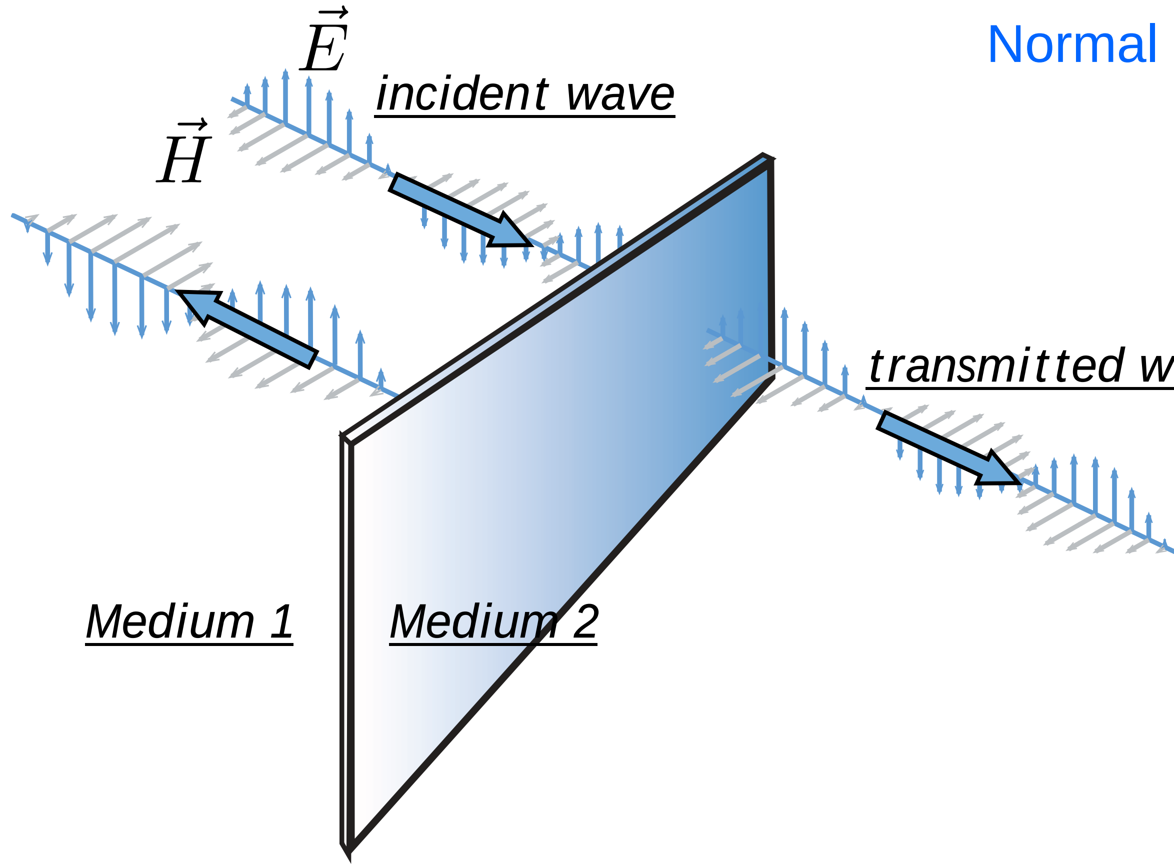
- Define the *transmission coefficient* as

$$t = \frac{E_o^t}{E_o^i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

What are the ranges for *r* and *t*? Is energy conserved?

Reflection & Transmission of EM Waves at Boundaries

Normal Incidence



Additional Java simulation at
http://phet.colorado.edu/new/simulations/

Reflectivity & Transmissivity of EM Waves

- Note that $1 + r = t$
- The definitions of the reflection and transmission coefficients do generalize to the case of lossy media.
- For loss-less media, r and t are real:

$$-1 \leq r \leq +1 \qquad 0 \leq t \leq 2$$

- For lossy media, r and t are complex:

$$|r| \leq 1 \qquad |t| \leq 2$$

- Incident Energy = Reflected Energy + Transmitted Energy

R = $|r|^2$ … fraction of incident power that is reflected

T = 1 – R … fraction of incident power that is transmitted

Reflectivity of Dielectrics

Consider nearly-lossless optical materials. For typical dielectrics, $\mu_1 \approx \mu_2 \approx \mu_0$.

$$r = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}} - \sqrt{\frac{\mu_1}{\varepsilon_1}}}{\sqrt{\frac{\mu_2}{\varepsilon_2}} + \sqrt{\frac{\mu_1}{\varepsilon_1}}} \approx \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = \frac{n_1 - n_2}{n_1 + n_2}$$

↑ Result ↑ $\mu_1 \approx \mu_2$ ↑ $\mu_1 \approx \mu_2$

Image by Will Montague http://www.flickr.com/photos/willmontague/3787127610/ on flickr

Reflection of EM Waves at Boundaries

$$r = \frac{E_o^r}{E_o^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{n_1 - n_2}{n_1 + n_2}$$

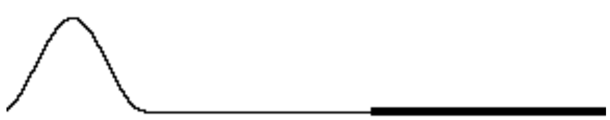
In terms of the characteristic impedances

In terms of the index of refraction, assuming $\mu_1 = \mu_2$

REMEMBER:

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\frac{\tilde{\epsilon}}{\epsilon_o} = \tilde{n}^2$$

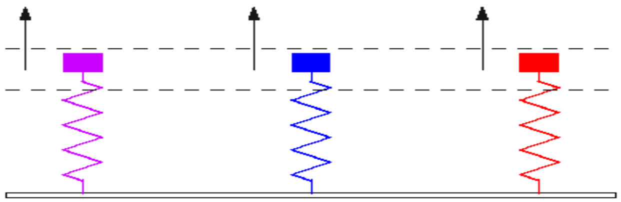


What is different in the two reflected waves ?

Which side is air and which side is glass ?

Why does metal reflect light?

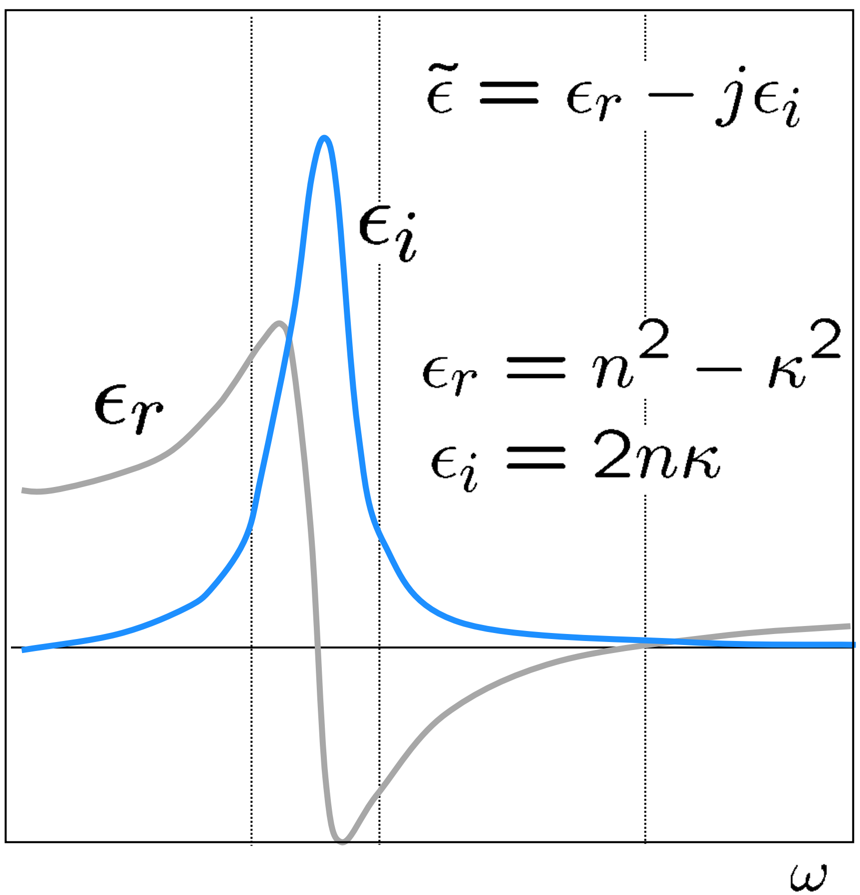
Microscopic Lorentz Oscillator Model

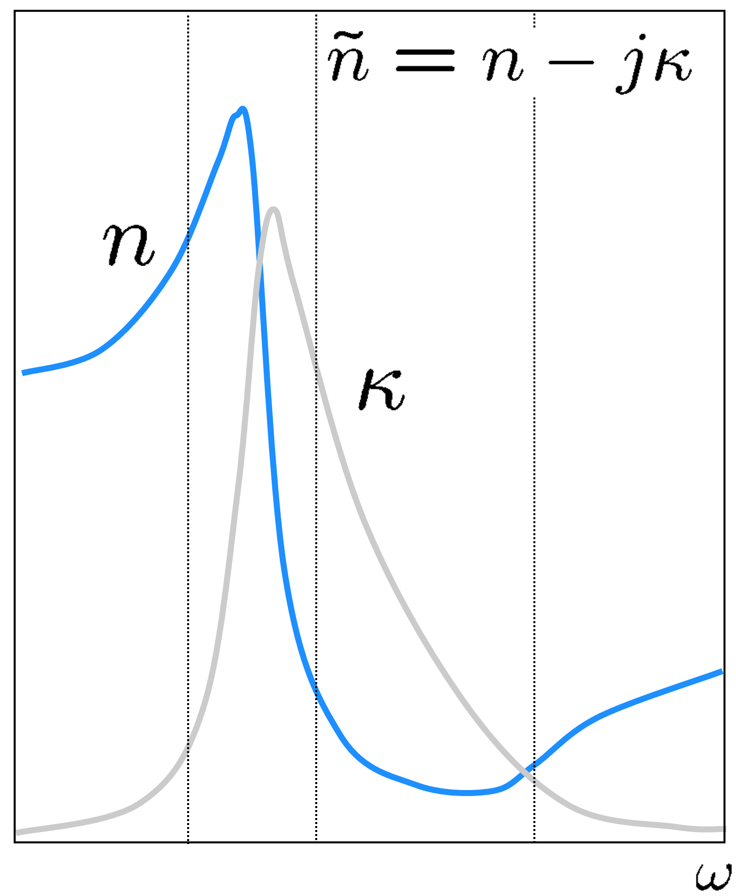


$$\epsilon = \epsilon_o \left(1 + \frac{\omega_p^2}{\omega_o^2 - \omega^2 + j\omega\gamma} \right)$$

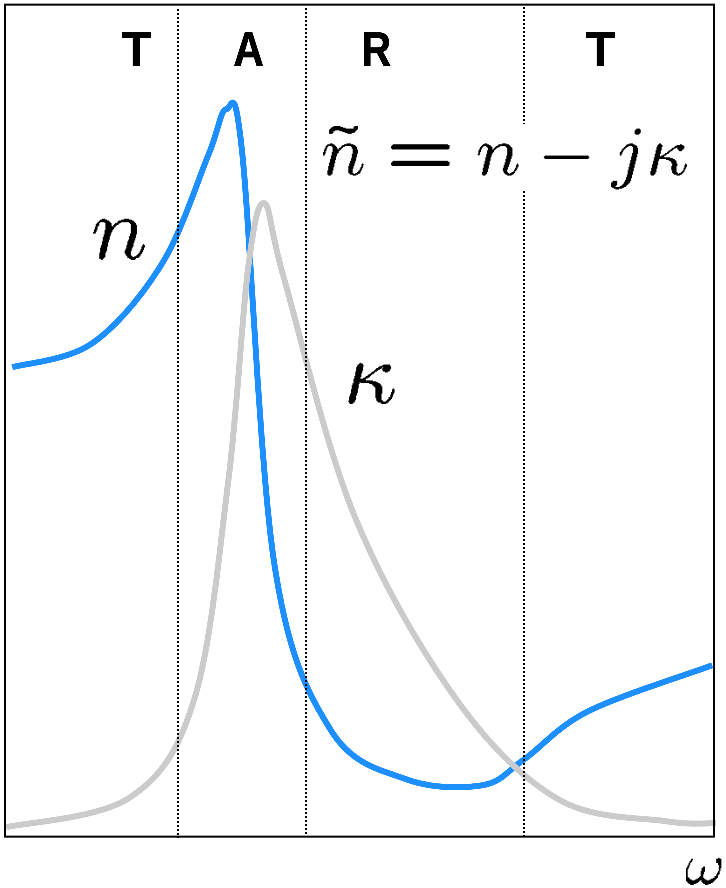
$$\omega_p^2 = \frac{Nq^2}{\epsilon_o m}$$

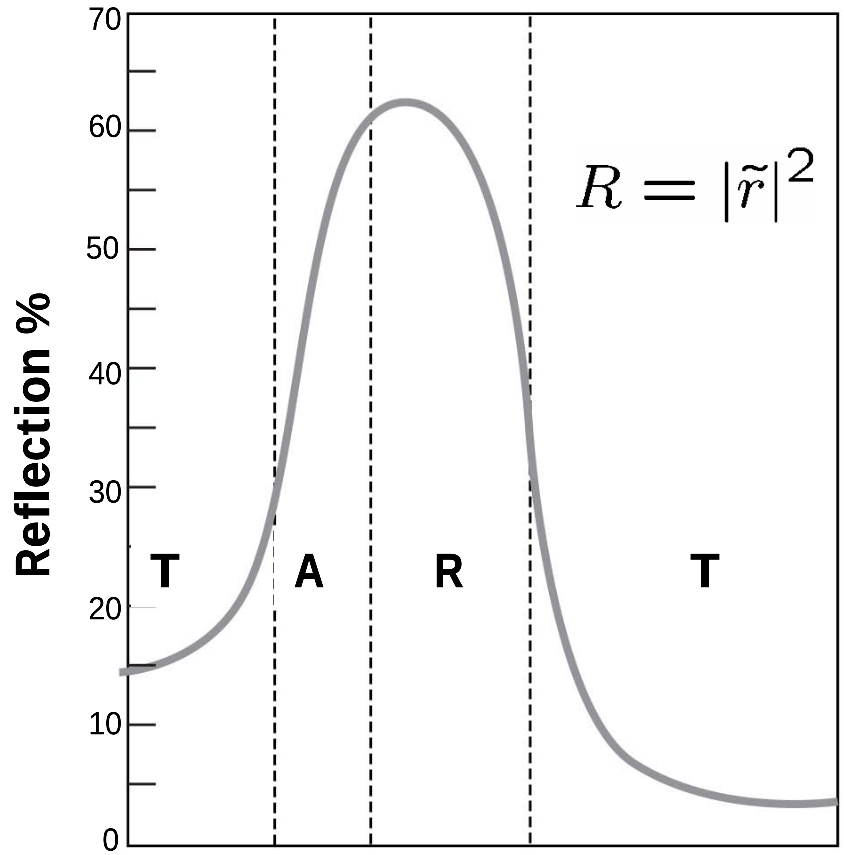
$$\omega_o^2 = \frac{k}{m}$$





T-A-R-T





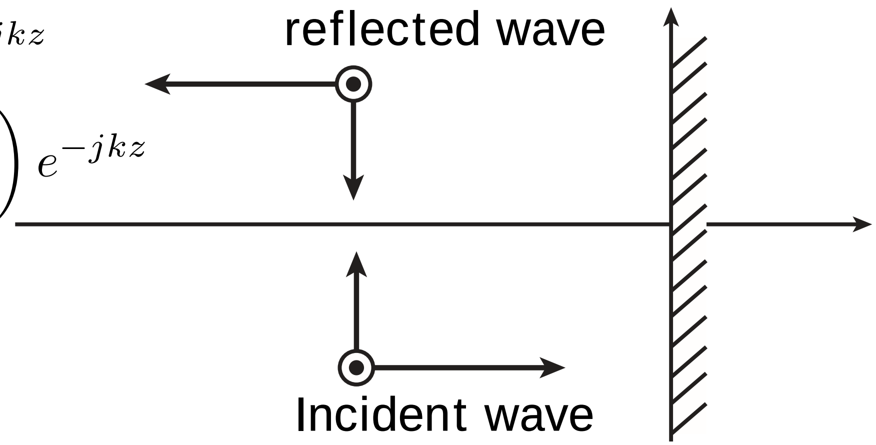
T-A-R-T **-** the material has four distinct regions of optical properties:

- **Transmissive,** $\omega < \omega_0 - \gamma/2$,
- **Absorptive,** $\omega_0 - \gamma/2 < \omega < \omega_0 + \gamma/2$
- **Reflective,** $\omega_0 + \gamma/2 < \omega < \omega_p$
- **Transmissive,** $\omega > \omega_p$

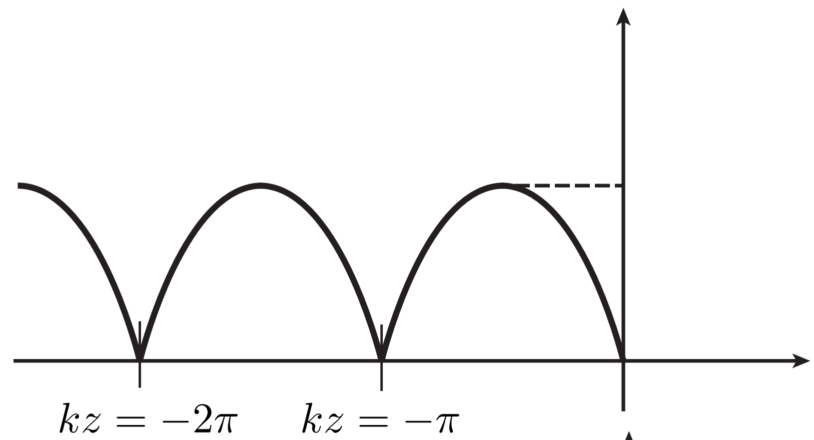
Reflection of a Normally Incident EM Wave from a Perfect Conductor

$$\vec{E}^i = \hat{x} E_o e^{-jkz}$$

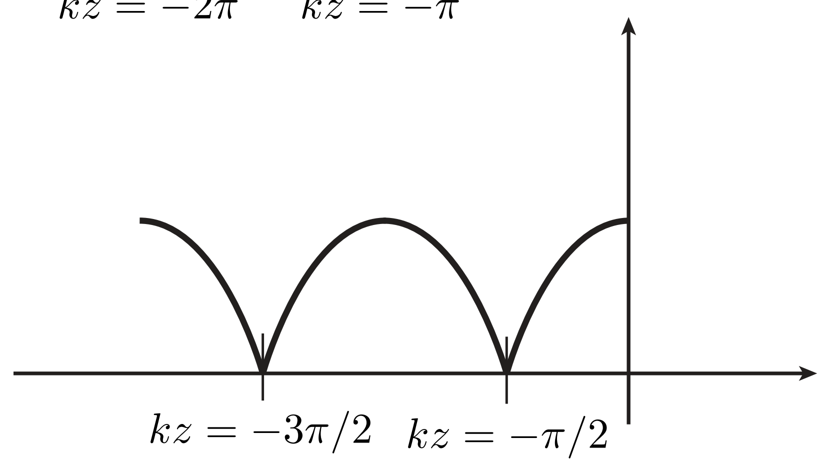
$$\vec{H}^i = \hat{y} \left(\frac{E_o}{\eta_o} \right) e^{-jkz}$$



Standing wave pattern of the E-field

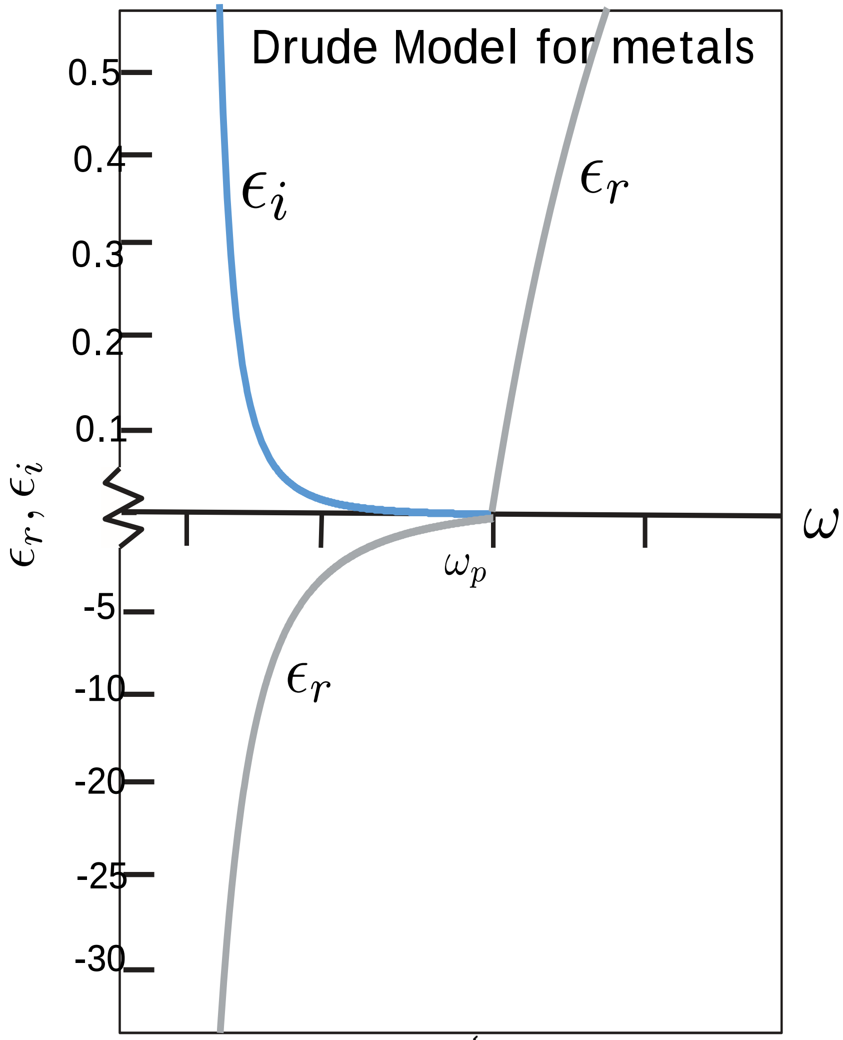


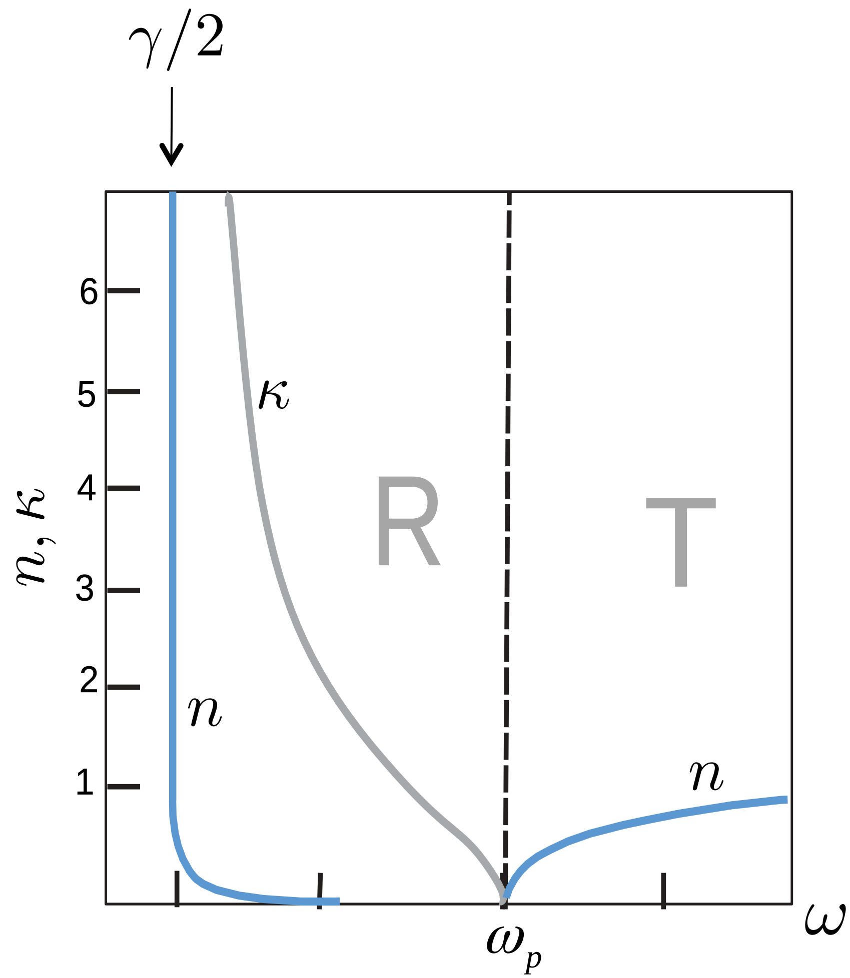
Standing wave pattern of the H-field



Microscopic Lorentz Oscillator Model

... for FREE ELECTRONS IN METALS $\omega_o \rightarrow 0$



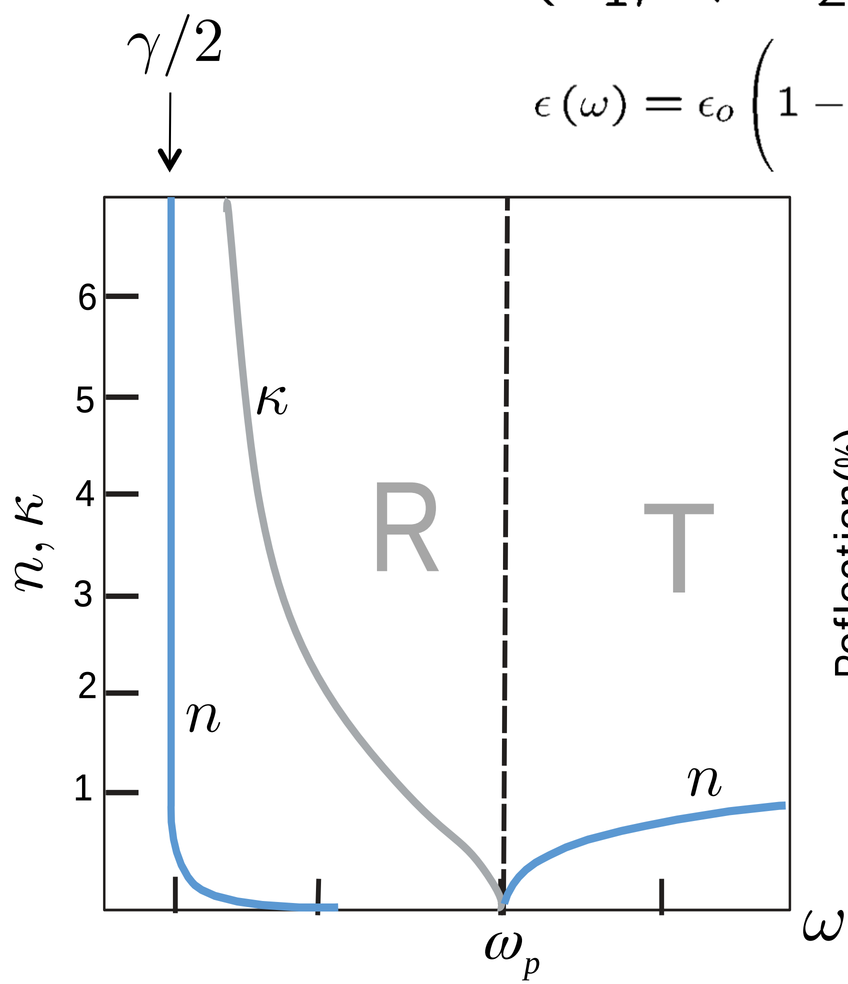


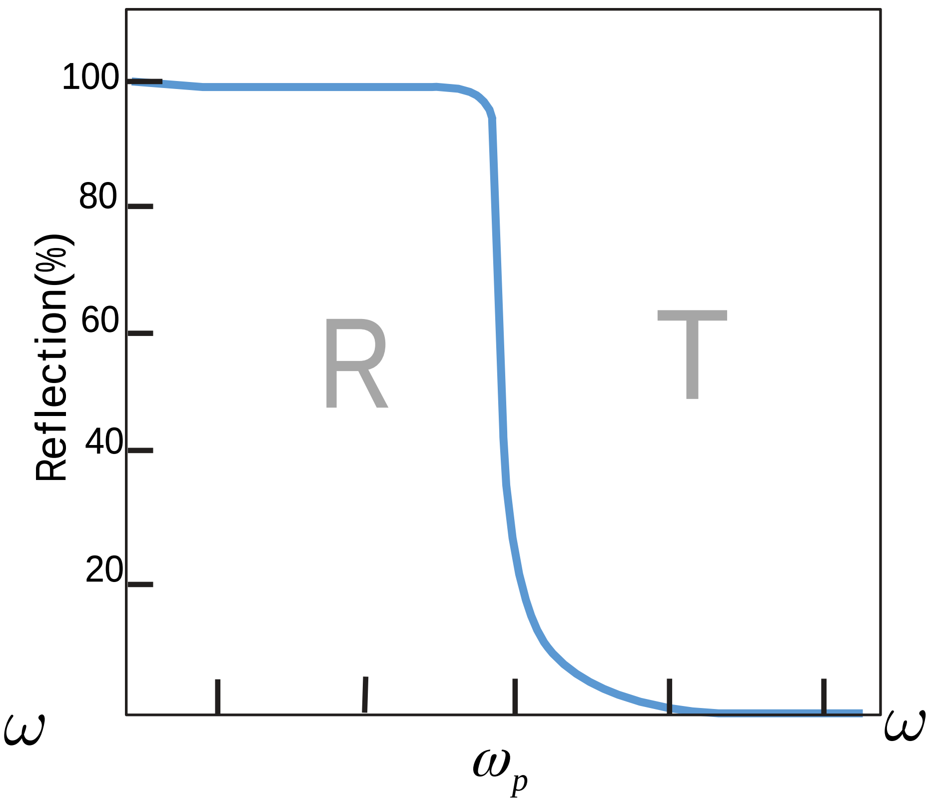
$$\epsilon(\omega) = \epsilon_o \left(1 + \frac{\omega_p^2}{\omega_o^2 - \omega^2 + j\omega\gamma}\right) \Rightarrow \epsilon(\omega) = \epsilon_o \left(1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma}\right)$$

Reflectivity of Silver

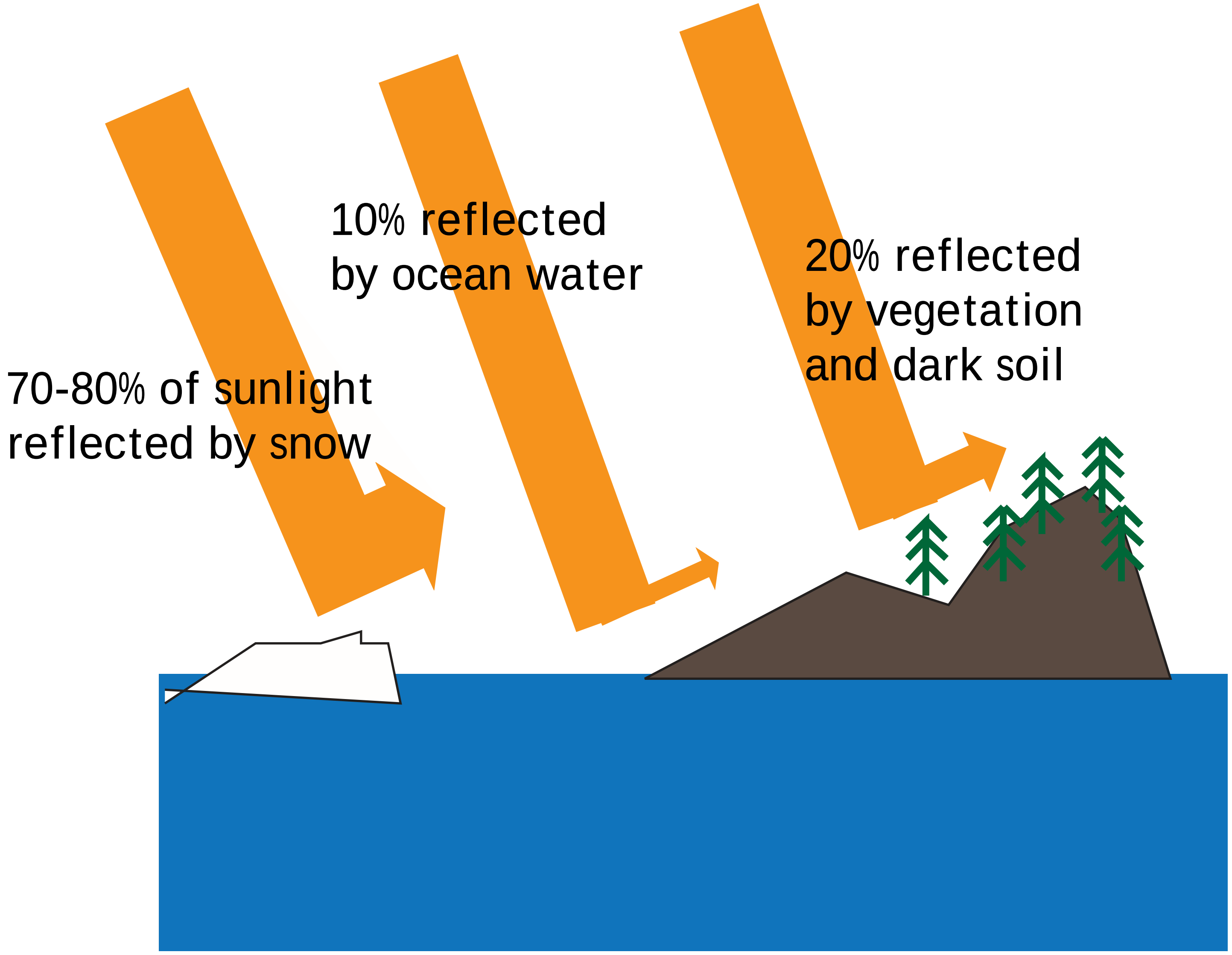
$$\tilde{r} = \frac{(n_{1r} - n_{2r}) + j(n_{1i} - n_{2i})}{(n_{1r} + n_{2r}) + j(n_{1i} + n_{2i})}$$

$$\epsilon(\omega) = \epsilon_o \left(1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma}\right)$$





Ice is more reflective than water



Thin Film Interference

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